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REINFORCED CONCRETE SECTION DESIGN TO BENDING ACCORDING TO EN 1992-1-1/2004–EUROCODE2

BY

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Abstract. In the next decades the design of the civil structures will be ruled by Eurocodes. For reinforced concrete structures the Eurocode2 will became of paramount importance to the design of the structural members. An important aspect of designing represents the relation between stress and strain to the design of the section for structural members. In this respect the EC2 defines 14 classes of concrete with different stress–strain relationships. The first stress–strain relation mentioned of EC2 for design is the parabola–rectangle stress distribution on concrete section, and further the simplified rectangle stress–strain shape is mentioned. Therefore, an aspect of the design, poorly emphasized in the national rule, is the design considering parabola–rectangle stress distribution on concrete section.

The exposed issues in this paper concern with the design of the reinforced concrete section subjected to bending using two stress–strain relationships mentioned by EC2, and the differences are underlined. The design to bending using parabola–rectangle stress distribution for rectangular section is largely presented, and also the reliability for a fast designing is emphasized. Design relations for a parabola–rectangle stress distribution on section are mentioned. Also, aspects about the boundary between the single reinforcing domain and the double reinforcing domain are emphasized.

Key words: Reinforced concrete; design section; bending; Eurocode2.

1. Introduction

In the last two decades the design of reinforced concrete/prestressed structures was based on the national standard STAS 10107/0-90 which became national law in 1991 [1]. Also, in early 1990’s an important book for designers was published, in which are largely presented the design of reinforced concrete members subjected to the mostly efforts which appear within reinforced concrete structures [2]. The design relations used in this guide are based on the design criteria presented in the above mentioned national rule which is still underway.

Beginning with 2004 the European Committee for Standardization approved a new EC2's version for design of the reinforced concrete structures, which is valid throughout Europe. This version contains many improvements regarding the last version [3]. Considering the imminent replacement of the national standard to Eurocodes in 2010, and because there are some differences between these two design standards, this paper underlines some significant aspects.

In comparison with the national standard within the last version of EC2 the strength and deformation characteristics are presented for 14 classes of concrete. The weakest strength class of concrete is C12/15 and the strongest strength class of concrete became C90/105. For all these classes of concrete EC2 defines stress–strain relationships used for: a) *structural analysis* – a relationship defined by the compressive strains in concrete, ε_{c1}, ε_{cu2} and the mean value of concrete cylinder compressive strength, f_{cm}; b) *design of cross-section* – a relationship defined by the strains ε_{c2}, ε_{cu2} for parabola−rectangle stress distribution on concrete section or by ε_{c3}, ε_{cu3} for rectangular stress distribution, and the design value of concrete compressive strength, f_{cd}; c) *confinement of concrete* – a relationship which consists in modification toward higher values of the effective stress–strain mentioned above for designing.

2. Design of Concrete Section to Bending Considering Parabola–Rectangle Stress Distribution for Concrete in Compression

It is well known that the plasticization of the concrete section to bending starts from the most compressed fiber toward the inside of the section. To model accurately this phenomenon one of the assumptions is that which considers the parabola–rectangle stress block for concrete section. This assumption is considered as base of a rigorous analysis of the reinforced concrete section. Thus, the European rule EC2 provide to §3.1.7.1 for the design of cross-section to ultimate limit state (ULS) the following stress–strain relationship

$$\sigma_c = \begin{cases} f_{cd}\left[1-\left(1-\dfrac{\varepsilon_c}{\varepsilon_{c2}}\right)^n\right], \text{ for } 0 \le \varepsilon \le \varepsilon_{c2}, \\ f_{cd}, \text{ for } \varepsilon_{c2} \le \varepsilon \le \varepsilon_{cu2}. \end{cases} \tag{1}$$

For the strength classes of concrete higher than 50 MPa the EC2 provide different power degrees, n , and values of the strain, ε_{c2} , ε_{cu2} , as is shown in Table 1.

