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EQUIVALENT VISCOUS DAMPING MODELS IN DISPLACEMENT BASED SEISMIC DESIGN

BY
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The paper reviews some equivalent viscous damping models used in the displacement based seismic design considering the equivalent linearization. The limits of application of the models are highlighted, based on comparisons existing in the literature. The study is part of a research developed by the author, aimed to determine the equivalent linear parameters in order to predict the maximum displacement response for earthquakes compatible with given response spectra.

1. Introduction

Traditionally, the seismic design approaches are force-based, and remain the basis of most current codes [2]. Even if the ductility is considered in the computation of the seismic forces and this approach implies assuming displacement capacity, the design is still carried out in terms of required strength. In the last decade, however, several researchers proposed displacement-based approaches for earthquake engineering evaluation and design, which are dealing directly with displacement demands. The aim of this paper is to provide improved reliability in the engineering process by more directly relating the computed response and expected structural performance. From the wide variety of displacement based design procedures that have been developed, for the purpose of the present paper, only the substitute structure method proposed by S h i b a t a and S o z e n [14] will be described here. In this approach, the real non-linear MDOF (Multidegree of Freedom) structure is replaced by an elastic SDOF (Simpledegree of Freedom) structure, having an equivalent stiffness (or period) and equivalent viscous damping ratio (equivalent linearization). This equivalent elastic structure is supposed to have the same response as the real non-linear structure under earthquake excitation. The substitute structure being elastic, its response to a particular earthquake can be determined from elastic response spectra calculated for the given equivalent damping ratio, function of the equivalent period. As shown in Fig. 1 [12] assuming an allowable ultimate displacement, Δu, and estimating the yielding displacement, in the initial step, the ductility ratio, $\mu \doteq \Delta u/\Delta y$, can be computed. Function of the ductility ratio, the equivalent viscous damping, ξ_e, is then computed on the basis of the most adequate hysteretic model. From the displacement response spectra, the target displacement, Δu, and the equivalent viscous damping, ξ_{eff}, give the equivalent period of the linear system, T_{eff}. Using the equivalent stiffness, K_{eff}, the structure design forces can be obtained and the design of the structure is performed accordingly. The yield displacement, Δu, is then revised and subsequent iterations occur until convergence is reached.

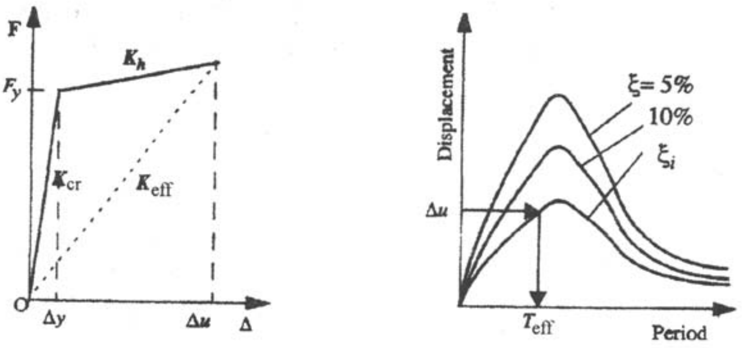

Fig. 1.– Design using equivalent linearization:
a – equivalent stiffness; b – displacement response spectra.

The main problem in this approach, is the determination of the equivalent viscous damping ratio for the elastic equivalent system. Several equivalent damping models exist in the literature; some of the most used are reviewed in this paper.

2. Equivalent Viscous Damping Models

The first author to introduce the equivalent viscous damping concept was J a c o b s e n [6]. The approach was initially used to approximate the steady-state response of a SDOF system with a non-linear damping function. The equivalent damping coefficient was determined so that the real non-linear damped oscillator and the equivalent linear system dissipate the same amount of energy per cycle of response to sinusoidal excitation. In this study, the stiffness of the equivalent linear oscillator was considered equal to the stiffness of the real system.

