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BUCKLING ANALYSIS FOR MEMBERS WITH VARIABLE RIGIDITY BY USING THE FINITE ELEMENT METHOD

BY

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Abstract. The stability analysis for compressed bars with variable rigidity is studied, by using the finite element method (FEM) in Galerkin’s variant. The general shape of elastic and geometrical stiffness matrices are obtained for stepped and tapered bars. In a study case, the critical buckling load obtained by using FEM, for three columns with constant and variable sectional stiffness (in steps and continuous variation), is evaluated in comparison with others analytical and numerical methods.

Key words: stability analysis; critical buckling load; variable rigidity; analytical and numerical methods; finite element.

1. Introduction

The use of high strength materials has offered the possibility of designing slender structural elements and structures. The problem which can arise is that of their stability.

The structural design must assure the stability of the initial equilibrium shape that presumes to correctly evaluate the critical loads corresponding to the neutral and instable equilibrium.

From different reasons (technological, economical, etc.) many such structural elements have variable rigidity (stepped members or tapered members) and the present paper approaches the problem of their analysis from buckling point of wiew.

2. The Governing Differential Equations

It is considered a member subjected to both transverse, $p(x)$, and axial, P, loads (Fig. 1 *a*) and a differential element cut from it (Fig. 1 *b*).

By expressing the equilibrium equations for the infinitesimal element dx in length, the differential equations between loads and internal forces are obtained [1]

$$\text{(1)} \qquad \text{a)} \quad \frac{\mathrm{d}V}{\mathrm{d}x} = -p(x); \qquad \text{b)} \quad \frac{\mathrm{d}M}{\mathrm{d}x} = V + N\frac{\mathrm{d}w}{\mathrm{d}x},$$

where the internal forces are: $N = P$ – the axial force, $V = V_z$ – the shear force, $M = M_y$ – the bending moment; $w(x)$ – the deflection of the section of abscissa x, along z-axis.

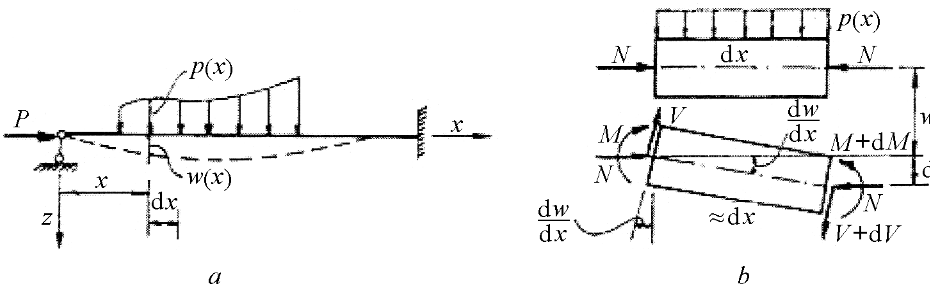

Fig. 1 – *a* – a structural element subjected to transverse and longitudinal loads; *b* – a differential element

When the shear force effect exerted upon deformations is ignored and the axial deformations too, the approximate differential equation of the elastic curve is

$$\text{(2)} \qquad EI\frac{\mathrm{d}^2 w(x)}{\mathrm{d}x^2} = -M(x),$$

where EI represents the cross-sectional flexural rigidity in xOz plane (E – Young's modulus of the constitutive material, $I = I_y$ – moment of inertia with respect to y-axis). In a more general case, the rigidity, EI, can be continuous variable or variable in steps along the beam axis.

By differentiating twice the eq. (2) and taking into account relations (1) it results [1], [4]

$$\frac{d^2}{dx^2}\left[EI\frac{d^2 w(x)}{dx^2}\right]+N\frac{d^2 w}{dx^2}=p(x). \tag{3}$$

To obtain the analytical solution of eq. (3) for the case of variable rigidity is a tedious and difficult task (by using Fredholm equations [2], Bessel functions or by tests [3]). In these circumstances the solution comes from numerical methods (the finite differences method – FDM, the finite element method – FEM) that provide enough accurate engineering results without using complex mathematical tools.

