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NUMERICAL METHODS FOR DETERMINING THE DYNAMIC BUCKLING CRITICAL LOAD OF THIN SHELLS STATE OF THE ART

BY

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Abstract. The problem of dynamic stability is substantially more complex than the buckling analysis of a shell subjected to static loads. The fundamental aim of this paper is to present criteria for determining the critical load of dynamic buckling of thin shell. Another purpose of establishing such criteria is to guide engineers scientists and researchers dealing with such problems, for a better comparison verification and a validation of their experimental or numerical results. To illustrate the application of these criteria, two examples have been studied.

Key words: dynamic instability; critical buckling load; thin shells; numerical methods.

1. Introduction

Buckling is a phenomenon of instability which can be observed specifically for thin slender structures of low bending stiffness subjected to compressive stress, beyond a certain value. The applied load leads to a significant change in the shape of the structure that results in a gradual or sudden onset of wrinkles or ripples.

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Thin shells are often subjected to structural instabilities more or less "catastrophic". Thus, the design of shell structures requires at first to have an understanding of these instabilities.

2. Dynamic Instability

The area of “dynamic” instability of structures has been the subject of numerous research studies over the past 40 years (Sahu & Datta, 2007). Similarly to the static analysis of thin shells (Touati & Barros, 2008) and the analysis of non-linearities of the problem (Moussaoui & Benamar, 2002), the choice of stability criteria is an essential element. In 1788 Lagrange proposed a criteria called *energy criterion*, where the minimum potential energy was refered to be a stable equilibrium. If this energy is a maximum then the equilibrium is considered as being unstable. This criteria has been rejected but was extended afterwards by Dirichlet in 1846 to cases of bounded disturbances. This requirement does not apply to cases such as: dynamic loads, non-conservative loads or to large displacements. Notice that the term dynamic stability (Simitses, 1987) encompasses a large number of common cases for structural thin shells under impacts, periodical loads or non-periodical earthquakes (to name a few).

2.1. Parametric Resonance

In the case of a simple (mass-spring) oscillator, resonance occurs when the structure is excited with a harmonic force with the natural frequency. A force excitation collinear with the motion of the mass, causes the oscillation of the mass with a continuous increase in displacements. The parametric resonance is a similar phenomenon but the difference is that the force exciter causes resonance in a second vibration mode or in another mode with natural frequency rather different from that directly excited by the exciting force. The first observation of parametric resonance is attributed to Faraday in 1831.

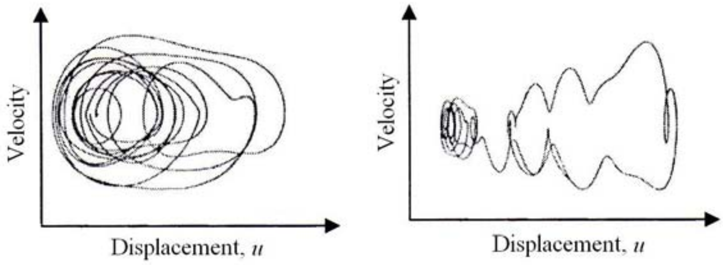
When the load excitation is periodic, the problem reduces to a system of equations of the Mathieu-Hill type as shown by Budiansky (1965). The resolution of the problem determines the areas of instability according to the theorem of Mathieu-Hill. Significant studies were conducted on the parametric resonance of cylindrical thin shells by Yao (1963, 1965).