J e n n i n g s [7] made a review of six proposals of equivalent linearization methods, based on Jacobsen (*op.cit.*) approach, for the case of steady-state response of yielding SDOF systems. The author noted that the different methods of treating the period shift are the reasons for the different behaviour of equivalent viscous damping factors of yielding structures. The linear equivalent models for determining the steady-state yielding response of SDOF systems is described by the general equation:

$$m(x_0)\ddot{x} + c(x_0)\dot{x} + k(x_0)x = F_0 \sin \omega t, \tag{1}$$

where $m(x_0)$, $c(x_0)$ and $k(x_0)$ are, respectively, the mass, the damping and the stiffness of the equivalent linear system, function of the steady-state response amplitude, x_0. For most methods, the mass of the associated linear oscillator will not vary, but will equal the mass of the yielding oscillator ($m(x_0) = m$), leaving the other two parameters to be determined. The associated equivalent linear system is subjected to the same sinusoidal excitation as the yielding system. Fig. 2 shows the force - displacement relation for the elasto-plastic hysteretic model, for which the equivalent linear oscillator was determined using the methods reviewed by Jennings (*op. cit.*). In this figure, x_y is the yield displacement, P_y - the yielding force and k - the stiffness of the real oscillator. As in Jacobsen approach, the equivalent viscous damping of the linear oscillator is obtained by equating the dissipated energy per cycle of the real oscillator (E_p) to that of the equivalent linear system (E_e). The energy dissipated per cycle of vibration by the elasto-plastic system is the area of the hysteresis loop, and it is easily found as:

(2) $$E_p(x_0) = 4kx_y(x_0 - x_y).$$

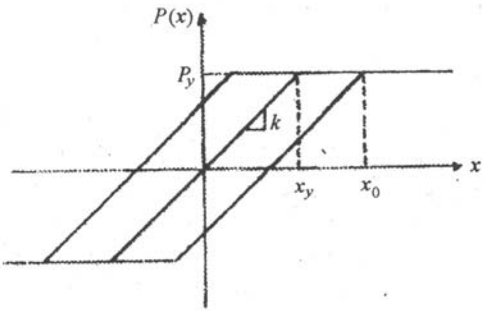

Fig. 2.- Elasto-plastic hysteretic model.

For the associated linear oscillator of equation (1), it is necessary to know, respectively, the critical damping coefficient, the damping ratio and the energy dissipated per cycle, function of the amplitude of the steady-state response [7]

(3) $$c_{cr}(x_0) = 2\sqrt{m(x_0)k(x_0)}, \;\; \xi(x_0) = \frac{c(x_0)}{c_{cr}(x_0)}, \;\; E_e(x_0) = 2\pi\xi(x_0)k(x_0)x_0^2.$$

For all the six methods considered, the quantities from equations (2), (3) are the same. The difference stands in the way that the three parameters of the linear equivalent system (mass, damping and stiffness) are varied. From these methods, the only one analysed here is the first one to propose the secant stiffness at maximum amplitude as the basis for selecting the period shift, by Rosenblueth and Herrera [13]. In this method, the stiffness of the associated linear system is determined by the geometry of the elasto-plastic force–displacement relation presented in Fig. 2, as the slope of the line joining the ends of the hysteretic loop. That means, according to the notations in Fig. 2, that the stiffness of the equivalent linear system becomes:

$$k(x_0) = k\frac{x_y}{x_0}. \tag{4}$$

By equating the dissipated energy per cycle of the equivalent linear system from equation (3) to the dissipated energy per cycle of the real oscillator from equation (2), and considering the expression of the equivalent stiffness from equation (4), it results the following equivalent damping ratio (function of ductility ratio, μ, and adding the viscous damping ratio of the yielding system, ξ_0)

$$\xi_{\text{eff}} = \xi(x_0) = \xi_0 + \frac{2}{\pi}\left(1 - \frac{x_y}{x_0}\right) = \xi_0 + \frac{2}{\pi}\left(1 - \frac{1}{\mu}\right). \tag{5}$$