The paper presents the finite element method in Galerkin's variant. The general shapes of elastic and geometrical stiffness matrices are obtained for stepped and tapered bars.

3. Numerical Solution by Finite Element Method

3.1. Finite Element Modeling

A global coordinate system *xyz* is adopted for the structure that is divided into finite elements connected at nodes, *n* in number (Fig. 2 *a*). Each node is provided with two degrees of freedom (the deflection, $D_{iz} = w_i$ and the slope $D_{i\theta} = \theta_i$). The column vectors of nodal displacements and corresponding nodal forces are considered for the whole beam

$$\{D\}=\{D_{1z}\ D_{1\theta}\ \ldots D_{iz}\ D_{i\theta}\ \ldots D_{nz}\ D_{n\theta}\}^T,\ \{P\}=\{P_{1z}\ P_{1\theta}\ \ldots P_{iz}\ P_{i\theta}\ \ldots P_{nz}\ P_{n\theta}\}^T. \tag{4}$$

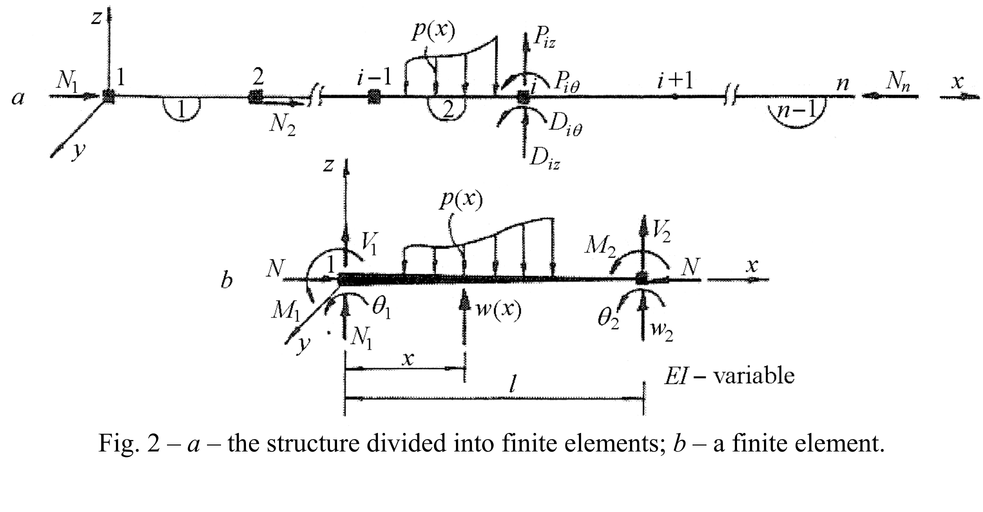

Fig. 2 – *a* – the structure divided into finite elements; *b* – a finite element.

3.2. The Galerkin Approach

The beam is discretized into one-dimensional finite elements subjected to both bending and compression. The finite element analysis is performed with respect to a local coordinate system *xyz* (Fig. 2 *b*).

Each finite element has four degrees of freedom, two at each node, so that the column vectors of nodal displacements and forces are

$$\{d_e\} = \{w_1\ \theta_1\ w_2\ \theta_2\}^T;\ \{S_e\} = \{V_1\ M_1\ V_2\ M_2\}^T. \tag{5}$$

As an approximate solution, $w_e(x)$, of the differential eq. (3) a polynomial of third degree is chosen; which accompanied by its first order derivative represent the displacement field

$$w_e(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3,\ w_e^{'}(x) = \theta_e(x) = \alpha_1 + 2\alpha_2 x + 3\alpha_3 x^2. \tag{6}$$

The boundary conditions, based on relations (6), yield to the following system of algebraic equations:

$$w_e(x) = \alpha_0 + \alpha_1 x + \alpha_2 x^2 + \alpha_3 x^3,\ w_e^{'}(x) = \theta_e(x) = \alpha_1 + 2\alpha_2 x + 3\alpha_3 x^2. \tag{7}$$

