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MACROSCOPIC SPEED-FLOW MODELS FOR CHARACTERIZATION OF FREEWAY AND MANAGED LANES

BY

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Abstract. As part of an effort to estimate operating speeds on freeway managed lanes as a function of predicted demands, speed-density models were calibrated using data at a freeway site. Nine different speed-density models, including four conventional models (Greenshields, Greenberg, Underwood, and Drake) as well as five modifications of these models were calibrated. The conventional Drake model proved to be the best fit model with reasonable estimates of free flow speed and speed and density at capacity for freeway conditions, followed by a Taylor Series expansion version of the Underwood model.

Key words: traffic flow models; freeway managed lanes; speed density; macroscopic freeway models.

1. Background and Study Objective

The classical speed-density-flow macroscopic models were generally developed based on data from two-lane roads and were not considered to be applicable to characterizing freeway flow conditions, where lane changing is allowed. As such, attempts were made to model freeway flow and lane changing using fairly complex and difficult to calibrate models (Prigogine & Herman, 1971; Rorbech, 1976; Hurdle & Datta, 1976; Dillon & Hall, 1987;

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Kyte *et al.*, 1989; Daganzo, 1999). With the advent of demand management strategies such as congestion pricing and managed lanes, freeway operators are increasingly in need of macroscopic freeway models that could accurately estimate speed conditions as a function of anticipated demand. In managed lane operations, for example, toll amounts could be varied at different times of day (Federal Highway Administration, 2008; Yin & Lou, 2009). However, facility operators may be obligated to guarantee a minimum operating speed on the managed lanes for the toll amount charged or provide a refund (NorthCentral Texas…, 2007). Therefore, once a toll amount is set, econometric models based on consumer sensitivity to price are used to predict the demand on managed lanes (Li & Govind, 2002). The predicted demand must then be used as input to macroscopic models to predict prevailing speeds. The goal of this study has been to identify classical speed-flow-density models or modified versions of them that are best suited to freeway flow conditions and may therefore be used in managed lane applications.

Four conventional traffic flow models (Greenshields, Greenberg, Underwood, and Drake) as well five proposed modifications of these models are calibrated for data from a freeway site in Dallas, Texas. Among the nine models analysed two are found to have the accuracy and consistency across all practical ranges of demand to be suitable for modeling freeway flow.

A brief description of each of the four classical models analysed is presented in what follows.

1.1. The Greenshields Model

The model was proposed by Greenshields in 1935 as a linear model to analyse the relationship between speed, flow and density. The model is simple and satisfies all boundary conditions, (u = 0 at $k = k_j$ and $u = u_f$ at k = 0), but the goodness of fit is generally not high, particularly for freeway data. The Greenshields formulation is as follows:

$$u = u_f \left(1 - \frac{k}{k_j}\right), \tag{1}$$

where: u is the speed, k – density, u_f – free-flow speed, k_j – jam density.

1.2. The Greenberg Model

Proposed by Greenberg in 1959, the model uses a fluid flow analogy and data from the Lincoln Tunnel in New York to establish a logarithmic relation between speed and density, namely

$$u = u_c \ln \frac{k_j}{k}, \tag{2}$$

where u_c is the speed at capacity.

The model does not satisfy the boundary condition at the low concentration regime ($u \to \infty$ at $k = 0$) but behaves well under congested conditions ($u = 0$ at $k = k_j$).

1.3. The Underwood Model

Developed by Underwood in 1961, the model hypothesizes an exponential relationship between density and speed. The model is observed to generally have a better fit than the Greenshields and Greenberg models for the uncongested traffic conditions, but does not present a good fit to the data for congested conditions. The Underwood model is given as

$$u = u_f \mathrm{e}^{-k/k_c}, \tag{3}$$

where k_c is the density at capacity.

1.4. The Drake Model (Bell-Shaped Curve Model)

This model was developed by Drake in 1961. He studied the various macroscopic traffic models postulated at that time and did not find any of them statistically significant. In developing his model, he estimated the density from speed and flow data, fitted the speed *vs.* density function and transformed the speed *vs.* density function to a speed *vs.* flow function. His model generally yields a better fit than the above three models for uncongested conditions. However, as in the Underwood case, it is not a good fit for congested conditions. The formulation of the Drake model is as follows:

$$u = u_f \exp\left[-\frac{1}{2\left(k/k_c\right)^2}\right]. \tag{4}$$

1.5. The Modified Greenberg Model

Considering that even under very light traffic conditions there are always some vehicles on the freeway, the modified Greenberg model introduces a non-zero average minimum density, k_0, in the Greenberg model (Ardekani & Ghandehari, 2008). The modified Greenberg model formulation is

$$q = u_c k \ln\frac{k_j + k_0}{k + k_0}, \tag{5}$$

where: k_0 is the average minimum density, u_c – speed at capacity.