Together with the equivalent damping ratio, the second parameter utilized in the equivalent linearization methods using the secant stiffness at maximum amplitude is the equivalent period, T_{eff}. Considering the secant stiffness definition from equation (4) the equivalent period is determined, function of the initial period of the yielding system and of the ductility ratio, by the formula:

$$T_{\text{eff}} = T_0\sqrt{\mu}. \tag{6}$$

Gulkan and Sozen [3] showed that the ductility by itself is not sufficient to interpret the behaviour of reinforced concrete structures. Two systems having the same ductility, may not have the same response to a cyclic excitation if the hysteretic properties of the two systems differ. Until Jennings review (*op. cit.*), all the equivalent linearization methods considered only the elasto-plastic model. Moreover, all these methods were based on harmonic loadings. Under a random excitation, which is the case of earthquake loading, the response of the yielding system is more complex. Gulkan and Sozen emphasized that two basic characteristics of reinforced concrete structures play an important role in determining response to strong ground motions: the changes in stiffness and energy dissipation capacity. Both can be related to the maximum displacement. The authors proposed a new formula for the equivalent damping ratio, based on shaking table tests of a series of reinforced concrete frames subjected to steady-state dynamic base motion and simulated earthquake motion. Data from different tests with different base motions were not strictly comparable, but it was observed a discernible evolution of the ductility ratio, a trend consistent with Jacobsen (*op. cit.*) approach, using Takeda model for degrading-stiffness-hysteresis-response [15]. Considering a symmetrical loop as shown in Fig. 3, the degrading stiffness is defined by the slope of line BC, function of the slope corresponding to fully cracked section for linear response, γ, the ductility ratio, μ, and a parameter, α, calibrated from test results. The slope AB represents the secant stiffness (equivalent stiffness of the linear system). Considering the symmetry of the loop and according to Jacobsen (*op. cit.*) approach, the equivalent damping ratio is computed by equating the area EBC (the dissipated energy of the real oscillator) to area ABF (the dissipated energy of the equivalent linear system). The following formulas are given for the equivalent damping ratio and, respectively, equivalent period

$$\xi_{\text{eff}} = \xi_0 + 0.2\left(1 - \frac{1}{\mu^{0.5}}\right), \quad T_{\text{eff}} = T_0\sqrt{\mu}. \tag{7}$$

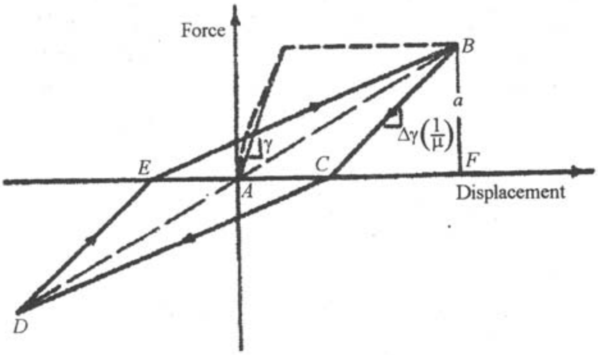

Fig. 3.– Takeda degrading-stiffness model.

In a later study, I w a n [5] used results from time-history analyses of 12 recorded earthquake ground motions, in order to calibrate empirical formulas for the equivalent damping ratio and period shift of the equivalent linear system. The hysteretic model used in this study for the time-history analysis is derived from a combination of linear elastic and Coulomb slip elements, which are divided in three groups: a single elastic element, an elasto-plastic group and a group able to model the stiffness degrading (cracking and crushing). In this way, six specific systems are considered, covering a wide range of hysteretic load–deformation behaviour. The post-yield stiffness was also considered, and set in all cases as 5% from the elastic one. The response of each structural model to each earthquake was calculated by numerical integration of the differential equation of motion, and the yield level of structural model was varied until a specified ductility ratio was obtained. In this way, the resulting maximum displacements, determined function of the ductility ratio, were used to construct inelastic displacement response spectra for each hysteretic system and earthquake, as a function of ductility ratio. Ductility ratios of 2, 4 and 8 were used, and a period range of 0.4...4 s, with a step of 0.1 s was investigated. By converting the spectral displacements to a normalized pseudo-velocity spectra, it was observed that the overall shape of the inelastic spectrum for a given value of ductility ratio would closely resemble with that of some linear spectrum, if that spectrum were shifted in period by a certain factor. The next step was to estimate the damping and period shift of linear system, which gave the "best" fit to the inelastic response data. By representing the optimal damping ratio *versus* optimal period shift ratio and the optimal period shift ratio *versus* ductility ratio, an empirical set of formulas was determined, to fit the available data. The following expressions were determined for equivalent damping ratio and, respectively, the equivalent period

$$(8) \qquad \xi_{\text{eff}} = \xi_0 + 0.0587(\mu - 1)^{0.371}, \quad T_{\text{eff}} = T_0[1 + 0.121(\mu - 1)^{0.939}].$$