The vector $\{\alpha\}$ can be easily obtained namely

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & l & l^2 & l^3 \\ 0 & 1 & 2l & 3l^2 \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} = \begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix}, \text{ or } [L]\{\alpha\} = \{d_e\}. \tag{8}$$

By introducing (8) in (6), the deflection expression becomes

$$w_e(x) = \begin{bmatrix} 1 & x & x^2 & x^3 \end{bmatrix} \begin{Bmatrix} \alpha_0 \\ \alpha_1 \\ \alpha_2 \\ \alpha_3 \end{Bmatrix} = [P(x)]^T [L]^{-1} \{d_e\} = [N_i(x)]^T \{d_e\} \tag{9}$$

where $[N_i(x)]$, (i = 1, 2, 3, 4), is the vector of shape functions (l'Hérmite functions)

$$\begin{cases} N_1(x) = 1 - 3\dfrac{x^2}{l^2} + 2\dfrac{x^3}{l^3};\ N_2(x) = x - 2\dfrac{x^2}{l^2} + \dfrac{x^3}{l^3}; \\ N_3(x) = 3\dfrac{x^2}{l^2} - 2\dfrac{x^3}{l^3};\ N_4(x) = -\dfrac{x^2}{l^2} + \dfrac{x^3}{l^3}. \end{cases} \tag{10}$$

The approximate solution (9) is introduced in the differential eq. (3) and the residual

$$\varepsilon(x) = \frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2}\right) + N\frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2} + p(x) \neq 0 \tag{11}$$

is obtained.

Galerkin's functionals can be expressed as

$$\begin{aligned} \Pi_i &= \int_0^l N_i(x)\varepsilon(x)\mathrm{d}x = \\ &= \int_0^l N_i(x)\left[\frac{\mathrm{d}^2}{\mathrm{d}x^2}\left(EI\frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2}\right) + N\frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2} + p(x)\right]\mathrm{d}x = 0, \end{aligned} \tag{12}$$

where $i = 1, 2, 3, 4$. They must be equal to zero in order to minimize the residual.

Integrating by parts it results

$$\begin{aligned} \Pi_i &= N_i(x)\frac{\mathrm{d}}{\mathrm{d}x}\left(EI\frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2}\right)\Bigg|_0^l - N_i^{'}\frac{\mathrm{d}^2}{\mathrm{d}x^2}EI\frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2}\Bigg|_0^l + \int_0^l N_i^{''}(x)EI\frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2}\mathrm{d}x + \\ &+ NN_i(x)\frac{\mathrm{d}w_e}{\mathrm{d}x}\Bigg|_0^l - \int_0^l NN_i^{'}(x)EI\frac{\mathrm{d}w_e}{\mathrm{d}x}\mathrm{d}x + \int_0^l N_i(x)p(x)\,\mathrm{d}x = 0,\ (i = 1, 2, 3, 4). \end{aligned} \tag{13}$$

From relations (1 *b*) and (2) it can be written that

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(EI\frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2}\right) = -\frac{\mathrm{d}M}{\mathrm{d}x} = -V - N\frac{\mathrm{d}w_e}{\mathrm{d}x}, \tag{14}$$

according to the sign convention from Strength of Materials.

In the finite element method sign convention, the sign "–" in front of V becomes "+" and taking into account (2), relation (13) becomes

$$\Pi_i = N_i(x)V(x)\Big|_0^l - NN_i(x)\frac{\mathrm{d}w_e}{\mathrm{d}x}\Big|_0^l + N_i^{'}(x)M(x)\Big|_0^l +$$

