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ABOUT SOME SUSPENSION ROOFS MADE OF THE ORTHOGONAL CABLE NETWORKS

BY

N. UNGUREANU, STAN-DRAGOŞ CĂCIULĂ and M. VRABIE

Abstract. Some types of structures made of orthogonal cable networks are considered. Improved technique for analysis regarding the determination of the axial force in the cables are proposed. Some modification of the outline shape and the anchoring mode meant to increase the safety rank are also mentioned. Aspects of dynamic effects generated by the wind on these roofs, referring to the flutter and resonance phenomena, are discussed.

Key words: suspension roofs; cable networks; elliptical contour; dynamic wind's action; flutter effect; resonance effect.

1. Introduction

Suspended roofs can have different shapes and various constructive systems, ways of supporting, cables tying.

Specifications are given concerning the analysed systems. We are talking of roofs made of orthogonal networks, cables which generate convex surfaces with a positive total curvature (Gauss curvature). If the networks meshes are relatively small compared to the general dimensions, then the covered surface can be assimilated with help of a membrane, also mentioning that when it comes to gravitational distributed actions the surface is subject to tension. This assimilation allows a parallel between the shells which have the same behavior as the membranes and the cabled systems [1], [2].

We have to mention that shells in the membrane state work essentially on compression and are made of materials with a good resistance to compression, for example, concrete, glass or materials whose resistance is near at tension and compression, such as steel, aluminum and its alloys, used in construction, some composite materials, the stratified ones being the most adequate.

These types of roofs can also have a certain capacity to resist to tension or the lack of it in a sufficient measure to be replaced by the reinforcement. Concrete can be reinforced, some as plastic materials, glass, etc.

Cabled roofs distinguish themselves through the fact that they only work at tension (when they are alone in a resistance structure) even if the material itself has a sufficient resistance to compression, which cannot be used due to the particular way in which are made the cables without a resistance to compression or bending [3].

These brief characterizations lead to the idea that cabled systems must be complementary to the classic shells type, in the meaning of a tensioned membrane's accomplishment. They can be conceived assuming a modification in the actions sense. Since they are gravitational they cannot be modified by inversing the ones made of shells that work on compression.

If we remain in the bordered background of the surfaces with a positive total curvature, than we can speak, for example, of turned cupolas: spherical, elliptical, and parabolic-shaped and through extension and other types of roofs where the loads induce tension in the cables [3],…, [5].

2. Assimilations in the Achievement of the Cabled Roofs with the Ones Made of Compressed Shells

If the roofs achieved as shells in the membrane state are compressed, through overturn the membrane state changes a tension one and can be taken over by the cables. But we must mention that in the case of the analysed roofs surfaces must enter in the category of the shallow ones. If other actions intervene, for example the wind or the earthquake, that do not have the directions of the gravitational actions at shells in a membrane state, tension internal forces can appear and they can be taken over through the capacity of resistance to tension of the basic material or through the use of reinforcing which takes over the tension.

In the case of the cabled structures, the appearance of compression no longer is allowed and it is necessary to use elements of covering to take over these efforts or initial tension efforts (pretension) which are to be introduced to cover the effects of compression through a partial discharge of the tension. Through pretensionning the constructive deflections of the cable are diminished (limited), that present the disadvantage of the useful space's reduction on the altitude.

We now describe a type of roof with an elliptical form on an ellipsis-shaped contour (Fig. 1). This surface presents a great advantage, for example in the case of the shells roofs in a membrane state on the orthogonal directions, parallel to the ellipsis planes, which also contains the surface, only compression stresses appear, the sliding stresses being null. Since for a cabled complementary roof, they cannot take over the sliding, the situation complicates both theoretically and practically if there are any sliding.

This has been a reason to mention the types of surfaces the article is about.

If the surface made on cables is similar to that of the shells, the cables in the network are to be directed on the mentioned directions and so we will be able to establish equivalence between the two types of roof.

Even more, on these directions the horizontal components in the cables will be the same on a direction and similarly on the other direction.