2.2. Dynamic Buckling (Dynamic Fast Loading)

A second type of dynamic instability refers to structures with post-critical unstable behavior, of the type snap-trough, subjected to very fast time-varying loads (explosions, crashes, ...). The dynamic instability appears when a slight disturbance arises on a structure under dynamic fast loading. The latter initiates a major deformation shift from the original undisturbed position. This phenomenon is characterized by: a) a finite jump from an initial stable state to a final stable state; or b) by an unstable infinite jump. Several approaches can be used to calculate the instability conditions for this type of problem (Huyan, 1996).

a) ***Total Energy-Phase Plane Approach*** (Hoff & Bruce, 1954)

The phase-plane is drawn in a displacement–velocity coordinate system (u, $\dot{u}$) as shown in Fig. 1. When the loading parameters are small, the stable movements describing closed paths are limited and focused around the solution of a static equilibrium (Fig. 1 *a*). When the loading factor increases, a value is reached at which a movement of the structure gets away from the pole without any oscillation around him. In this case (Fig. 1 *b*) the structural system is in a condition of instability to which corresponds a critical load value.



a *b*

Fig. 1 – Phase plane: *a* – stable motion; *b* – unstable motion.

b) ***Total Potential Energy Approach*** (Simitses, 1965)

The process of transfer of potential energy into kinetic energy reflects the phenomenon of instability of an elastic system. When the value of the critical load factor is reached then a steady state, represented by a local minimum point of the curve of the potential energy, changes position and moves to another local minimum along the same curve of the potential energy. This movement of the steady state representative point releases potential energy accompanied by large deformations in bending, and the potential energy released is transformed into kinetic energy, which in turn accelerates the preceding deflections.

The Fig. 2 *a* shows the qualitative variation of the potential energy function with respect to degree of freedom, u_p , and the static equilibrium is represented by the point *B* of the energy curve. If the applied load factor, λ, is less than the dynamic buckling load factor, λ_{cr}, then the structural movement oscillates harmonically around the point *O*. Thus, an oscillation measure is represented by the distance *AC*; the maximum amplitude is represented by the distance *OC*; the dynamic movement occurs between the two limits, *A* and *C*. This latest corresponds to a dynamic stability situation.

The complementary figure (Fig. 2 *b*) shows the qualitative variation of the potential energy function when the applied load factor, λ, tends towards the dynamic buckling load factor, λ_{cr}. In fact when the movement oscillates around the point *O*, at point *C* the total energy at this level corresponds to the buckling load factor of the dynamic movement. In this situation the movement escapes from point *O* to point *C* and becomes uncontrollable (since the representative point *C* no longer corresponds to a local minimum) and an unlimited dynamic instability occurs by buckling.

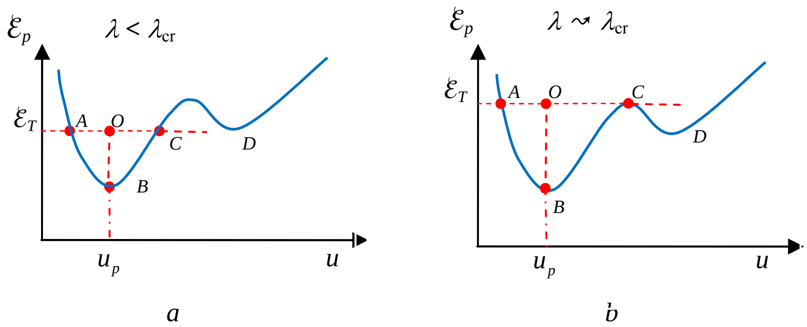


Fig. 2 – Variation of potential energy as a function of some displacement variable: *a* – stable dynamic movement; *b* – unstable dynamic movement

The curve of the displacement response over time, for each value of the applied load, λ ($\lambda < \lambda_{cr}$), oscillates harmonically around the static equilibrium position of maximum amplitude *OC* as shown in Fig. 2 *a*. The offset of the

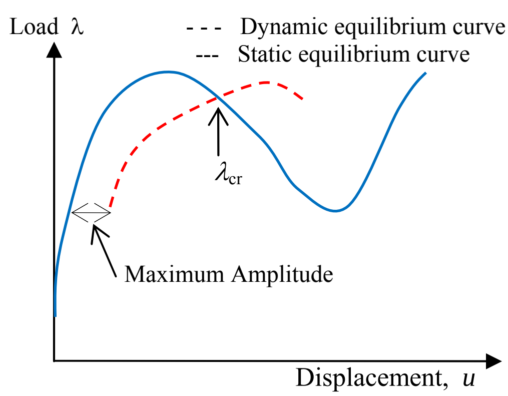


Fig. 3 – Critical load factor at the intersection of the two curves of static and dynamic equilibrium.