Unlike the classic Greenberg model, the modified version yields a finite free flow speed of $u_f = u_c \ln(1 + k_j/k_0)$ when density approaches zero. It is also worth to note that solving for $\mathrm{d}q/\mathrm{d}k = 0$ yields a density at capacity of $k_c \approx 0.4k_j$, a value close to that of the classical Greenberg model, namely $k_c \approx 0.368k_j$.

1.6. The Underwood Model with Taylor Series Expansion

The Underwood model does not yield a solution for the jam density when speed approaches zero. But the exponential function can be expanded in a Taylor series obtaining a numerical approximation for the jam density

$$u = u_f \mathrm{e}^{-k/k_c} = u_f \left(1 - \frac{k}{k_c} + \frac{k^2}{2k_c^2} - \frac{k^3}{6k_c^3} + \frac{k^4}{24k_c^4} - \frac{k^5}{120k_c^5} + \ldots\right).$$

Taking up the expansion to the term containing k^3 yields

$$u = u_f \mathrm{e}^{-k/k_c} = u_f \left(1 - \frac{k}{k_c} + \frac{k^2}{2k_c^2} - \frac{k^3}{6k_c^3}\right). \tag{6}$$

For $u = 0$, the solution of eq. (6) gives an estimate for the jam density, k_j.

1.7. The Polynomial and Quadratic Models

We can also express the relationship between density and speed in terms of a second degree polynomial equation namely

$$u = u_f + bk + ck^2, \tag{7}$$

where b and c are additional model parameters.

Alternatively, the speed *vs.* density relationship may be expressed as a quadratic equation of the form

$$u = u_f \left(1 - \frac{k^2}{k_j^2}\right). \tag{8}$$

1.8.The Drake Model with Taylor Series Expansion

As in the case of the Underwood, the Drake model also does not yield a solution for the jam density when speed approaches zero. Hence, we can use the Taylor series expansion to obtain a numerical approximation for the jam density, as follows:

$$u = u_f \left(1 - \frac{k^2}{2k_c^2} + \frac{k^4}{8k_c^4} - \frac{k^6}{48k_c^6} \right). \quad (9)$$

Again, at $u = 0$ the solution of eq. (9) would yield an estimate for the jam density, k_j.

2. Data Description and Model Calibration

Data from a freeway site in Dallas, Texas,are used to calibrate the above models and determine which models in their existing or modified form best fit the data. Special attention is paid to models which provide a strong fit to the data and yield reasonable estimates of free-flow speed, capacity, and jam concentration for freeway operations.

A five minute volume and speed loop detector data collected in the utmost left lane of the southbound Loop 12 at Irving Blvd in Dallas, Texas, are used for model calibration purposes. The data were collected on April 19th, 2004,by the Texas Department of Transportation.

Table 1

Comparison of Traffic Model Parameters and R^2 Values for Southbound Loop 12 Freeway at Irving Blvd in Dallas, Texas

Model	Equation	Calibration	u_f mph	k_j vpmpl	u_c mph	k_c vpmpl	Adj. R^2
Greenshields	$u = u_f\left(1 - k/k_j\right)$	$u = 62.8(1 - k/120.8)$	62.8	120.8	31.4	60.5	0.86
Conventional Greenberg	$u = u_c \ln\left(k_j/k\right)$	$u = 8.83\ln\left(4,461/k\right)$	–	4,461	8.83	1,646	0.43
Modified Greenberg	$u = u_c \ln\dfrac{k_j + k_0}{k + k_0}$	$u = 14.3\ln\left[759/(k+5)\right]$	71.8	754	14.3	274	0.55
Conventional Underwood	$u = u_f e^{-k/k_c}$	$u = 72.4e^{-k/58.2}$	72.4	–	26.6	58.2	0.85
Underwood with Taylor series	$u = u_f(1 - k/k_c + k^2/2k_c^2 - k^3/6k_c^3)$	$u = 52.7(1 - k/34.2 + k^2/2,339.2 - k^3/240,010.9)$	52.7	54.5	17.5	34.2	0.96
Polynomial	$u = a + bk + ck^2$	$u = 58.1 - 0.15k - 0.0041k^2$	58.1	102	35.9	57.6	0.91
Quadratic	$u = u_f\left(1 - k^2/k_j^2\right)$	$u = 56(1 - k^2/10,201)$	56	101	37.4	58.3	0.90
Conventional Drake	$u = u_f \exp\left[\dfrac{1}{2(k/k_c)^2}\right]$	$u = 58.2\exp(-k^2/5,000)$	58.2	–	35.3	50	0.96
Drake with Taylor series	$u = u_f(1 - k^2/2k_c^2 + k^4/8k_c^4 - k^6/48k_c^6)$	$u = 57.8[1 - k^2/6,340 + k^4/(80.4 \times 10^6) - k^6/(152.85 \times 10^{10})]$	57.8	100.5	34.9	56.4	0.95

