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CONSTRUCŢII. ARHITECTURĂ

EXTENDING MOHR'S THEORY OF LIMIT STATES IN DETERMINING SOIL SEISMIC LOADS ON RETAINING WALLS

BY

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Mohr's theory concerning the stress limit state and its use in determining the active and passive soil pressures on retaining walls is being developed. Rankine's hypothesis, according to which the pressures on surface elements that are parallel to the free surface of the ground have a vertical direction; the horizontal acceleration effect which generates inertia forces in the soil are accounted for. On these grounds the normal and tangential stresses on planes parallel to the back–of–wall ground surface are being expressed. By using the intrinsic curve corresponding to the sliding plane and the stresses expressed on these planes, Mohr's limit circles, as well as the active and the passive pressures generated by the seismic event on the retaining wall are determined. The solving process is an essentially graphic one and allows for the finding of the sliding plane and the main normal stresses as well as the extreme tangential ones in the points of this plane in which the tension state has reached the limit.

1. Introduction

Determining the active and passive pressures on the face of the retaining wall, in terms of seismic action effects, is an important issue in the design of these retaining structures. The pseudo-static Mononobe Okabe method and its developments are used to correct the results obtained for the static pressures with an effect equivalent to the rotation of the gravitational axis by an angle, θ, given by the direction of the resultant of the gravitation force of the sliding soil prism and its corresponding inertia force. This way we can obtain the total pressure of soil on the retaining wall.

By extending Rankine's hypothesis from the static case, we can determine the active and passive seismic pressures on the retaining wall. The use of limit pressure states, based on Mohr's circles, makes possible to determine the sliding planes, the main normal stresses, the extreme tangential stresses, as well as other useful values.

Extending the method to the seismic case allows for the finding of normal and tangential stresses in a point on the sliding plane and on planes that are parallel to the back-of-wall ground surface.

2. Extending Rankine's Hypothesis to the Seismic Action Case

Similarly to the static case, the pressures on a point of the soil located on the plane parallel to the back-of-wall ground surface, are vertical (Fig. 1 *a*).

Let us consider the effect of the soil inertia forces given by the horizontal acceleration on the same sections. From the elementary equilibrium equation we can determine the normal and tangential forces on sections that are parallel to the back-of-wall ground surface, in points on the sliding surface, particularly in a point at the base of the wall in which the sliding plane intersects the retaining surfaces (Fig. 1 *b*).

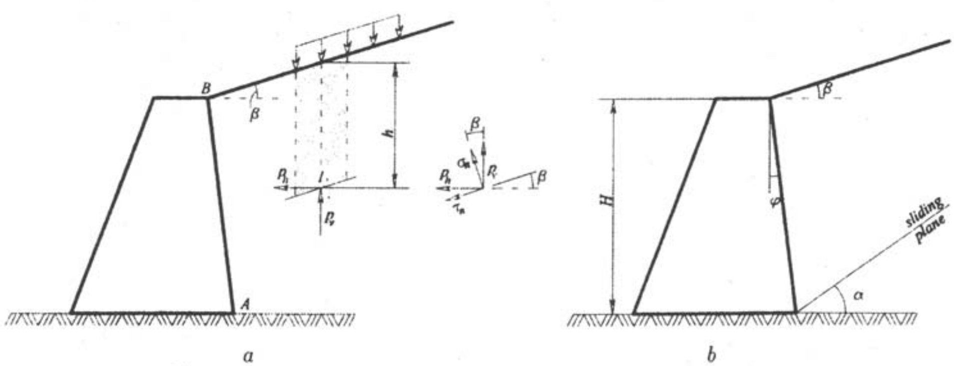

Fig. 1.– Rankine's hypothesis in the case of seismic action.

Considering a soil column of height h, measured from the ground surface, and unitary skew section, the pressure, p_ν, according to Rankine's hypothesis, is

$$p_\nu = \gamma h \cos\beta, \tag{1}$$

which, if there is an overload of intensity q, becomes

$$p_\nu = (\gamma h + q)\cos\beta, \tag{2}$$

where: γh is the weight of the column with unitary section in horizontal plane; for the unitary section, parallel to the ground surface inclined at an angle β, and amplified by $\cos\beta$; q – the overload intensity in vertical direction, in horizontal plane.