static equilibrium curve for a distance equal to the corresponding maximum amplitude *OC*, for every load factor, λ, gives the curve of dynamic equilibrium. The intersection of these two curves plotted simultaneously, determines the value of the dynamic buckling load factor, λ_{cr}, for the maximum vibration amplitude *OC*, as shown in Fig. 3.

c) ***Equations of Motion Approach*** (Budiansky & Roth (1962))

This criteria is applied with the knowledge of the response calculated for different loading parameters from the numerical solution of the motion eqs. By drawing the curve of some selected displacement *vs*. time, while varying the intensity of the applied load, a jump of the curve is found from the curves drawn for neighbouring values. Under the criteria of Budiansky & Roth (1962), a particular value of the load causing this remarkable leap corresponds to the critical value of dynamic buckling, as shown in Fig. 4 *a*.

A refinement of this criteria was proposed by Ari Gur & Simonetta (1997) to be applied in the case of existence of smaller peaks on deflections curves due to pulses of very short duration with very high intensities. In this case the dynamic instability corresponds to a reduction in the peak of the maximum deflection caused by a slight increase in load intensity as shown in Fig. 4 *b*.

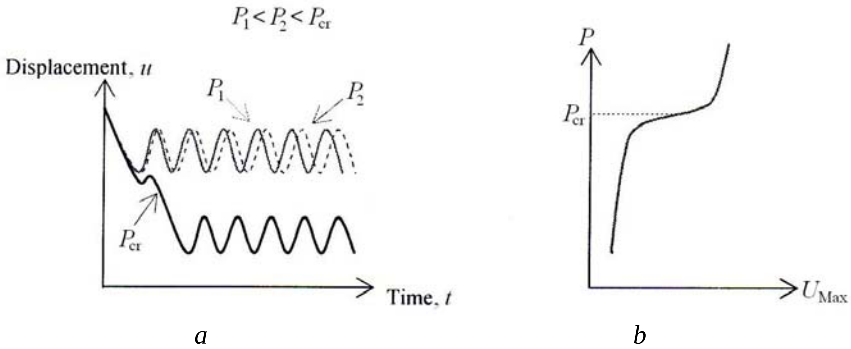


Fig. 4 – Critical load: *a* – criteria of Budiansky & Roth;
b – criteria of Ari Gur & Simonetta.

d) ***Criteria of two Dynamic Curves*** (Chamis & Abumeri (2005))

The principle of this criteria is based purely on geometry curves which rely on updating values after each step of time (Δt) by solving the eq. of dynamic equilibrium. Then the critical load is determined from a solution among obtained eigenvalues, that generally look like the ones given in Fig. 5.

This figure shows that for every value of positive dynamic load there is a buckling load whose variation decreases monotonically, until it reaches a value for which occurs an increase of the dynamical load in over time at almost constant buckling load.

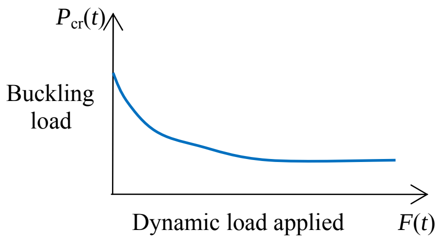


Fig. 5 – Buckling load *vs*. dynamic load.

By drawing on the same benchmark two curves – the curve of the buckling load *vs*. time, $P_{cr}(t)$, and the line of the dynamic load increase applied over time, $F(t)$ – it is possible to obtain the dynamic buckling by the intersection of these two curves, as shown in Fig. 6.