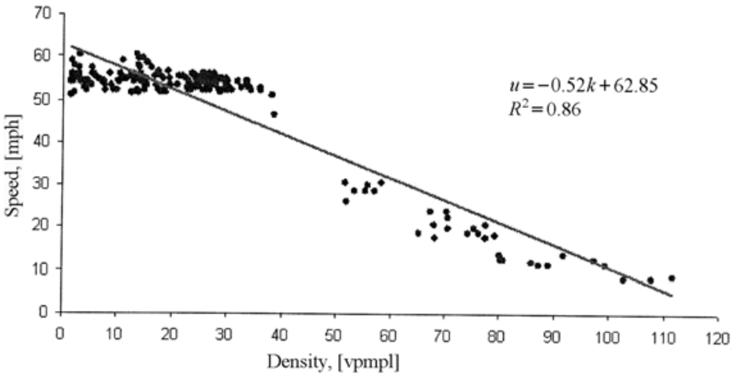
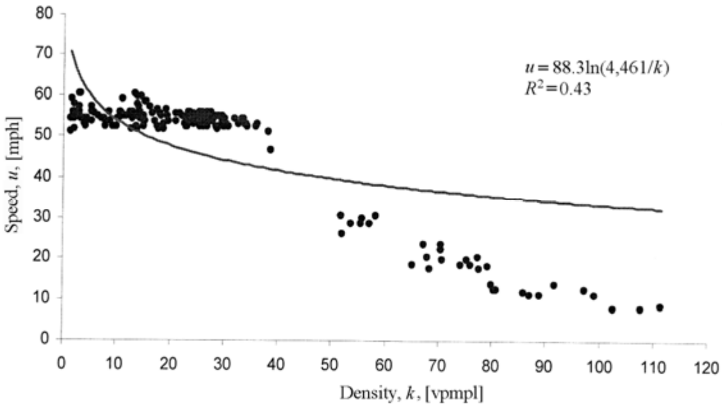


Fig. 1 – The Greenshields model fit.



a

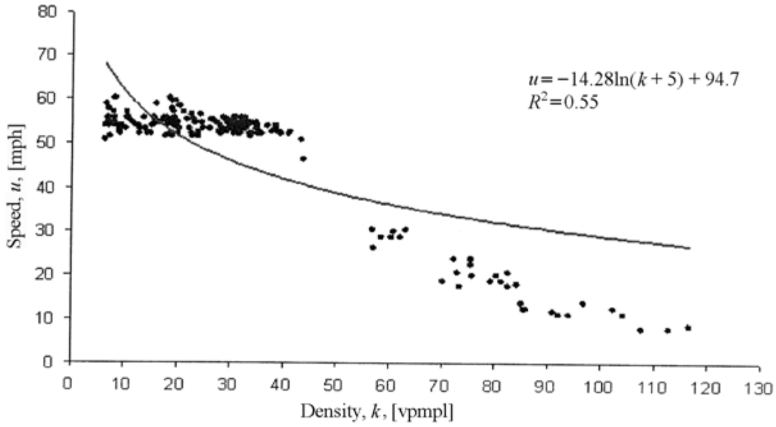


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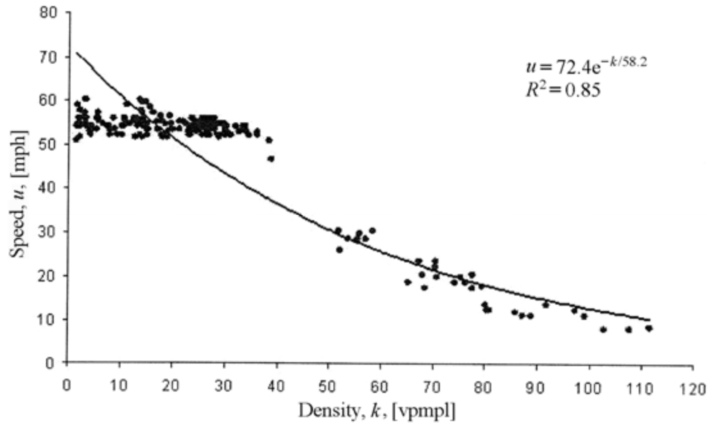
Fig. 2 – The Greenberg model fit: *a* – the conventional Greenberg model fit; *b* – the modified Greenberg model fit.