Table 1
Parabola–Rectangle Stress–Strain Curve Parameters Specified in EC2 [3]

Strength classes	≤50/60	55/67	60/75	70/85	80/95	90/105
f_{cd} , [Mpa]	8…33	36.7	40	46.7	52.3	60
ε_{c2} , [‰]	2.0	2.2	2.3	2.4	2.5	2.6
ε_{cu2} , [‰]	3.5	3.1	2.9	2.7	2.6	2.6
n	2	1.75	1.6	1.45	1.4	1.4

Based on relation (1) the parabola–rectangle relationships for all fourteen concrete classes are illustrated in Fig. 1. The relationships are drawn using the design strength, f_{cd}, and the strain, ε_{c2} and ε_{cu2} , for concrete mentioned in EC2.

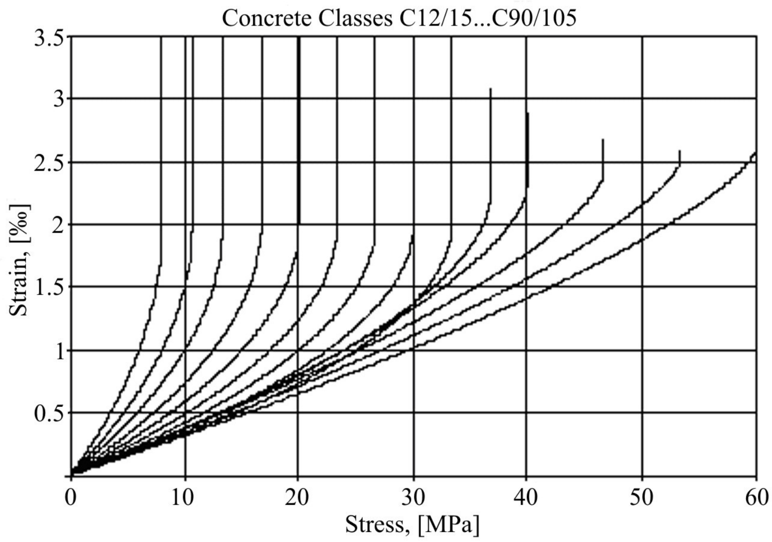

Fig. 1 – Parabola–rectangle stress–strain curves for the all strength classes of concrete according to EC2.

Considering the critical deformation of the reinforced concrete section and the stress distribution above mentioned, it can be admitted the following deformation domains which describe failure stages to bending without axial force $N \approx 0$. Thus, in Fig. 2 are shown the possible failure stages on section which can be encountered when the depth neutral axis, x , increases from zero to a critical limit, $x_{\lim}$.

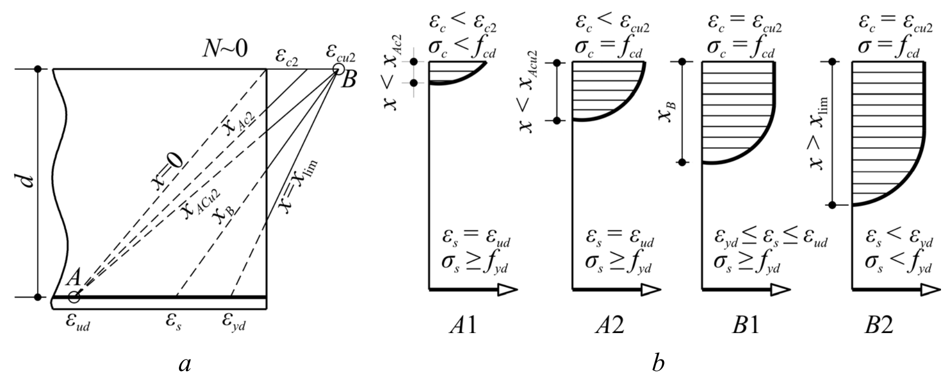

Fig.2 – *a* – Pivot diagram for possible strain distributions in the ULS ($N \approx 0$);
b – failure stages to bending for the domains of strain of the pivot diagram.

Generally, for a reinforced concrete section the failure is accomplished, when the strain ε_{cu2} is reached in concrete or the strain ε_{ud} is reached in the reinforcement in tension. The diagram from Fig. 2 *a* is a particular case, axial force is neglected ($N \approx 0$), of the pivot diagram mentioned in [4] for a section in bending with axial force. Thus, four possible deformation stages to failure may be encountered.