(15)
$$+\int_0^l N_i^{''}(x)EI\frac{\mathrm{d}^2 w_e}{\mathrm{d}x^2}\mathrm{d}x + NN_i(x)EI\frac{\mathrm{d}w_e}{\mathrm{d}x}\Big|_0^l - \int_0^l NN_i^{'}(x)\frac{\mathrm{d}w_e}{\mathrm{d}x}\mathrm{d}x +$$

$$+\int_0^l N_i(x)p(x)\mathrm{d}x = 0,\ (i=1,2,3,4).$$

Finally it results

(16)
$$\Pi_i = -\begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix} + \left(\int_0^l N_i^{''}(x)EI\begin{bmatrix} N_1^{''} & N_2^{''} & N_3^{''} & N_4^{''} \end{bmatrix}\mathrm{d}x -\right.$$

$$\left. -\int_0^l NN_i^{'}(x)\begin{bmatrix} N_1^{'} & N_2^{'} & N_3^{'} & N_4^{'} \end{bmatrix}\mathrm{d}x\right)\begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix} + \int_0^l N_i(x)p(x)\mathrm{d}x = 0,\ (i=1,2,3,4),$$

where the deflection $w_e(x)$ expression is given by relation (9).

The final results of relation (16) can be shortly written

(17)
$$\left(\left[k_e\right] - \left[k_e^G\right]\right)\left\{d_e\right\} + \left\{R_e\right\} = \left\{S_e\right\},$$

that represents the finite element force–displacement relation, where the involved terms have the following significances: $\left[k_e\right]$ – finite element elastic stiffness matrix; $\left[k_e^G\right]$ – finite element geometric stiffness matrix, which considers the axial force effect on bar deformation; $\left\{R_e\right\}$ – vector of reactive forces for a double fixed beam subjected to bending, produced by the transverse distribute loads $p(x)$, acting along the finite element; $\left\{S_e\right\}$ – vector of internal forces at the finite element nodes.

An element, k_{ij}, of the elastic stiffness matrix, $\left[k_e\right]$, and geometric stiffness matrix, $\left[k_e^G\right]$, respectively, is computed with the following relations:

(18) $$k_{ij} = \int_0^l EIN_i^{"} N_j^{"} \mathrm{d}x,\quad k_{ij}^G = \int_0^l NN_i^{'} N_j^{'} \mathrm{d}x,\quad (i, j = 1,2,3,4).$$

For the particular case of the finite element of constant flexural cross-sectional stiffness (EI = const.), acted by a uniformly distributed load $p(x) = p$ and by an axial force $N(x) = N$ along its whole lenght, relation (16) has the shape

(19) $$\left(\frac{EI}{l^3}\begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} - \frac{N}{30l}\begin{bmatrix} 36 & 3l & -36 & 3l \\ 3l & 4l^2 & -3l & -l^2 \\ -36 & -3l & 36 & -3l \\ 3l & -l^2 & -3l & 4l^2 \end{bmatrix}\right)\begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix} + \\ + \begin{Bmatrix} l/2 \\ l^2/12 \\ l/2 \\ -l^2/12 \end{Bmatrix} = \begin{Bmatrix} V_1 \\ M_1 \\ V_2 \\ M_2 \end{Bmatrix}.$$

For the bars with variable cross-section along their longitudinal axis, the moment of inertia with respect to the neutral axis is described by a function [5], …, [7]

(20) $$I(x) = I_0 (a + cx)^m,$$

where I_0 is a moment of inertia considered as a reference one (Fig. 3); a and c are numerical coefficients that depend on the section geometrical properties.

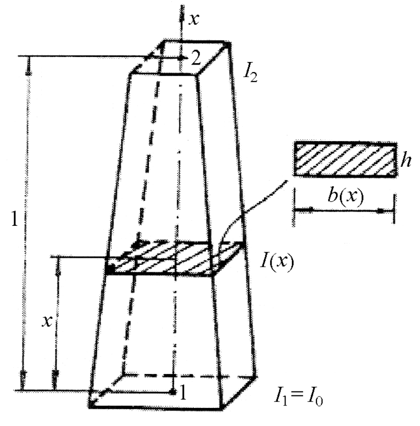

Fig. 3 – A finite element with variable cross-section.

As concerns the parameter m, it has different values for different possibilities of conceiving the variable in cross-section member, for instance

a) $m = 1$ for a bar with rectangular cross-section, having a constant height and a variable width, its variation being described by a function of first degree;

b) $m = 3$ for a bar with rectangular cross-section, having a constant width and a variable height, its variation being described by a first degree function.