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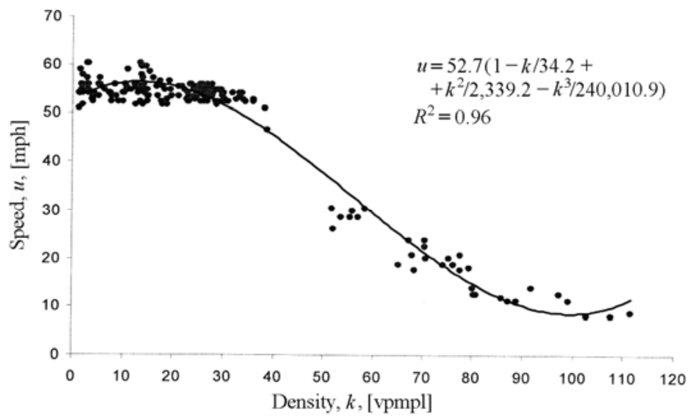

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Fig. 3 – The Underwood model fit: *a* – the conventional Underwood model fit; *b* – the Underwood model with Taylor series expansion fit.

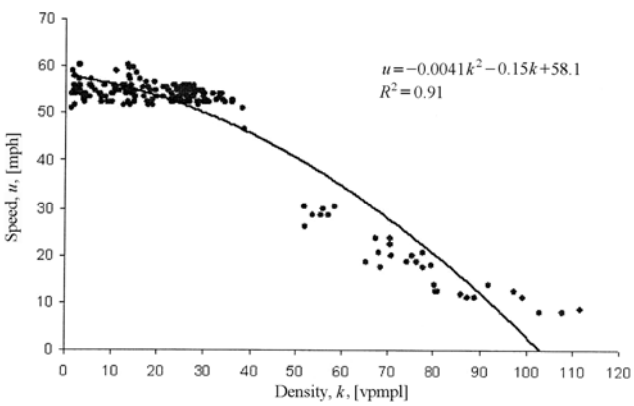


Fig. 4 – The two-degree polynomial fit.

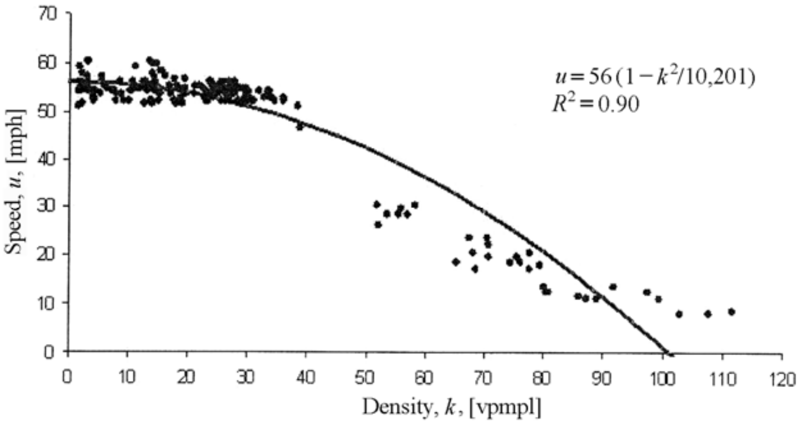
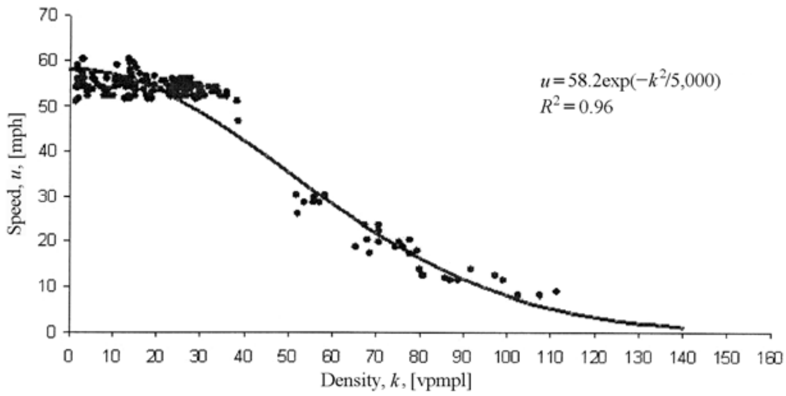


Fig. 5 – The quadratic model fit.