In a more recent theoretical study, K o w a l s k y [8] used also the secant stiffness at maximum deformation for defining the period shift, together with the theoretical Takeda hysteretic model for degrading-stiffness-hysteresis-response [15]. Using the Jacobsen (*op. cit.*) approach, *i.e.* equating the energy dissipated by one cycle of the real oscillator by one cycle of sinusoidal response of the equivalent linear system (with the secant stiffness defined at maximum deformation), considering an unloading stiffness factor, $\alpha = 0.5$ in the Takeda model (with reference to Fig. 3) and a post yield to initial stiffness ratio, r, the equivalent damping ratio and, respectively, period are:

$$(9) \qquad \xi_{\text{eff}} = \xi_0 + \frac{1}{\pi}\left(1 - \frac{1-r}{\mu^{0,5}} - r\mu^{0.5}\right), \quad T_{\text{eff}} = T_0\sqrt{\mu}.$$

It should be noted that equations (9) derived for steady-state under harmonic excitation, therefore he became an approximation for earthquake excitation. It may be observed that relations (9) are similar to (7), excepting the constant term in front of bracket, which leads, in Kowalski's method, to a higher value of the equivalent damping.

3. Comparison between Equivalent Viscous Damping Models

Several studies evaluated the methods based on equivalent linearization to estimate the maximum inelastic displacement demands of SDOF systems, but as show by M i r a n d a and R u i z-G a r c i a [11] their scope has been limited. Of particular interest to practicing engineer is to know which method produces better results for specific periods of vibration or at least for specific spectral regions, as well to know which method provides better results for levels of inelastic behaviour expected to occur in the structure. This was the purpose of Miranda and Ruiz-Garcia (*op. cit.*) study, in which the authors evaluated the four methods described above. A comparison between the "exact" results computed with non-linear time-history analyses with those computed with the approximate methods was made. In this evaluation, three types of hysteretic behaviour are considered: the elasto-plastic model, the modified C l o u g h stiffness-degrading model [1] and the Takeda (*op. cit.*) model. The post-elastic stiffness, for all models, was set equal to zero and the damping ratio at 5%. A set of 50 periods of vibration between 0.05 and 3 s were considered, and a total of 264 earthquake acceleration time-histories recorded in the state of California, USA, in 12 different earthquakes, were used in this evaluation.

It was concluded that Rosenblueth and Herrera method (*op. cit.*) lead to significant underestimations of the maximum inelastic displacement for all three types of hysteretic models considered in this study. This result was an expected one, considering that a comparison between the normalized damping ratios of the four methods (product of equivalent damping ratio with the ratio of initial to equivalent stiffness) showed significant higher value in case of Rosenblueth and Herrera method, based on elasto-plastic hysteretic model and steady-state harmonic response. From the other methods, based on degrading stiffness hysteretic models and developed for seismic loading, Iwan's procedure yield the best estimations of maximum displacements, with the mention that, for periods lower than 0.4 s, it underestimates the maximum displacement, which leads to non-conservative results. The conclusion of the study was that, despite having relatively small mean errors, the dispersion of the results in some cases is substantial, in particular for large levels of inelastic behaviour. Hence, when applied to individual earthquake ground motions, any of these methods could lead to significant errors in the estimation of the maximum displacement.