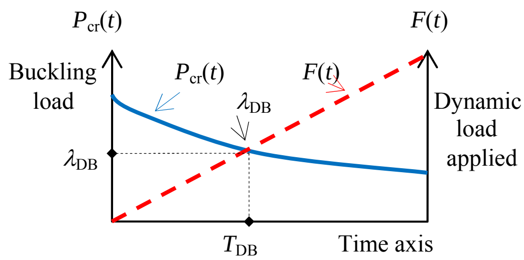


Fig. 6 – Critical load determined by the geometric process.

e) ***Pseudo-Dynamic Method***

Consider an unperturbed motion, x, and a neighbourhood disrupted movement, x_v, and also $x^* = x - x_v$. The dynamic eq. of free motion is

$$M\ddot{x}^* + C\dot{x}^* + K_T x^* = 0. \tag{1}$$

Eq. (1) can be written in matrix form as

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{Bmatrix} \dot{x}^* \\ x^* \end{Bmatrix} = \begin{bmatrix} -M^{-1}C & -M^{-1}K_T \\ I & 0 \end{bmatrix}\begin{Bmatrix} \dot{x}^* \\ x^* \end{Bmatrix}, \tag{2}$$

or

$$\frac{\mathrm{d}}{\mathrm{d}t}\left\{X^*\right\} = \left[A\right]\left\{X^*\right\}. \tag{3}$$

The form of eq. (3) represents the first approximation of Lyapunov (Aggarwal, 1972; Nguyen, 2000) for the phenomenon of stability. The solutions of eq. (3) may be given following an exponential form as

$$X^*(t) = X_0^* \mathrm{e}^{\omega_r t}. \tag{4}$$

The system stops vibrating and becomes undefined when $\Re e(\omega) = 0$, where $\omega = 0$. This transition is characterized by the singularity of a tangent stiffness matrix, K_T, that is

$$\omega = 0 \text{ implies } \det[K_T] = 0. \tag{5}$$

The load factor, λ, is the ratio of the critical load to the applied load, given by $\lambda = P_{\mathrm{cr}} / P$. In the neighbourhood $\lambda \cong 1, P$ leads to λP, and

$$K_e + K_g \text{ becomes } K_e + \lambda K_g. \tag{6}$$

The characteristic eq. become

$$\left[K_e + \lambda K_g - \omega^2 M\right]\left\{x\right\} = 0. \tag{7}$$

In the critical case, $P_{\mathrm{cr}}=\lambda P$, where the load factor assumes a critical value, λ_{cr}, and $\omega = 0$. The above equation takes now the form

$$\left[K_e + \lambda K_g\right]\left\{x\right\} = 0, \tag{8}$$

which represents an eigenvalue-eigenvectors problem (λ , x).

The flowchart in Fig. 7 shows a possible fluxogram to be followed for solving problems of dynamic buckling by the pseudo-dynamic method.

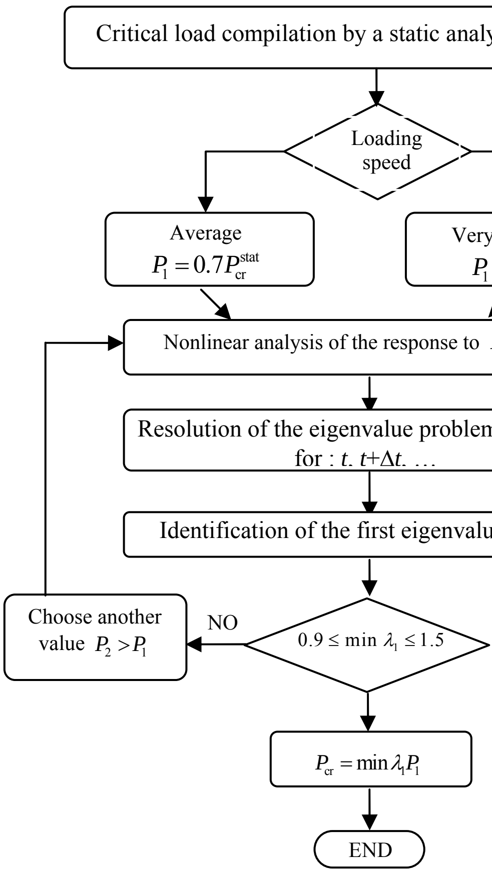


Fig. 7 – Schematic application of the pseudo-dynamic method.