($A1$) The pivot point for the strain diagram is A. The section is bent, in steel the strain $\varepsilon_s = \varepsilon_{ud}$, and the stress takes values of $\sigma_s \geq f_{yd}$. In concrete the strain takes values of $\varepsilon_c < \varepsilon_{c2}$, and the stress, of $\sigma_c < f_{cd}$, respectively. As a consequence the neutral axis depth verifies the inequality $0 < x < x_{Ac2}$, in which x_{Ac2} is the neutral axis depth when the strain section turns on point A and the strain in the most compressed fiber for concrete is equal to ε_{c2}.

($A2$) The pivot point is A. The strain and stress in steel takes values of $\varepsilon_s = \varepsilon_{ud}$, $\sigma_s \geq f_{yd}$, respectively. In concrete the strain takes values of $\varepsilon_{c2} < \varepsilon_c < \varepsilon_{cu2}$, the stress $\sigma_c = f_{cd}$. As a consequence the neutral axis depth satisfies the inequality $0 < x < x_{Acu2}$, where x_{Acu2} is the neutral axis depth when the strain section turns on point A and the strain in the most compressed fiber is equal to ε_{cu2}. The stages of failure emphasized to ($A1$) and ($A2$) are a feature of the bent members with small percentages of reinforcement, and the failure of the section occurs by an excess of plastic strain in the reinforcement.

($B1$) The pivot point is B. In steel the strain take values of $\varepsilon_{yd} < \varepsilon_s < \varepsilon_{ud}$, and the stress $\sigma_s \geq f_{yd}$, respectively. In concrete the strain $\varepsilon_c = \varepsilon_{cu2}$ and the stress $\sigma_c = f_{cd}$ are recorded. The neutral axis depth is limited to $x_B \leq x_{\lim}$, which depends on the concrete class and the steel grade. This deformation stage of failure to bending is encountered at the mostly reinforced concrete members. The failure of the section occurs by an excess of plastic strain in concrete.

($B2$) The pivot point is B. The strain in steel takes values of $\varepsilon_s < \varepsilon_{yd}$, and the stress, of $\sigma_s < f_{yd}$, respectively. In concrete the strain $\varepsilon_c = \varepsilon_{cu2}$ and the stress $\sigma_c = f_{cd}$ are recorded. The neutral axis depth is $x_B > x_{\lim}$. This deformation stage of failure may be encountered at the bent reinforced concrete members with large percentages of reinforcement on section. The failure of the section occurs by an excess of plastic strain in concrete.

Customary the EC2 made for design to ULS the same assumptions which were made in the reinforced concrete theory from the beginning of the age. The plane sections remain plane, *i.e.* Navier-Bernoulli assumption is satisfied. No slipping between steel and concrete, that meaning $\varepsilon_{\text{concr.}} = \varepsilon_{\text{steel}}$ in the surrounding concrete. Also, the tensile strength of concrete is ignored. In addition the stress–strain relationships for concrete and steel are well established, and are included in the design assumption.

Accordingly to [3] the stress distribution for concrete in compression may be a parabola–rectangle defined by (1), as is shown in the Fig. 3.

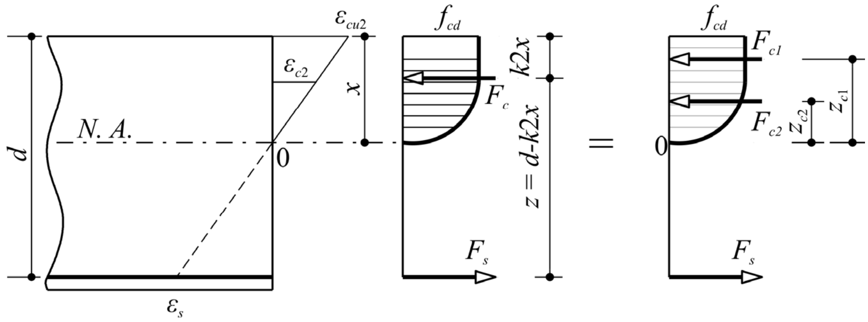

Fig. 3 – Parabola–rectangle stress block for rectangular singly reinforced section.