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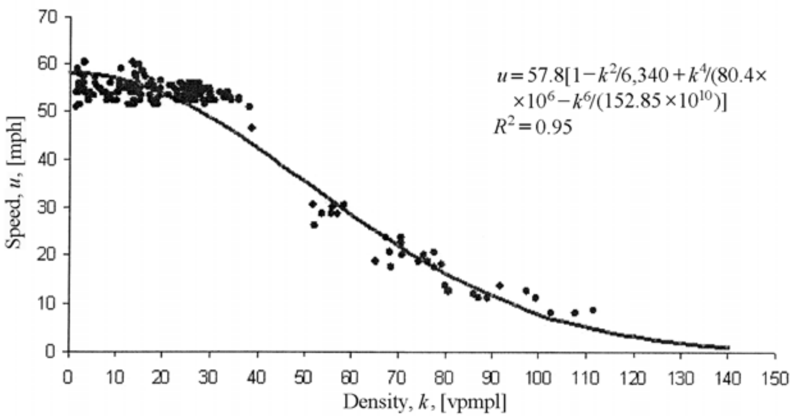

b

Fig. 6 – The Drake model fit: *a* – the conventional Drake model fit;
b – the Drake model with Taylor series expansions fit.

The calibration results from these nine models, summarized in Table 1, are compared in terms of values of free-flow speed (u_f), jam density (k_j), speed at capacity (u_c), density at capacity (k_c) and the goodness of fit (R^2). The above mentioned models are plotted in Figs. 1,…,6. As shown, the Underwood model with Taylor series expansion (Fig. 3 *b*) and the conventional Drake model (Fig. 6 *b*), both with R^2 = 0.96, yield the best fits to the data.

Given the above, the use of the conventional Drake model can be recommended as a macroscopic *q*-*k*-*u* model for freeway flow. The Drake model not only consistently results in the highest observed R^2 among the nine models for both study sites, it also yields reasonable values of free flow speed and speed and density at capacity. It is also more mathematically tractable than its closest competition, the Taylor series expansion of the Underwood model.

3. Conclusions and Recommendations

The Greenshields, Greenberg, Underwood and Drake models were calibrated for a freeway site in Dallas, Texas. Similarly, the modified Underwood model with Taylor series expansion, a modified Greenberg model, a polynomial model, a quadratic model and the Drake model with Taylor series expansion were also calibrated. The Drake model and the Underwood model with Taylor series expansion gave the best fit with an R^2 = 0.96 in both cases. Furthermore, estimates of the free-flow speed and speed and density for both models are at reasonable values for typical freeway sections. The modified Greenberg model also gives a realistic free-flow speed but the jam density is estimated to be too high and the density at capacity too low. The polynomial and quadratic models give realistic values for the free-flow speed and jam density, but the R^2 values are fairly low.

Based on these results, the Drake model is adopted for use in a software program developed for estimating managed lane speeds as a function of toll amounts and the resulting managed lane demand (Sinprasertkool *et al.*, 2009).

It should be emphasized that the analyses in this study have been based on the data regarding the utmost left lane of the freeway site since managed lanes are typically the utmost left lanes. However, calibration parameters for other lanes of the same freeway sections may yield different values. It has been observed, for example, that the right lanes of a freeway section generally have lower free-flow speeds (Uddin & Ardekani, 2002). Speeds may also be generally lower in proximity to ramp junctions, resulting in different parameter values. Hence, in using macroscopic models for freeways, the models may need to be calibrated separately for the inside *versus* outside lanes. Care must also be taken so that the data used for calibration are not affected by factors such as proximity to ramp junctions, weaving sections, or other such geometric factors or anomalies.

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MODELE MACROSCOPICE ALE VITEZEI DE CURGERE PENTRU CARACTERIZAREA CIRCULAȚIEI PE BENZILE LIBERE ŞI DIRIJATE

(Rezumat)

Ca parte a unui efort de a estima vitezele de operare ale benzilor dirijate ca funcție de densitatea de trafic presupusă, un număr de modele matematice sunt calibrate folosind date concrete provenite din operarea unei autostrăzi. Mai precis, un număr de nouă modele diferite descriind densitatea de trafic ca funcție de viteză, incluzând patru modele convenționale (Greenshields, Greenberg, Underwood şi Drake) şi cinci modificări ale acestora sunt calibrate folosind aceste date. Din punctul de vedere al estimării vitezei de circulație liberă, vitezei şi densității la capacitatea maximă de operare, modelul Drake convențional este observat a produce cele mai bune aproximări, urmat de o versiune a modelului Underwood ce utilizează aproximări de tip serie Taylor.