3. Applications

3.1. Historical Overview (Sahu & Datta, 2007)

One of the first contributions to the dynamic buckling analysis using numerical methods is due to Ari-Gur *et al*. (1982). It is an investigation on the behaviour of columns subjected to impacts. The computational part of the work uses the finite difference method.

The application of the finite element method in the analysis of dynamic buckling of shells began in the 1970's. Ari-Gur and Elishakoff (1997) and Yaffe and Abramovich (2003) have undertaken the study of a series of experimental works, starting from simple structures such as columns before addressing more complex structures such as stiffened shells.

Nakagawa *et al*. (1995) studied experimentally the behavior of thin cylindrical shells under seismic biaxial loading.

Ren *et al*. (1983) conducted an experiment on the influence of impact velocities on the dynamic buckling of cylindrical shell, by testing an aluminum cylinder, built-in at one end and impacted by a mass of 54 g at the free end. The cylinder deforms axially when impacted by the mass falling from different heights. The free end that receives the shock is reinforced by a block of variable mass depending on the height of impact. It was verified that the buckling mode changed when the impact velocity increased. Before reaching a critical speed, the cylindrical hull deformed with a sinusoidal axial symmetry and the total strain remained generally small; consequently the shell retained its sinusoidal shape. But when exceeding the critical speed, the cylinder bends quite sharply and the non-uniform buckling shape loses its axial symmetry with the onset of large deformations. Since the meaningless sinusoidal shape of the tested cylinders only occurs for small strains and small deformations, the authors concluded that the loss of the cylinder's bearing capacity defines such state as being the critical state of stability.

Michel *et al*. (2000) designed an experiment to study the dynamic buckling under shear loading. This work has been validated numerically, particularly for the thin and thick vessels constructed in nuclear engineering.

The first numerical studies of dynamic buckling dealt with simple structures such as rods and beams. Clough & Wilson (1971) addressed the case of thin shells in a broader context of nonlinear dynamics.

More recently Karagiozova & Jones (2000, 2001, 2002, 2004) have examined the dynamic elastic buckling of elastoplastic cylindrical shells using a discrete method called "Backward Differentiation Formula (BDF)". The results were also compared with those theoretically and experimentally obtained by Lindberg & Florence (1987). Their work also revealed a strong dependence of the characteristics of buckling on the dynamic load speeds.

Simitses (1983) showed the effect of a static preload on the critical force. The effect of certain phenomena on the dynamic buckling were also investigated, but since the results were still disparate Petry & Fahlbush (2000) and others researchers have studied the effect of imperfections for in-plane impacts.

3.2. A Cylindrical Roof under a Concentrated Load

Consider the example of a cylindrical roof subjected to a concentrated load in his center as shown in Fig. 8. The variables of this example – radius of the cylinder, thickness of the roof, elasticity modulus, Poisson ratio and mass density – were specified as $R = 10L = 2{,}540$ mm, $t = 6.35$ mm, $E = 3.1029$ GPa, $\nu = 0.3$ and $\rho = 7{,}800$ kg/m^3, respectively.