Cables are fixed in a marginal ring and so there aren't necessary any exterior anchors. We propose even the extension of this type of roof to other forms in a near-by plane, for example the rectangular one, in the conditions of maintaining the elliptical roof and the placement of the system of anchorage on a structure associated of the form in plan, which will complete and cover the rectangular surface (Fig. 2).

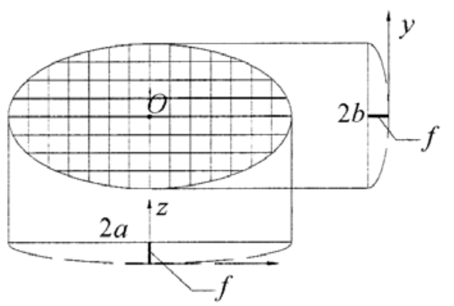

Fig. 1 – Roof with an elliptical form on an ellipsis-shaped contour.

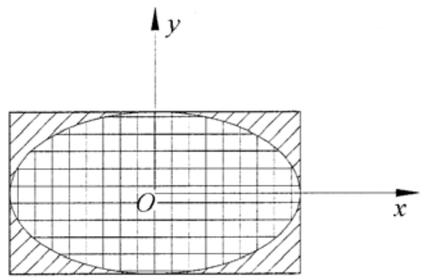

Fig. 2 – Roof with an elliptical form on a rigid contour.

To accomplish structural and aesthetic compatibility, there are necessary some developments on the ring's conception and for the entire support structure of the rectangular construction. Surely, suspended roofs can be built directly on a rectangular contour but, as we will see in what follows, some advantages are lost, for example the marginal system advantageously works and the handling of the cables pretensionning process.

3. Orthogonal-Cables Suspended Roof on an Elliptical Contour

Out of the surface's geometrical point of view we use the elliptical parabola whose surface equation is [4]

(1) $$z = \frac{f_1}{a^2}x^2 + \frac{f_2}{b^2}y^2.$$

The contour's projection in a horizontal plane will be

(2) $$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$

where a and b are the semi-axis of the ellipsis that represents the contour's projection in the xy horizontal plane (the origin of the triorthogonal reference system is the top of the cupola) and f_1 and f_2 are the surface's level towards the top for $x = \pm a$ and $y = \pm b$.

For $f_1 = f_2 = f$, the surface's equation becomes:

(3) $$\frac{z}{f} = \frac{x^2}{a^2} + \frac{y^2}{b^2}.$$

We immediately see that for $z = f$, the contour's equation is a plane curve. For a cabled roof, the anchoring frame follows the contour's ellipsis.

The orthogonal cables are projected on the contour ellipsis' plane following straight lines parallel with the axis. The coordinate planes xz and yz are also principal planes of the elliptical parabola (Fig.3).

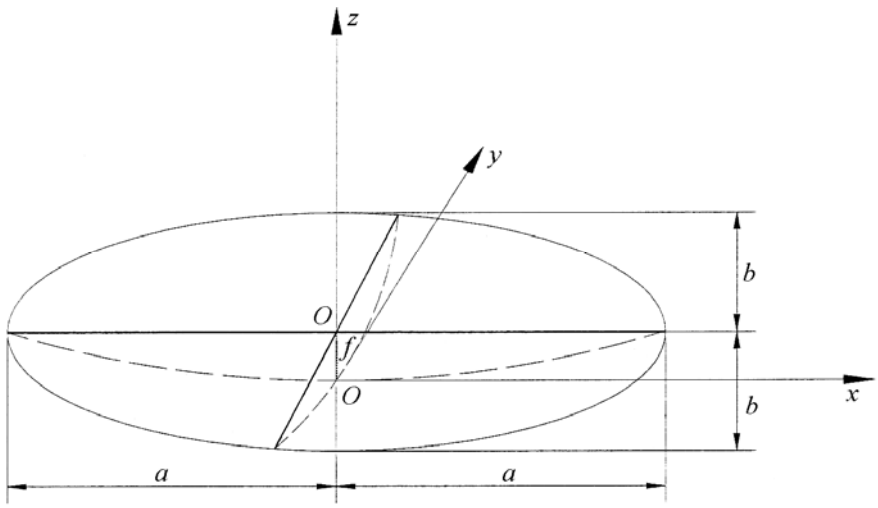

Fig. 3 – Orthogonal cables in the Cartesian coordinates system.