The seismic action on the considered soil prism generates inertia forces. The seismic coefficient in horizontal direction being k_h the seismic pressure on the horizontal as related to that of the acceleration will produce a pressure, p_h, on the inclined unitary surface, which has the expressions:

$$p_h = k_h \gamma h \cos\beta, \tag{3}$$

respectively:

$$p_h = k_h(\gamma h + q)\cos\beta, \tag{4}$$

when the overload q acts on the ground surface.

The normal stress, σ_n, and the tangential stress, τ_n, yields

$$\sigma_n = p_\nu \cos\beta + p_h \sin\beta, \quad \tau_n = -p_\nu \sin\beta + p_h \cos\beta. \tag{5}$$

If relations (1),...,(4) are taken into account, then we have:

a) ground surface is free,

$$\sigma_n = \gamma h \cos^2\beta + k_h \gamma h \cos\beta \sin\beta; \quad \tau_n = -\gamma h \cos\beta \sin\beta + k_h \gamma h \cos^2\beta; \tag{6}$$

b) ground surface is subjected to overload of intensity q,

$$\begin{cases} \sigma_n = (\gamma h + q)\cos^2\beta + k_h(\gamma h + q) cos\beta \sin\beta, \\ \tau_n = -(\gamma h + q)\cos\beta \sin\beta + k_h(\gamma h + q)\cos^2\beta. \end{cases} \tag{7}$$

Relations (6) and (7) may be also written as:

$$\sigma_n = \gamma h \overline{\sigma}_n, \quad \tau_n = \gamma h \overline{\tau}_n, \tag{8}$$

respectively,

$$\sigma_n = (\gamma h + q)\overline{\sigma}_n, \quad \tau_n = (\gamma h + q)\overline{\tau}_n, \tag{9}$$

where

$$\overline{\sigma}_n = \cos^2\beta + k_h \cos\beta \sin\beta, \quad \overline{\tau}_n = -\cos\beta \sin\beta + k_h \cos^2\beta. \tag{10}$$

The values $\overline{\sigma}_n$ and $\overline{\tau}_n$ are dimensionless and depend on the slope of the back-of-wall soil, given by the angle β and the seismic location given by k_h. These may be interpreted considering that

$$\gamma \overline{h} + q = 1 \text{ and consequently } \overline{h} = \frac{1-q}{\gamma}, \tag{11}$$

that is, $\overline{\sigma}_n$ and $\overline{\tau}_n$ are the normal and tangential pressures values at the depth $\overline{h}$ on the plane which is parallel to the ground surface.

3. The Intrinsic Curve Corresponding to Mohr's Theory on the Limit Stress States

In the points found on the intrinsic curve, the stress state is at its limit. This confines the domain of the admissible stress states. Two situations may be distinguished in the case of soils namely:

a) The soils are non-cohesive, between the stresses τ and σ is satisfied the relation $\tau = \sigma \tan\varphi$ and the curve is given by two lines tilted at an intrinsic friction angle,

φ, of the soil which originate in point O of the system of reference $\sigma O \tau$, the axis $O\sigma$ being the bisecting line of the angle 2φ (Fig. 2).

b) The soils are cohesive having the intrinsic friction angle, φ, and cohesion, c, therefore $\tau = c + \sigma \tan\varphi$. The intrinsic curve is made up of two lines tilted on both sides of the axis $O\sigma$ with the angle φ and which originate in point O' on the axis $O\sigma$ at the distance $-s$ (Fig. 3), where

$$s = \frac{c}{\tan\varphi}. \tag{12}$$

The distance, s, has the significance of a normal stress. If c and φ are constant and s is constant too, that is, in any point cohesion acts like a spherical tensor of uniform stress, s.

