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# PRACTICAL OPTIMIZATION OF COMPOSITE STEEL AND CONCRETE GIRDERS

BY

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**Abstract.** The composite action in members built up of different materials results in savings in construction cost. These savings can be further advanced by employing optimization techniques in the design of composite members. This paper presents an algorithm model for the cost optimization of composite beams based on the specifications of the SR EN 1994-1-1/2006: Design of composite steel and concrete structures. Design examples taken from the literature were analysed in order to validate the proposed model, to illustrate its use, and to demonstrate its capabilities in optimizing composite beam designs.

**Key words:** composite steel – concrete beam; EC4; nonlinear optimization.

## 1. Introduction

In common practice, composite plate girders are designed by a trail-and-error approach due to the complexity of the design rules. The design of a composite girder is a tedious and time-consuming job for the designer.

In the field of optimization of composite structures different optimization techniques and objective functions have been proposed. Bhatti (1995) introduced the structural mass minimization, in the context of a highway

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bridge composite welded plate girder. Adeli and Kim (2001) developed a cost objective function which includes the costs of concrete, steel beams and shear studs using neural dynamics model programming. Kravanja and Šilih (1992) applied the structural optimization method rather than classical structural analysis. The optimization was performed by the nonlinear programming approach (NLP). Their cost function includes the costs of concrete, structural steel, reinforcement, shear studs, anti-corrosion paint, fire protection paint, sheet-steel cutting costs, welding costs and the costs of the formworks. Neal and Johnson (1992) concludes that composite trusses of spans exceeding 18 m are generally the most economic structural systems, while for spans between 12 and 15 m, the cost is determined by floor height limitations.

This paper performs the optimization of the composite structures by the NLP approach. The formulation includes the cost steel beam. The use of the built-up section for the plate girder makes it more adaptable to optimization techniques and offers large savings in composite structures.

The design, resistance and deflection inequality constraints for composite **I** welded beams were defined in accordance with the Eurocodes (EN 1990; EN 1992-1-1; EN 1993-1-1, N 1994-1-1) in order to satisfy the requirements of both the ultimate and the serviceability limit states.

The optimization was performed for simply supported composite structures, for different combinations of spans and loads.

The optimization also considered different economic conditions: different structural steel grades, four defined spans from 4 to 10 m, various uniformly distributed imposed loads from 5 kN/m (dead weight) to 10 kN/m taking into account the variation of the value of the imposed load. In this formulation, bearing stiffeners, welded connections and shear connectors are not taken into account. The concrete slab is tacking in account by the compressive strength ($f_{ck}$).

A parametric study was conducted to compare the obtained optimal results. The task of the research was to define the spans and loads, at which each of the presented composite structures would show its advantages.

## 2. Research Methodology

The methodology of this research consists of five sequential stages as follows:

a) *Literature review*. Establish the concepts of composite steel–concrete structural element and its current design applications specified in the cited Eurocode normes. Then, the relevant equations, parameters and methods of calculation for the composite steel–concrete systems are established. This study is focusing on the beams, to demonstrate the approach of the proposed system.

b) *Cost parameters*. Define the parameters in the composite steel–concrete design process that affect the cost of materials. This would include the dimensions of the different steel elements, and the different steel and concrete grades available in the market.

c) *Design phase*. The design phase utilizes the automatic calculations and programming powers of the spreadsheet environment with its macro capabilities. The design phase consists of four stages as follows:

1. Structural design computer program. Implementation of design variables and design procedures and equations as per the Eurocodes. It automates the design process for accuracy and speed purposes.

2. Cost estimating computer program. The proposed program uses optimiztation solving methods like Newton-Raphson method, direct tangent methods, or descendant steps in order to estimate total cost of materials included Excell Microsoft spreadsheet programs.

d) *Validation and implementation*. The functionality of the cost optimization support system is then tested in an iterative mode to ensure reliability *prior* to final implementation. Upon successful testing, parametric cost studies are conducted to determine the relationship between the structural element dimensions and their costs.

## 3. Cost Optimization of Concrete Structures

The objective function in the optimization problem is associated with a set of constraints. In the context of structural optimization, these constraints fall into two categories: behavior constraints, that limit the plastic moment, shear force or stability conditions; and design constraints imposed on design variables to put practical limits on some dimensions of elements. Further, to simplify the formulation, it is necessary to introduce dependent design variables. These variables give rise to additional constraints, called *equality constraints*.