Through the sections x = const. and y = const., parabolas are obtained, them being exactly the cables' deformed lines (in the case of tensioned cables).

For a uniformly distributed load, the cables deformed lines are parabola. We observe that the horizontal components, H_x and H_y , in the cables have the expressions

$$H_x = \frac{pa^2}{4f}, \quad H_y = \frac{pb^2}{4f}. \tag{4}$$

If we go on with this analogy, with the shell in a membrane state, under a more general form, the projected efforts, S_x and S_y , have the expressions [2], [4]

$$S_x = -\frac{pa^2}{2\left(f_1 + f_2\right)}, \quad S_y = -\frac{pb^2}{2\left(f_1 + f_2\right)}. \tag{5}$$

For $f_1 = f_2$, replacing p with $-p$ in the relations (5) we obtain the expressions (4). This comparison is also useful for the determination of the cables' pretension forces. First, we observe that the S_x and S_y efforts are constant in sections that also contain the cables. So, all the cables on a direction are pretensioned with the same force, that is H_x and H_y, equivalent to a vertical load of constant intensity, p.

The calculation of the prestressing forces are made by determining the elongation of the cable under the action of the forces H_x and H_y ,

$$\Delta l_i = l_i - l_{0i} , \tag{6}$$

where: l_{0i} is the initial length of the cable i, equal with the distance between suspension points; l_i – cable length under the action of forces H_x , respectively H_y.

In the same time, the elongation of the cable i can be expressed as a lot of two terms

$$\Delta l_i = \Delta l_{ei} + \Delta l_{pi} , \tag{7}$$

with: Δl_{ei} – elastic elongation of the cable i; Δl_{pi} – permanent elongation of the cable i.

The components of the cable elongation, Δl_i , are

$$\Delta l_{ei} = \frac{H_{x,y} l_{0i}}{EA}, \quad \Delta l_{pi} = K_0 l_{0i} , \tag{8}$$

and finally

$$\Delta l_i = \frac{H_{x,y}}{EA} + K_0 l_{0i}\,, \tag{9}$$

where: E is the modulus of elasticity of the cable (Young's modulus of the cable); A – cross section area of the cable; K_0 – coefficient of permanent deformation.

To materialize the forces H_x, H_y in the cables (H_x in the direction of the x-axis and H_y in the direction of the y-axis) we determine the displacements which need to be measured. So, the total effort in the roof's pretensioned cables acted by the load p and pretensionning will be

$$H_{xt} = \frac{\left(p+\overline{p}\right)a^2}{4f}, \quad H_{yt} = \frac{\left(p+\overline{p}\right)b^2}{4f}. \tag{10}$$

The manner of analysis is useful also for the technique in which the cables' pretensionning is achieved. The cables must be pretensioned each before they form the network ties through their coupling, so that through the cables' introduction in the network they would maintain the introduced forces. Technically speaking, it's important the manner of introducing the pretensionning forces followed by the cables' coupling in the ties. The total tensions in the cables, T_x and T_y, are

$$T_x = \frac{H_{xt}}{\lambda}, \quad T_y = \lambda H_{yt}\,, \tag{11}$$

where

$$\lambda = \sqrt{\frac{1+y^2/b^4}{1+x^2/a^4}}\,. \tag{12}$$

This result has been obtained starting from the case of the shell for which projected efforts are calculated. Because the cables are sufficiently pretensioned, the resulted surfaces are shallow surfaces and the T_x and T_y tensions do not significantly differ from H_{xt} and H_{yt}. We therefore consider sufficient the loads due to the roofs own weight and to the snow, which is uniformly distributed in a horizontal plane.