The stiffness matrix of the finite element having a variable rectangular cross-section (h = const., $b = b(x)$) is

$$\left[k_e\right]=\frac{EI_0}{l^3}\begin{bmatrix} 12a+6cl & 6al+2cl^2 & -12a-6cl & 6al+4cl^2 \\ 6al+2cl^2 & 4al^2+cl^3 & -6al-2cl^2 & 2al^2+cl^3 \\ -12a-6cl & -6al-2cl^2 & 12a+6cl & -6al-4cl^2 \\ 6al+4cl^2 & 2al^2+cl^3 & -6al-4cl^2 & 4al^2+3cl^3 \end{bmatrix}. \tag{21}$$

If $a = 1$ and $c = 0$ we obtain the finite element elastic stiffness matrix for the finite element having a constant cross-section (s. relation (19)).

3.3. The Assembly Procedure

Based on the compatibility between the vector $\{d_e\}$, where $e = 1, 2,...,$ $n - 1$, and the vector of nodal displacements for the whole structure $\{D\}$, by expanding the element force–displacements relation, it results

$$\left(\left[k_e\right]^{\exp}-\left[k_e^G\right]^{\exp}\right)\{D\}+\left\{R_e\right\}^{\exp}=\left\{S_e\right\}^{\exp}. \tag{22}$$

By assembling these relations expressed for all component finite elements, the structural force–displacement relation results

$$\left(\left[K\right]-\left[K^G\right]\right)\{D\}+\{R\}=\{P\}, \tag{23}$$

where

$$\left[K\right]=\sum_{e=1}^{n-1}\left[k_e\right]^{\exp},\quad \left[K^G\right]=\sum_{e=1}^{n-1}\left[k_e^G\right]^{\exp},\quad \{R\}=\sum_{e=1}^{n-1}\left\{R_e\right\}^{\exp}. \tag{24}$$

$[K]$ is the elastic structural stiffness matrix and $[K^G]$ – the geometric structural stiffness matrix. They are both singular matrices.

The vector $\{R\}$ has as components the equivalent concentrated forces at nodes to the action of the distributed loads along the finite element.

In order to obtain non-singular elastic and geometric stiffness matrices the boundary conditions are imposed.

The column vector of nodal displacements is divided in two subvectors: $\{D_n\}$ – the subvector of free displacements and $\{D_r\}$ – the subvector of restrained displacements. This presumes a rearranging and partition of all component matrices and vectors of the structural force–displacement relation

$$\left(\begin{bmatrix} K_{nn} & K_{nr} \\ K_{rn} & K_{rr} \end{bmatrix} - \begin{bmatrix} K_{nn}^G & K_{nr}^G \\ K_{rn}^G & K_{rr}^G \end{bmatrix}\right)\begin{Bmatrix} D_n \\ D_r \end{Bmatrix} = \begin{Bmatrix} P_n \\ P_r \end{Bmatrix} - \begin{Bmatrix} R_n \\ R_r \end{Bmatrix}, \tag{25}$$

or, by developing it

$$\begin{aligned} &a)\ \left(\left[K_{nn}\right]-\left[K_{nn}^G\right]\right)\left\{D_n\right\}+\left(\left[K_{nr}\right]-\left[K_{nr}^G\right]\right)\left\{D_r\right\}=\left\{P_n\right\}-\left\{R_n\right\}, \\ &b)\ \left(\left[K_{rn}\right]-\left[K_{rn}^G\right]\right)\left\{D_n\right\}+\left(\left[K_{rr}\right]-\left[K_{rr}^G\right]\right)\left\{D_r\right\}=\left\{P_r\right\}-\left\{R_r\right\}, \end{aligned} \tag{26}$$

that represent two matrix equations; their solutions depend on the member constraints. Generally, there are three main situations:

$\{D_r\} = 0$ – for fixed ends of the member;

$\{D_r\} \neq 0$ – but the restrained displacements are known (support settlements);

$\{D_r\} = [K_r]\{P_r\}$ – the constraints are elastic ones and the restrained displacements are proportional to the reactive forces.