Thus, it can be calculated the resultant compressive force for concrete, F_c, which acts through the centroid of the effective area of concrete in compression (Fig. 3).

$$F_c = bk_1 x f_{cd}. \tag{2}$$

Considering the stress diagram represented in Fig. 3, the compressive resultant, F_c, will be evaluated as a sum between compressive resultant for the rectangle block stress, F_{c1}, and the parabolic block stress, F_{c2}, as follows:

$$F_c = F_{c1} + F_{c2}. \tag{3}$$

Therefore, for a rectangular section of width b, subjected to bending and having the neutral axis depth, x, the two components of the compressive resultant, F_c, are

$$F_{c1} = b\left(\frac{\varepsilon_{cu2} - \varepsilon_{c2}}{\varepsilon_{cu2}}\right) x f_{cd}, \quad F_{c2} = b\left(\frac{2\varepsilon_{c2}}{3\varepsilon_{cu2}}\right) x f_{cd}. \tag{4}$$

The resultant for the whole stress block is

$$F_c = bk_1 x f_{cd}. \tag{5}$$

where

$$k_1 = 1 - \frac{\varepsilon_{c2}}{3\varepsilon_{cu2}}. \tag{6}$$

The k_1 coefficient, expressed by (6), is used to calculate the value for the compressive stress resultant, F_c, considering the stress intensity constant for the whole block and equal with f_{cd}. Summarizing, the k_1 coefficient can be considered as a shape coefficient equal with ratio between an equivalent rectangle diagram with aria defined by x and f_{cd} and the real area of the parabola–rectangle diagram. The second equation which can be expressed with regard to the singly reinforced section is the positive bending moment action about the centroid of reinforcement in tension. Considering, z – the lever arm about centroid of reinforcement, this equation is

$$M_{rd} = F_c z = F_c (d - k_2 x). \tag{7}$$

The product $k_2 x$ from the relation (7) is the distance between the centroid of the parabola–rectangle stress block and the most compressed fiber of the section. The k_2 coefficient results from an equilibrium equation regarding the neutral axis, namely the point O, as follows:

$$F_c x(1 - k_2) = F_{c1} z_{c1} + F_{c2} z_{c2}. \tag{8}$$

If the relation (8) is developed using the foregoing relations (4),…,(6) and having in view the geometrical properties of the second degree parabola, then the relationship for k_2 results as

$$k_2 = 1 - \left(\frac{6\varepsilon_{cu2}^2 - \varepsilon_{c2}^2}{12\varepsilon_{cu2}^2 - 4\varepsilon_{cu2}\varepsilon_{c2}} \right). \tag{9}$$

It can be observed that both coefficients, k_1 and k_2 , are related to the deformation properties for every concrete strength class. Therefore, the values of these coefficients may be computed for every concrete strength class and used directly to the equilibrium equation on the section. Equally, for strength classes of concrete higher than 50 MPa, the k_1 and k_2 coefficients can be calculated using the geometrical properties of the n^{th} degree parabola. Thus, developing for n^{th} degree parabola the k_1 and k_2 coefficients take the following relationships:

$$k_1 = 1 - \frac{\varepsilon_{c2}}{(n+1)\varepsilon_{cu2}}, \tag{10}$$

$$k_2 = 1 - \frac{0.5(n+1)(n+2)\varepsilon_{cu2}^2 - \varepsilon_{c2}^2}{(n+1)(n+2)\varepsilon_{cu2}^2 - (n+2)\varepsilon_{cu2}\varepsilon_{c2}}. \tag{11}$$

In Table 2 based on relations (10) and (11), the values of k_1 and k_2 coefficients, for all classes of concrete specified in EC2, are calculated.