If we need to cover rectangular surfaces, especially with big spans, the rectangular contour frames are less economical and present a higher risk compared to the elliptical contour. In the case of a rectangular construction we can design a suspended roof on an elliptical contour which couples constructively to the rectangular contour with a specific treatment for the corner areas. This solution will be analysed by the authors in a subsequent paper.

4. Some Aspects Concerning the Wind's Action

It is well known the suspended roofs sensibility to the wind's dynamic action. Two problems concern the research and the design [6], [7] namely

a) The flutter effect that consists of the coupling of the vertical oscillations with the waving ones, as a result of the roof's deformation coupled with the direction of the wind's action.

b) The resonance effect, generated by the quasiperidiocity in the wind's fluctuation.

The flutter manifests especially at roofs with a double surface and is produced by the aerodynamic lifting (suction) forces, the roof's reduced weight favorizing the phenomenon. By increasing the roof's rigidity to torsion, the waving is diminished and the flutter effect is reduced.

At the roofs with a single network of cables, the flutter phenomenon can be damped especially through lesting, for example by using ceramic or concrete plates in the covering system. At these roofs, more important is the resonance effect. Different directions of the wind's speed and the speed of the punctual displacements on the roof's surface facilitate the production of whirlswind which maintain the periodic movement of the roof.

Starting from the Donnell - Mustari - Vlasov model and from the adequate equation of movement, K u n e i d a [7] has obtained a parametric non-linear equation of the suspended roof's movement with the help of which he has determined the stability diagram, in which we can situate the wind's speed fluctuation intensity. Kuneida proposes for the wind's speed a variation of the form

$$V(t) = V_m\left(1 - q\cos\omega_f t\right), \quad q < 1, \tag{13}$$

where: V_m is the medium wind's speed; t – the time; q – the parameter which measures the fluctuations intensity; ω_f – the speed's fluctuation's pulsation.

Out of the resonance condition, at a ω_f pulsation, we obtain

$$q = \sqrt{2S_v\left(\omega_f\right)}, \tag{14}$$

where $S_v(\omega_f)$ is the power density's specter of the wind's speed established by D a v e n p o r t [8].

The dynamic stability condition in the case of the parametric resonance is expressed through

$$q < q_0 , \tag{15}$$

q_0 being situated in the domain of stability determined as mentioned earlier.

The flutter and resonance effects on suspended roofs we consider them to be determined in a certain measure by the efforts variation in cables generated by the winds periodic action, which leads to modifications of the vibration frequencies in cables in a different way and even on segments, this being the way the structural response is produced in assembly. This issue is to be analysed both theoretically and experimentally.

5. Conclusions

1. The suspended roofs on cables, subjected to gravitational loads, may be replaced for design purposes with tensioned thin shells, where the axial forces (tension) take over by cables.

2. If other loads intervene (the wind, the seismic load) which don't have the direction of gravitational loads, compression appears in the roof. In this case elements of the shell must take over the compression or initial tensile force needs to be introduced (pretensionning axial force) which takes over the effects of compression by partial discharge of tension.

3. An advantageous roof structure is that of orthogonal cables with elliptic contour, which may be extended also to rectangular contour after adequate treatment of the corner areas.

4. The suspended roofs on cables are very sensitive at the dynamic action of the wind. The flutter effect and the resonance phenomenon are especially treated by research and design.

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„Gheorghe Asachi" Technical University of Iaşi,
Department of Structural Mechanics
e-mail: mihai_vrabie@yahoo.com

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ASUPRA UNOR ACOPERIŞURI SUSPENDATE PE REȚELE DE CABLURI ORTOGONALE

(Rezumat)

Se consideră câteva tipuri de structuri de acoperiş pe rețele de cabluri ortogonale şi se propun analize şi tehnici îmbunătățite privitoare la determinarea forței axiale din cabluri.

Se menționează, de asemenea, unele modificări ale formei conturului în plan şi ale modului de ancorare menite să sporească gradul de siguranță.

Se discută aspecte ale efectelor dinamice generate de vânt asupra acestor acoperişuri, referitoare la fenomenele de fluturare („flutter”) şi de rezonanță.