D w a i r i and K o w a l s k y [10] obtained the same conclusion, in a very recent study. The authors investigate the accuracy of the equivalent viscous damping concept, as stated by Jacobsen (*op. cit.*), if applied to real earthquake records. In this study, the Takeda hysteretic model (*op. cit.*) and Ring-Spring hysteretic model [4], were considered, in order to compute expressions between displacement ductility and equivalent damping ratios, using the Jacobsen approach. By determining the expressions of equivalent damping for Takeda model, two extreme cases were selected: the smallest and largest possible loop, by changing the model parameters. This is the main difference between this approach, aimed to obtain a low and, respectively, a high energy dissipation (low and high damping), and the Kowalsky's (*op. cit.*) method presented previously. For the Ring-Spring model only the largest possible loop was considered at determining the equivalent damping ratio expression. The results from two earthquake records, which have distinctly different response spectrums, considering all these three hysteretic models, show very wide scatter and varies from conservative to unconservative. By comparing the results of both records, the authors concluded that Jacobsen's approach is not only sensitive to earthquake characteristics, but also to the oscillator fundamental period and level of ductility. The best way to quantify the scatter, in authors opinion, in order to introduce any modification, would be through a large number of results from a large number of simulated earthquakes and utilizing a statistical analysis. Consequently, a number of 100 earthquakes were selected, for a number of 50 oscillators with fundamental periods range from 0.1 s to 5 s. The total number of the inelastic time-history analysis conducted in this part of the study was 125,000. The results of all 100 earthquakes were averaged and plotted as a function of equivalent damping against oscillator periods.

It was emphasized that the hysteretic models which predict less damping (Takeda small loop) have better results, which suggest that Jacobsen's approach, on average, overestimates damping and, consequently, underestimates actual displacements. This conclusion suggests that a reduction factor is needed. For this purpose, the author intends, as part of future research, to realize a more complex study, comprising four hysteretic models and 280,000 time-history analyses.

4. Conclusions

It may be concluded, that up to present time, no direct displacement-based design method, based on equivalent linearization, is able to provide satisfactory results, in the attempt to determine the maximum displacement response from real earthquake records. Most of the methods, like Rosenblueth and Herrera or Kowalsky, are using a theoretical approach to compute the equivalent viscous damping. These methods are assuming a sinusoidal steady-state response and are based on the arbitrary choice of the one cycle criterion to estimate the equivalent viscous damping. This means that the energy dissipation of the system will be approximated from the cycle corresponding to the maximum level of deformation, cycle supposed to be symmetric and with the same shape for the entire excitation period. For an earthquake loading, the response is most of the time smaller than the response amplitude, so the use of this criterion will overestimate the equivalent damping. In the same time, the equivalent stiffness is approximated also function of the maximum level of deformation. This leads, for a particular earthquake, to a wide dispersion in the estimation of the maximum displacement, from conservative to unconservative range.

Methods combining the theoretical approach with testing using recorded accelerograms, like Gulkan and Sozen proposal, or empirical determination of equivalent parameters based on numerical simulations of recorded accelerograms, would seem to be more reliable when applied to real earthquake records. However, this kind of approach is generally suitable only for earthquake records having similar spectral response with the one considered in determining the equivalent linear characteristics.

In the opinion of the author, the most suitable approach in order to obtain equivalent linear parameters for a particular earthquake record would be the Iwan empirical approach (*op. cit.*), but considering only one hysteretical model when determining the formulas for these parameters. For a family of earthquake records, compatible with a determined response spectra, and for a given structural model (hysteretic model), the response could be calculated by numerical integration for different ductility ratios, and empirical formulas for the equivalent period and damping ratio can be obtained, following similar steps as described in Iwan's method. The author is actually developing such a study, in which time-history motions, compatible with particular response spectra are considered. In this way, a direct displacement-based design method using equivalent linearization approach could be set up for different type of earthquakes and for different hysteretical models. The results of this study will be further published.

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MODELE PENTRU AMORTIZAREA VÂSCOASĂ ECHIVALENTĂ ÎN PROIECTAREA ANTISEISMICĂ BAZATĂ PE DEPLASARE

(Rezumat)

Se prezintă câteva dintre modelele de amortizare vâscoasă echivalentă utilizate în proiectarea antiseismică bazată pe deplasare, considerând modelul liniar echivalent. Sunt evidenţiate limitele de aplicare ale acestor modele, în baza comparaţiilor existente în literatură. Acest studiu face parte dintr-o cercetare în curs de desfăşurare, efectuată de către autor, pentru determinarea parametrilor modelului elastic echivalent, capabil să prezică deplasarea inelastică maximă pentru cutremure compatibile cu un anumit spectru de răspuns.