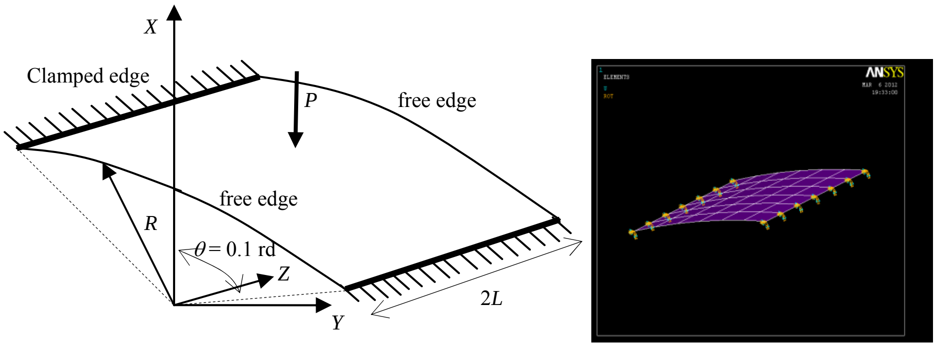


Fig. 8 – Example of a cylindrical roof under concentrated load.

This example has been studied by many known international researchers and engineers using a static analysis (Touati & Barros, 2008), where no application has been addressed for finding the critical load of dynamic bifurcation. But recent studies on the dynamic buckling of shells have been conducted by Djermane (2007) and published by Djermane & Chelghoum (2008) using a specific numerical approach.

The variation of transversal displacement of the center of the roof *versus* time is shown in Fig. 9, under the application of concentrated forces very close to the dynamic buckling load. It can be noticed that the displacement under the load increases slightly until a load value of P = 0.485 kN beyond which corresponds a more significant oscillation around the position of static equilibrium.

For the phase-plane representation from Fig. 10, the onset of bifurcation towards a dynamic equilibrium position is recorded for the value of P = = 0.486 kN to which corresponds a quite significant increase of the post-critical movement, as visually seen by the magnitudes of the displacements and velocities for neighbouring loads immediately adjacent to the dynamic buckling load (Figs. 10 *a* and 10 *b*). The critical value of the dynamic bifurcation load is then taken to be P_{cr} = 0.486 kN.

It is clearly seen that for P = 0.485 kN the trajectory of the motion is stable around the static equilibrium position (Fig. 10 *a*), but when the load, P, exceeds the value of 0.486 kN the trajectory (still stable in this example) makes more significant oscillations around the previous equilibrium position before launching into another stable post-critical equilibrium position (a focal central point, of saddle type) nevertheless of much higher amplitudes and velocities as shown in Fig. 10 *b*.

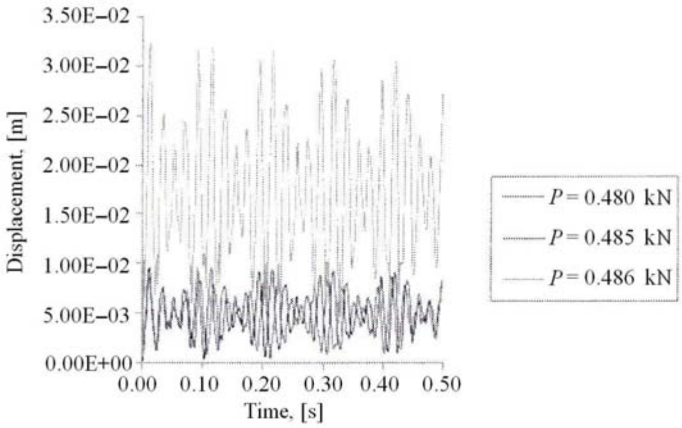


Fig. 9 – Dynamic response of the center of the cylindrical roof.

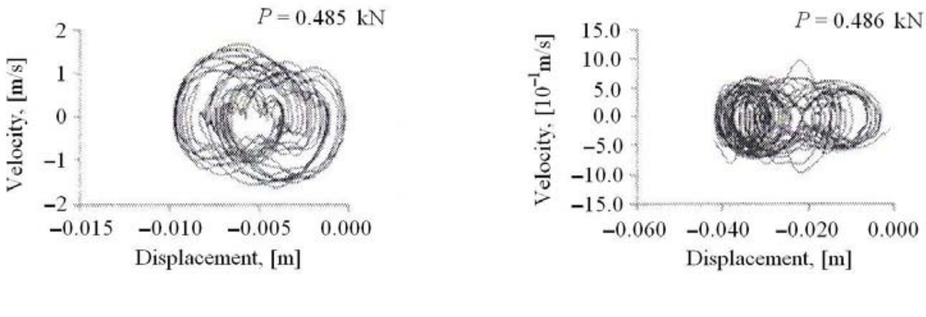


Fig. 10 – Phase-plane: *a* – stable pre-buckling motion; *b* – stable post-buckling motion.