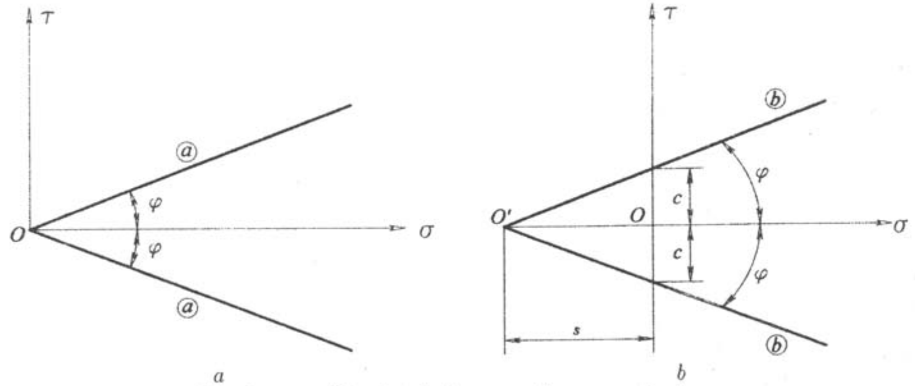

Fig. 2.– a – The intrinsic curve for non-cohesive soils; b – the intrinsic curve for cohesive soils.

The intrinsic curve for non-cohesive soils is a particular case of the cohesive ones, in which $c = 0$; the internal friction angle is specific to each specific soil and is determined experimentally, in laboratory.

4. Limit Stress State

The limit stress state in a point may be represented by Mohr's circle. Any circle tangent to the intrinsic curve represents a limit stress state.

In the case of a retaining wall, where a limit stress state has been reached in the soil, that is, the yielding plane has been formed, in a point which might be point A from the basis of the wall, the stresses σ_n and τ_n on the section through A, parallel to the ground surface plane, represent a limit stress state and the point $M(\sigma_n, \tau_n)$ will be situated on Mohr's circle corresponding to yielding.

It is admitted that the soil is cohesive and, by knowing the angle of internal friction, φ, and cohesion, c, the intrinsic curve can be plotted. From a geometrical point of view, we need to draw a circle that passes through the point M and is tangent to the two straight lines.

There are two solutions and through M will pass two circles tangent to the two lines which form the intrinsic curve. The small circle corresponds to the active pressures, while the big one corresponds to the passive ones (Fig. 3).

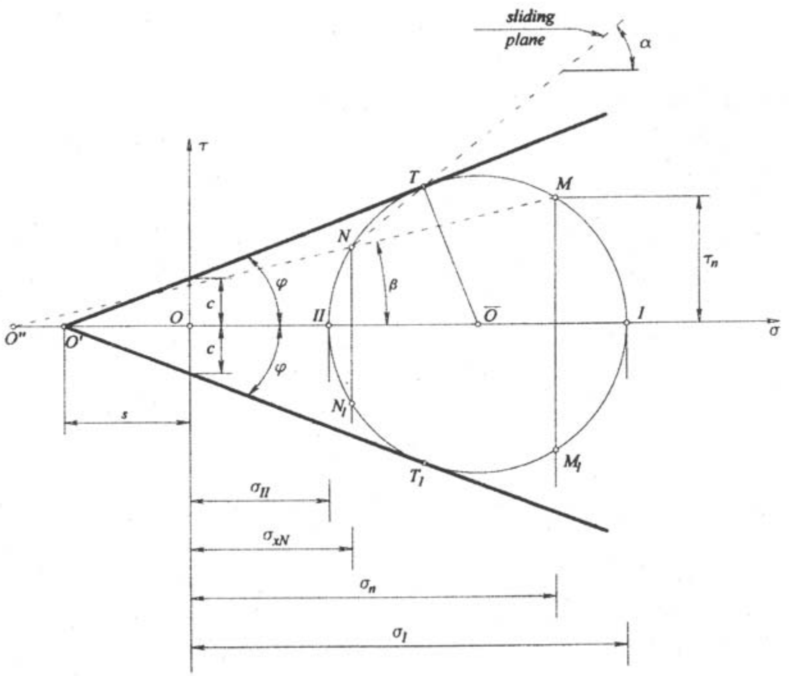

Fig. 3.– Representation of limit stress with Mohr's circle.