Table 2
Coefficients Used to Calculate the Value and Position of the Resultant F_c

Classes	≤50/60	55/67	60/75	70/85	80/95	90/105
k_1	0.810	0.742	0.695	0.637	0.599	0.583
k_2	0.416	0.392	0.377	0.362	0.355	0.353
k_2/k_1	0.514	0.528	0.542	0.568	0.593	0.605

The nondimensional relations which conduct to the reinforcement area or to the resisting moment affected by the k_1 and k_2 coefficients are

$$\omega = \frac{A_s}{bd} \cdot \frac{f_{yd}}{f_{cd}}, \tag{12}$$

$$\omega = k_1 \xi, \tag{13}$$

$$\mu = k_1 \xi (1 - k_2 \xi) = \omega \left(1 - \frac{k_2}{k_1} \omega \right), \tag{14}$$

$$\omega = \frac{k_1}{2k_2} \left(1 - \sqrt{1 - 4\frac{k_2}{k_1}\mu} \right). \tag{15}$$

The above mentioned relations result from the well known equations between internal forces on the reinforced concrete section. Looking at relations (14) and (15) it can be observed that designing using parabola–rectangle stress distribution is relatively easy if the k_1 and k_2 coefficients are known.

The design of reinforced members to bending moment is conceived as the ductile failure of section to be encountered at ULS instead of the brittle failure. In order to reach this goal the neutral axis depth is limited to a maximum, $x_{\lim}$, of which size is related to the strength class of concrete and the steel grade. According to the limit situation *B*2, specified in Fig. 2 *b*, concerning the reinforced concrete section the boundary between brittle and ductile failure is conditioned by the strain to the yield stress, f_{yd}/E_s, in reinforcement, and the ultimate strain in compression for concrete, ε_{cu2}. Thus, the balanced strain condition for parabola–rectangle stress distribution is

$$\xi_{\lim} = \frac{\varepsilon_{cu2}}{\varepsilon_{cu2} + f_{yd}/E_s}. \tag{16}$$

Considering the relations (13) and (14) the limit values for the mechanical reinforcement ratio and the reduced moment on section can be calculated as follows:

$$\omega_{\lim} = k_1 \frac{\varepsilon_{cu2}}{\varepsilon_{cu2} + f_{yd}/E_s}, \tag{17}$$

respectively

$$\mu_{\lim} = k_1 \frac{\varepsilon_{cu2}}{\varepsilon_{cu2} + f_{yd}/E_s}\left(1 - k_2 \frac{\varepsilon_{cu2}}{\varepsilon_{cu2} + f_{yd}/E_s}\right). \tag{18}$$

Table 3
Nondimensional Limits between Ductile and Brittle Failure

Classes	Steel	$\xi_{\lim}$	$\omega_{\lim}$	$\mu_{\lim}$
≤50/60	S400	0.668	0.541	0.390
	S500	0.618	0.500	0.371
	S600	0.572	0.463	0.353
55/67	S400	0.641	0.475	0.356
	S500	0.588	0.436	0.336
	S600	0.543	0.403	0.317
60/75	S400	0.625	0.434	0.332
	S500	0.572	0.397	0.312
	S600	0.526	0.366	0.293
70/85	S400	0.608	0.388	0.302
	S500	0.554	0.353	0.282
	S600	0.509	0.324	0.264
80/95	S400	0.599	0.359	0.283
	S500	0.545	0.326	0.263
	S600	0.499	0.299	0.246
90/105	S400	0.599	0.359	0.276
	S500	0.545	0.318	0.257
	S600	0.499	0.291	0.240

It can be remarked that the limit values for the mechanical reinforcement ratio, $\omega_{\lim}$, and for the reduced moment of the section, $\mu_{\lim,}$, depends on the mechanical properties of concrete and steel. Thus in Table 3, the limit values which mark the boundary to which the double reinforcing section became more rational, for each strength class of concrete and for the whole range of steel grade mentioned in EC2, are calculated based on the relations (16),…,(18). Therefore, beginning with the limits presented in Table 3 is more rational to consider reinforcement in compressed section. As is well known, the doubly reinforcing constitutes a non-economical solution because the increase of bending moment is supported by additional amount of reinforcement disposed equally in compressed zone as in tensioned zone.

The design of the section as double reinforced is made considering the neutral axis depth, $x_{\lim}$, resulted from the balanced strain condition, as is shown in Fig. 4.