For the first case, that is in fact the most common case ($\{D_r\} = 0$), the two equations given by (26) becomes

$$\begin{aligned} &a)\ \left(\left[K_{nn}\right]-\left[K_{nn}^G\right]\right)\left\{D_n\right\}=\left\{P_n\right\}-\left\{R_n\right\}=\left\{F_n\right\}, \\ &b)\ \left(\left[K_{rn}\right]-\left[K_{rn}^G\right]\right)\left\{D_n\right\}=\left\{P_r\right\}-\left\{R_r\right\}=\left\{F_r\right\}. \end{aligned} \tag{27}$$

3.4. Buckling Analysis

In this case the axial force, N, acting along the member longitudinal axis is unknown and it is required to determine its critical value, N_{cr}, which produces the member stability loss. The transverse loads can exist or not and the restrained displacements $\{D_r\}$ are considered to be nought.

When there are no transverse loads, relations (17), expressed for a standard finite element, can be written as

(28) $$\frac{EI_{et}}{l_{et}^3}\left(\left[k_{et}^*\right]-\lambda_{et}\left[k_{et}^G\right]\right)\left\{d_e\right\}=\left\{S_e\right\},$$

where

(29) $$\lambda_{et}=\frac{N_{et}l_{et}^3}{30EI_{et}}.$$

The force–displacement relations for the other finite elements can be expressed in terms of the standard element force–displacement relation

(30) $$\frac{EI_{et}}{l_{et}^3}\left(\left[k_{et}^*\right]-\lambda_{et}\left[k_{et}^G\right]\right)\left\{d_e\right\}=\left\{S_e\right\},$$

where the matrix $\left[k_{et}^*\right]$ is obtained by multiplying the elements of matrix $\left[k_{et}\right]$ with the coefficient

(31) $$\alpha_e=\frac{I}{I_{et}}\left(\frac{l_{et}}{l}\right)^3$$

and the matrix $\left[k_{et}^{G*}\right]$ is obtained in the same manner, but the coefficient is

(32) $$\beta_e=\frac{N}{N_{et}}\left(\frac{l}{l_{et}}\right)^2\frac{I_{et}}{I}.$$

By assembling the finite element force–displacement relations and by introducing the support conditions, relation (27 *a*) becomes:

(33) $$\frac{EI_{et}}{l_{et}^3}\left(\left[K_{nn}\right]-\lambda_{et}\left[K_{nn}^G\right]\right)\left\{D_n\right\}=0,$$

that represents a system of algebraic homogeneous equations (eigenvalues and eigenvectors problem).

The system admits solutions different from zero if and only if

(34) $$\det\left\|\left[K_{nn}\right]-\lambda_{et}\left[K_{nn}^G\right]\right\|=0,$$

that is the matrix $\left[K_{nn}\right]-\lambda_{et}\left[K_{nn}^G\right]$ is singular.

The minimum eigenvalue leads to the critical load value

$$N_{\text{cr}} = \frac{30EI_{et}}{l_{et}^3}\lambda_{et}^{\min} \tag{35}$$

that produces the bar stability loss by equilibrium bifurcation.

Where transverse loads exist too, the system of algebraic equations (27 *a*) has the form

$$\frac{EI_{et}}{l_{et}^3}\left(\left[K_{nn}\right] - \lambda_{et}\left[K_{nn}^G\right]\right)\left\{D_n\right\} = \left\{P_n\right\} - \left\{R_n\right\} = \left\{F_n\right\}. \tag{36}$$

In this case, the bar stability loss is produced by the equilibrium divergency.

The stability equation has the same shape as in the previously discussed case, so that the critical load has the same value.

4. Study Case

It is determined the critical load, N_{cr}, for a column conceived in three different variants:

a) with constant rectangular cross-section b=40 cm; h=15 cm (Fig. 4 *a*);

b) with rectangular cross-section variable in steps (h = 15 cm, b_1 = 50 cm, b_2 = 30 cm) (Fig. 4 *b*);

c) continuous variable rectangular cross-section (Fig. 4 *c*), having a constant height h = 15 cm and a variable width, from 60 cm at the lower end up to 20 cm at the upper one.