The line passing through the point M and making the angle β with the horizontal direction $(O\sigma)$ is parallel to the soil surface plane. This line intersects the circle in the second point, N, which is called the circle's pole. This pole has the property that the direction given by the line which connect a point on the circumference with the point N is parallel to the section plane on which act the stresses representing the point's coordinates. The symmetrical of N against the axis $O\sigma$, the point N_1 on the circle, determines the limit stresses $\sigma_{xN_1}, \tau_{xN_1}$ on the face of the wall. As a consequence of equality $\sigma_{xN_1} = \sigma_{xN}$, there follows that:

$$\sigma_{xN} = \sigma_{xN_1} = \overline{\sigma}_{xN}(\gamma + q) \tag{13}$$

and $\overline{\sigma}_{xN} = \overline{\sigma}_{xN_1} = k_{as}$, therefore

$$p_{as} = \tfrac{1}{2}\, H K_{as}(\gamma H + 2q). \tag{14}$$

The direction of the yielding plane can be obtained by connecting N with T, and the second plane corresponds to the direction NT_1.

The parallel to NT which passes at the basis of the wall and makes the angle α with the horizontal, represents the trace of the sliding plane.

The points I and II determine the main planes on which the tangential stresses are null; σ_I is the maximum stress, and σ_{II} – the minimum one.

The circle centre has the coordinates $((\sigma_I + \sigma_{II})/2, 0)$, and the radius is equal to $\tau_{\max}$; for the active pressures, it results

$$R_a = \frac{\sigma_I - \sigma_{II}}{2}. \tag{15}$$

In the case of non-cohesive soils, for which $c = 0$, we can draw the circle using $\overline{\sigma}_n$ and $\overline{\tau}_n$ so that to obtain $\overline{\sigma}_{xN} = \overline{\sigma}_{xN_1} = K_{as}$. All the previous reasoning stands valid, with benefic practical consequences. The coefficients K_{as} may be easily determined for different categories of non-cohesive soils.

The cohesive soil can be treated to determine the pressures on the face of the walls as for a non-cohesive soil with the same internal friction angle as of the cohesive one. Using the correspondence theorem, the spherical tensor is superposed on the stress, s, applied on the whole contour of the yielding prism, plus a uniform load of intensity, s, on the back-of-wall free soil surface.

5. Conclusions

Similarly to the static case, the pressure on a point of the soil located on the plane parallel to the back-of-wall ground surface, are vertical. The solving is relatively simple and general for the gravity and assimilated retaining walls. It may be used in both cohesive and non-cohesive soils. The coefficients of earth pressure may be determined for both active and passive pressures.

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EXTINDEREA TEORIEI STĂRILOR LIMITĂ DE TENSIUNE A LUI MOHR PENTRU DETERMINAREA PRESIUNILOR SEISMICE ALE PĂMÂNTULUI ASUPRA ZIDURILOR DE SPRIJIN

(Rezumat)

Se dezvoltă teoria stărilor limită de tensiune a lui Mohr pentru determinarea presiunilor active şi pasive ale pamântului asupra zidurilor de sprijin.

Se foloseşte ipoteza lui Rankine, conform căreia presiunile pe elemente de suprafaţă paralele cu suprafaţa liberă a terenului au direcţia verticală; se ia în considerare efectul acceleraţiei orizontale, care generează forţe de inerţie în sol. Pe baza acestora se exprimă tensiunile normale şi tangenţiale pe planuri paralele la suprafaţa terenului din spatele zidului.

Folosind curba intrinsecă corespunzatoare planului de lunecare şi tensiunile exprimate pe acest plan, se determină cercurile lui Mohr limită şi presiunile active şi pasive generate de cutremur pe zidul de sprijin.

Procesul de rezolvare este esenţial grafic şi permite determinarea planului de lunecare precum şi a tensiunilor principale şi tangenţiale extreme în punctele acestui plan, în care starea de tensiune este limită.