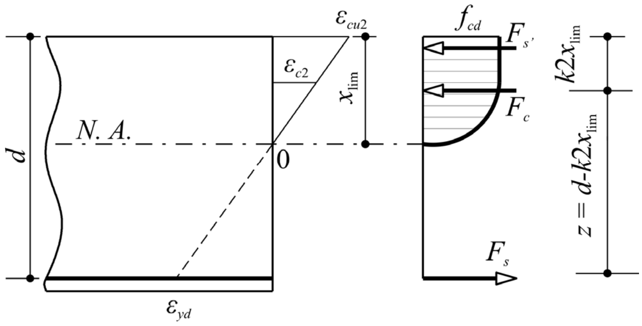

Fig. 4 – Stress and strain diagram for doubly reinforced section.

The relation between mechanical reinforcement ratios of steel in tension *versus* steel in compression is

$$\omega = \omega_{\lim} + \omega'. \tag{19}$$

This relation is a consequence of the internal forces equilibrium in the considered section.

Considering that the neutral axis depth defines the limit situation mentioned above, the increase of reduced bending moment is given by

$$\mu - \mu_{\lim} = \omega'\left(1 - \frac{a'}{d}\right). \tag{20}$$

The relation (20) represent the result of the equilibrium equation between external and internal forces.

3. Design of Concrete Section Considering Rectangular Stress Distribution for Concrete in Compression

The European design rule also provides other simplified stress distribution equivalent to the parabola–rectangle. A rectangular stress distribution may be assumed on concrete section, as shown in Fig. 5.

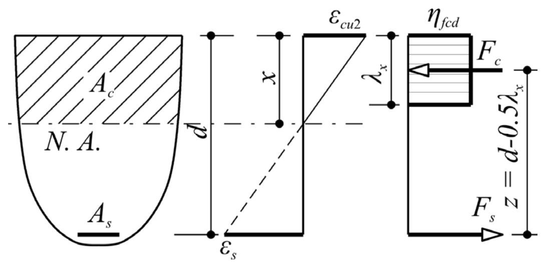

Fig.5 – Rectangular stress distribution according EC2 §3.1.7.3.

As it can be seen in Fig. 5, the stress distribution in concrete section is established on the design value of the concrete compressive strength, f_{cd}, and other two factors. The factor λ, defining the effective height of the compression zone, and the factor η, defining the effective strength, both specified in EC2, are shown in Table 4.

Table 4
The Factors λ and η Specified in EC2

Concrete strength	λ	η
$f_{ck} \leq 50$ Mpa	0.8	1
$50 < f_{ck} \leq 90$ Mpa	$0.8 – (f_{ck} – 50)/400$	$1.0 – (f_{ck} – 50)/200$

The relations for design, considering the rectangle stress distribution in the section, are

$$\omega = \lambda\eta\xi, \tag{21}$$

$$\mu = \lambda\eta\xi(1 - 0.5\lambda\xi). \tag{22}$$

Since the EC2 leaves at designer's latitude the assumption concerning the stress distribution used in design, in Figs. 6,…,8 are illustrated the differences which exist in the design of a RC section when the parabola–rectangle or the rectangle stress distribution is adopted. The relationships are established between the reduced moment in section, μ , and the mechanical reinforcement ratio, ω, for the steel grade S400 and concrete classes below 50 MPa and above.

Also, considering the rectangle stress distribution for a rectangular section, in Fig. 9 is illustrated the design diagram of RC section by the variation of reinforcement percentage of section, p, at the increasing of the bending moment. The variation is illustrated for two strength classes of concrete and two steel grades.

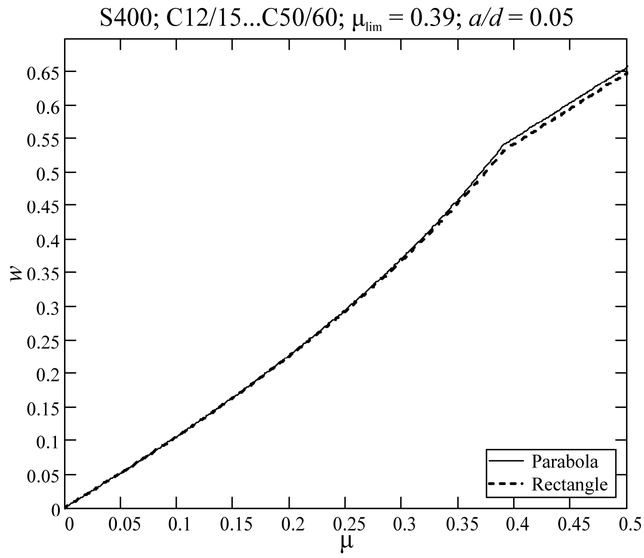

Fig. 6 – μ *vs.* ω interaction diagram for concrete classes less than or equal to 50 MPa; parabola–rectangle *vs.* rectangle stress distribution.