The element volume is the same in all three variants.

The critical load obtained by using FEM is presented in Table 1, in comparison with the values computed by using other analytical and numerical values.

Table 1

The Values of Critical Load Obtained by Different Methods

Variant	Method	Source	Critical load, [kN]
a)	Analytical	Euler formula	346.978
	Numerical	Finite Element Method (FEM)	347.625
b)	Analytical	Formula VIII.59 [5]	386.863
		Table VIII.11 [5]	385.547
		Table 15.3 and 15.5 [6]	326.953
		Formula for case 32 in Table 38 [7]	386.863
		Table for case 32 [7]	385.547
		Formula for case 33 [7]	394.229
	Numerical	Finite Element Method (FEM)	386.016
c)	Numerical	Finite Element Method (FEM)	307.660

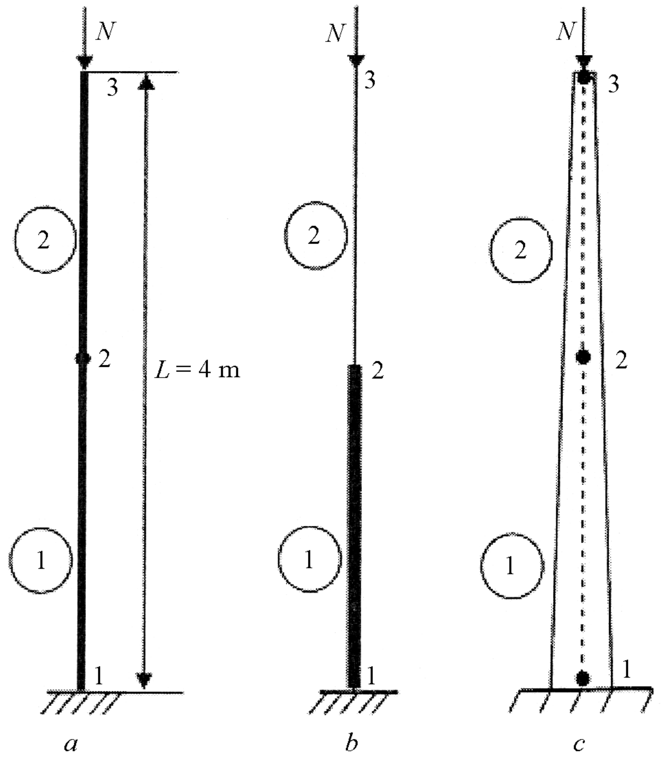

Fig. 4 – A column conceived in three variants.

4. Conclusions

1. For the bar with constant cross-section the critical load obtained by using FEM does not significantly differ from the analytical solution given by Euler.

2. In case of the stepped bar, the relation given in work [6] leads to a critical load 18% lower than that resulted by using in proceedings from [5] and [7], while FEM provides a critical load very closed to them.

3. For the tapered bar, the analytical solutions are quite difficult obtained and the finite element method represents an alternative that provides enough accurate results.

4. This last discussed bar that has a similar shape to the constant stress bar subjected to compression is not the best solution as concerns the stability, because it is characterized by the lowest critical load.

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ANALIZA STABILITĂŢII ELEMENTELOR CU RIGIDITATE VARIABILĂ PRIN UTILIZAREA METODEI ELEMENTELOR FINITE

(Rezumat)

Se efectuează analiza de stabilitate pentru bare cu rigiditate variabilă supuse la compresiune utilizând metoda elementelor finite (MEF) în varianta Galerkin. Se obţine forma generală a matricelor de rigiditate elastică şi de rigiditate geometrică pentru bare cu secţiune variabilă în trepte şi cu secţiune variabilă continuu. În studiul de caz, forţa critică de flambaj obţinută prin utilizarea MEF pentru trei stâlpi cu rigiditate secţională constantă şi variabilă (în trepte şi cu variaţie continuă), este evaluată în comparaţie cu alte metode analitice şi numerice.