3.3. A Cylindrical Tank under Horizontal Load

The cylindrical tank structure investigated in this example has a clamped condition at the base and a free open top (Fig. 11). The dynamic load model established for the dynamic buckling analysis is the suddenly applied horizontal load on the top of tank, as a finite duration impulsive loading.

For comparison with computational results associated with the parametric study of anchored metallic circular tank shells, designed under various seismic codes (Barros, 2007, 2008, 2010), the geometries of the tank used herein are shown in Fig. 11. Maintaining the meaning of the variables already used in the previous example of section **3.2**: R = 6 m, H = 10 m, t = = 3.125 mm, E = 2.10 GPa, ν = 0.3 and ρ = 7.7 kN/m^3.

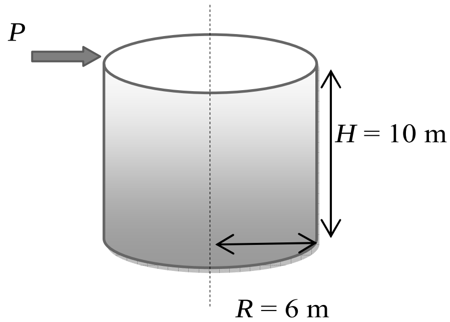


Fig. 11 – Example of the cylindrical tank under a concentrated load at the top.

Fig. 12 illustrates the results obtained for the dynamic buckling of the tank for aspect ratio H/R = 5/3 and for the stiffness ratio R/t = 1,920, as used earlier by Barros (2007, 2008, 2010) in the parametric study of the seismic response of circular metallic tanks. Until P = 994 kN, the radial horizontal displacement of the top of the tank, obtained by transient solution, oscillates around a fixed value. When the force P reaches and slightly exceeds the value

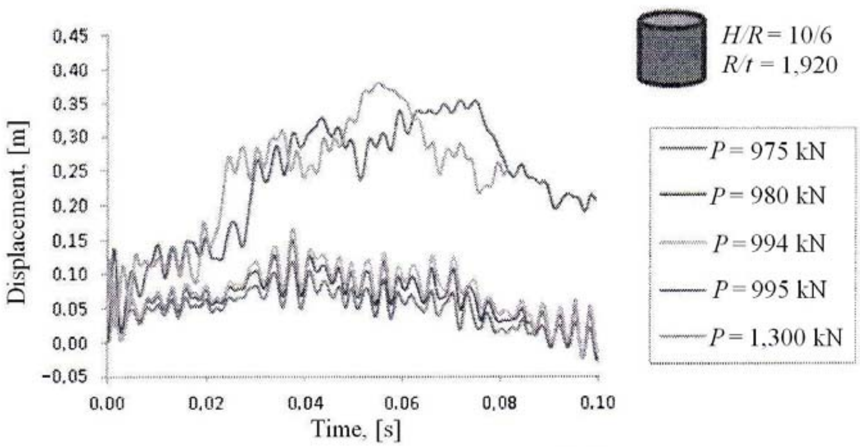
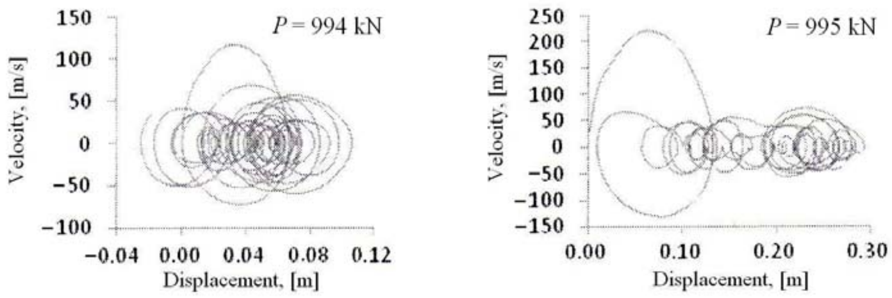


Fig. 12 – Dynamic response ot the radial displacement at the top of tank.