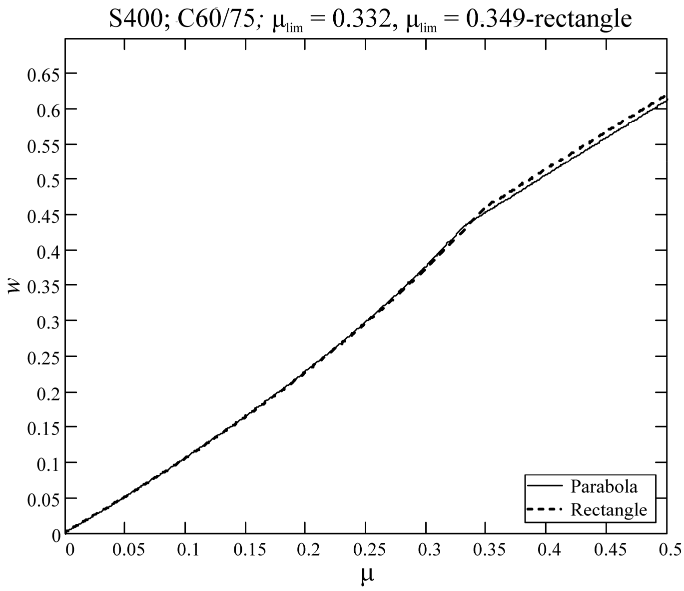

Fig. 7 – μ *vs.* ω interaction diagram for C60/75 concrete class; parabola–rectangle *vs.* rectangle stress distribution.

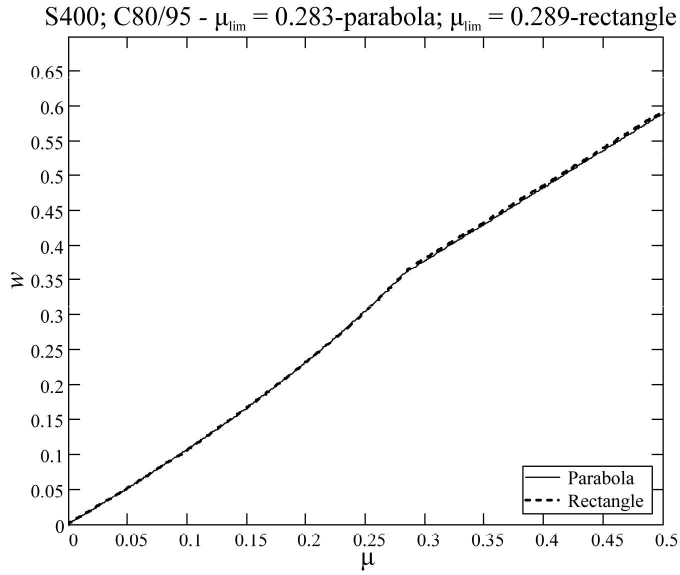

Fig. 8 – μ *vs.* ω interaction diagram for concrete class C80/95; parabola–rectangle *vs.* rectangle stress distribution.

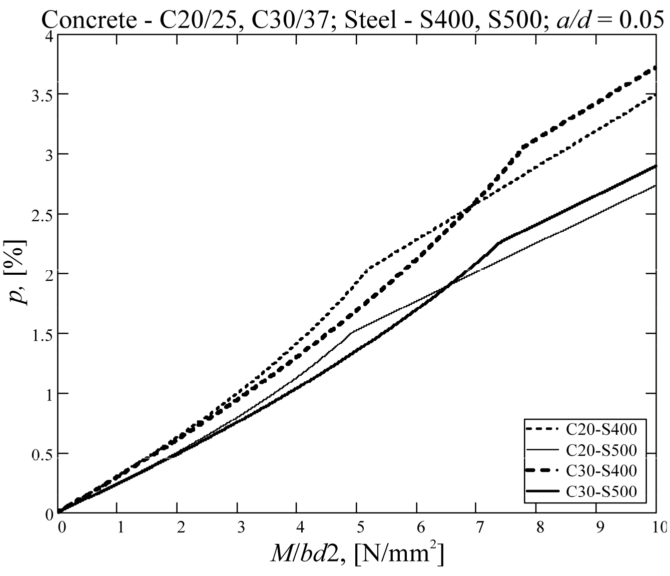

Fig. 9 – Design diagrams of RC section.

4. Conclusions

Considerable progress has been achieved in the last 50 years in the design of reinforced concrete structures. Nowadays the Eurocodes constitute the most laborious documentation for designing of reinforced/prestressed concrete structures. The EC2 has introduced the design of section for 14 strength classes of concrete, each class of concrete having a well established relationship between stress–strain mentioned in it. Moreover, the design of the section can be made considering the parabola–rectangle stress distribution on concrete section or a simplified rectangular distribution. The stress distribution of concrete section for the parabola–rectangle assumption is ruled by the stress–strain relationship, which is established for each strength class of concrete. An important feature of the strain–stress relationship is the adopted power degree, n.

Generally, the design of the reinforced concrete section based on parabola–rectangle stress distribution was considered more difficult in application. Nevertheless, its application is relatively easy to fulfill, if the k_1 and k_2 coefficients are known. Besides, the design based on parabola–rectangle stress distribution can be applied fast enough, if designing charts or interaction diagrams which give the relation between reduced moment, μ , and the mechanical reinforcement ratio, ω , are constructed.

The parallel calculation using both the stress distributions mentioned in the paper, namely, parabola–rectangle and rectangle shows that, as for strength classes of concrete smaller than 50 MPa, as for classes higher than 50 MPa, the differences between the amounts of reinforcement are less than 1% for singly reinforcing section and less than 2% for doubly reinforcing section.

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PROECTAREA SECŢIUNII DE BETON ARMAT SOLICITATĂ LA ÎNCOVOIERE DUPĂ NORMA EUROPEANĂ EN 1992-1-1/2004 – EUROCODE2

(Rezumat)

În următoarele decenii proiectarea construcţiilor civile va fi realizată în conformitate cu normele europene cunoscute şi sub denumirea de Eurocoduri. În proiectarea elementelor structurale ale construcţiilor de beton armat Eurocod 2 (EC2) este de maximă importanţă. Un aspect important al proiectării îl constituie curba tensiuni–deformaţii utilizată la proiectarea secţiunilor. In această privinţă EC2 defineşte 14 clase de rezistenţă pentru betonul structural şi pentru fiecare clasă parametrii adimensionali ce definesc relaţiile dintre tensiuni şi deformaţii sunt, în general, diferiţi.

Prima distribuţie tensiuni–deformaţii recomandată in EC2 privind proiectarea secţiunilor este distribuţia parabolă–dreptunghi pe secţiunea de beton. Ulterior acestei distribuţii este menţionată distribuţia dreptunghiulară a tensiunilor de compresiune pe secţiunea de beton. Astfel, un aspect al calculului ce a fost puţin menţionat de norma de proiectare aflată în vigoare îl constituie proiectarea secţiunii de beton considerând pe secţiunea comprimată de beton o distribuţie de tensiuni de tip parabolă–dreptunghi.

Probemele studiate în acestă lucrare sunt legate de aspecte ale calculului secţiunii de beton armat supuse la încovoiere considerând distribuţii diferite de tensiuni pe secţiunea comprimată de beton recomandate de EC2. Mai întâi a fost considerată o distribuţie de tip parabolă–dreptunghi şi apoi o distribuţie dreptunghiulară, iar rezultatele calculului au fost comparate şi analizate. Calculul secţiunii în ipoteza unei distribuţii parabolă–dreptunghi este pe larg prezentat fiind puţin utilizat până în prezent şi câteva elemente ce simplifică calculul sunt propuse. De asemenea pentru calculul secţiunii la încovoiere în ipoteza unei distribuţii parabolă–dreptunghi, sunt comentate câteva aspecte legate de înălţimea limită a zonei comprimate de beton de unde armarea dublă devine mai raţională.