995 kN, then, beyond such threshold load value, the displacement undergoes a sudden increase in magnitude (a jump). This case corresponds to the occurrence of a critical value for the dynamic buckling load, as it is emphasized by the change in the phase-plane diagrams from stable (Fig. 13 *a*) into unstable (Fig. 13 *b*) situations.



a *b*

Fig. 13 – Phase-plane: *a* – stable motion; *b* – unstable motion.

4. Conclusions

The criteria of the phase-plane–total potential energy approach, as well as of Budiansky & Roth, are mainly used by the authors in the investigation of critical conditions of dynamically loaded structures.

Use of the total energy criteria and phase plane is very simple, however, it requires laborious work in solving equilibrium eqs. for different values of the loading parameter. Although this leads to a large computational time, the usefulness and efficiency of these criteria and methodologies in verifying results is quite considerable when compared with the direct way solution by other approaches.

The Budiansky & Roth criteria is the most used in practice, either in determining the critical value of the excitation or in determining the magnitude and nature of displacement. It is also based on robust and sound mathematics, even in the neighbourhood of dynamic buckling. On the other hand, it requires hard work by calculating a transient response for different nonlinear loads, with very high computational time.

Concerning the use of the criteria of total potential energy, one must be careful to the possible occurrence of parametric resonance. The corresponding studies indicate that a structure subjected to dynamic loadings can not produce other unstable conditions of displacement, but precisely should induce more displacements corresponding to the primary exciting function.

Viscous damping affects dynamical characteristics of buckling. The effect of response decay on a dynamic system is an essential process of energy dissipation. It is obvious that the occurrence of viscous damping always increases dynamic buckling loads.

For a better modelization of dynamic buckling phenomenon, it is important to have reliable criteria of dynamic stability and the pseudo-dynamic tests should meet these requirements. It is worth noting also that the use of conventional criteria of dynamic buckling is sensitive and risky for the cyclic nature of seismic loading. Such use may even give misleading results in overestimating the critical level of seismic excitation.

It is important to notice that if the static buckling of structures seems nowadays to be well understood, the dynamic buckling remains misunderstood and it continues to attract a lot of questions and confusions for a great number of researchers. Besides, the misunderstanding of the origin and of the possible aspects of such structural phenomenon gives rise to divergent opinions. On the other hand, dark areas of research still exist; namely: the interaction between the modes of vibration and the buckling modes, and the importance of decoupling the phenomenon of dynamic buckling and parametric resonance, for determining the critical load for dynamic buckling with improved efficiency of practical criteria available.

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METODE NUMERICE PENTRU DETERMINAREA SARCINII DINAMICE CRITICE DE FLAMBAJ A PLĂCILOR SUBȚIRI
Stadiul actual al cunoaşterii

(Rezumat)

Problema stabilității dinamice este considerabil mai complexă decât analiza flambajului plăcilor subțiri supuse la încărcări statice. Scopul acestei lucrări este de a prezenta nu numai criteriile de bază pentru determinarea sarcinii dinamice critice de flambaj a plăcilor subțiri, dar şi a criteriilor de verificare a rezultatelor. Un alt obiectiv este cel de a îndruma inginerii, oamenii de ştiință şi cercetătorii care se ocupă cu astfel de probleme, spre verificarea şi validarea rezultatelor lor experimentale sau numerice. Pentru a ilustra aplicarea acestor criterii, au fost studiate două exemple.