To find the mathematical formulation, two major steps are formulated (Arora, 1997; Farkas & Jármai, 1997) namely

a) To determine the major decision variables affecting the design of composite beams.

b) To formulate the objective of cost optimization of composite beams in optimization model.

### 3.1. Model Formulation of Composite Girder

The primary purpose of this development stage is those to formulate an optimization model that supports the cost minimization of composite beams. In this formulation, bearing stiffeners, welded connections, and shear connectors are not taken into account. The mathematical programming can be expressed as follows:

$$\text{Min } z = f(\mathbf{x})$$
$$\text{subjected to: } \begin{cases} h(\mathbf{x}) = 0, & (\text{NLP}); \\ g(\mathbf{x}) \le 0, \end{cases} \tag{1}$$

where **x** is a vector of continuous variables, defined within the compact set, $X$.

The optimization of the composite structures was performed by the NLP approach included in Excell Toolbox.

### 3.2. Decision Variables

The present model is designed to consider all relevant decision variables that may have an impact on the cost optimization of composite beams. These include for the steel section: the nominal yield strength for the section ($f_y$), the cross-sectional area ($A_a$), the depth ($h_w$), steel depth ($d$), the web thickness ($t_w$), the width of the compression flange ($b_1$), the thickness of the compression flange ($t_{f1}$), the width of the tension flange ($b_2$), the tension flange thickness ($t_{f2}$), the moment of inertia ($I_a$). These variables are shown in Fig. 1.

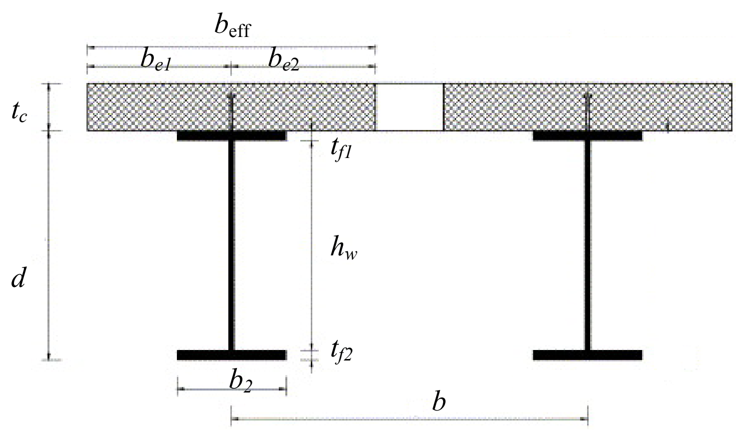


Fig. 1 – Notations for composite **I** welded beam.

For the concrete slab only the compressive strength ($f_{ck}$), the characteristic cylinder compressive strength of the concrete) is tacking into account, while the height of the concrete slab ($t_c$) is given and fixed at the onset of each design.

### 3.3. Optimization Objectives

The present optimization model is formulated in order to provide the capability of cost optimization of composite beams. The model is also designed to quantify and measure the impact of various decision variables that affect the cost optimization of composite beams. It incorporates the following objective eq.:

$$\text{minimize composite beam weight:} \quad G(\mathbf{x}) = G_c + G_a(\mathbf{x}), \tag{2}$$

where $G_c$ and $G_a$ are the weight of concrete and steel beam, respectively. The terms used in the objective eq. are defined as follows:

$$G_c = \rho_c L b t_c\,, \tag{3}$$

$$G_a = \rho_a A_a L\,, \tag{4}$$

where $L$ is the beam span, $\rho_a$ – the unit weight of steel section and $\rho_c$ – the unit weight of concrete section. The minimization of the objective function is subjected to the constraints prescribed by the Eurocodes specifications. These constraints are described briefly in the following section.

### 3.4. Design Constraints Derived from EC3

The moment, shear, and concentrated load bearing resistances of beams whose plate elements are slender, may be significantly influenced by local buckling considerations. The moment, shear and concentrated load bearing resistances of beams whose plate elements are slender may be significantly influenced by local buckling considerations. Because of this, beam cross-sections are classified as Class 1, 2, 3, or 4, depending on the ability of the elements to resist local buckling. Sections are classified by comparing the slenderness $\lambda = (c/t)\sqrt{235/f_y}$ of each compression element with the appropriate limits of Table 5.2 of EC3 (EN 1993-1-1). In this example, only cross-section of Class 1 and 2 are considered.

The local web buckling constraint is expressed by

$$t_w \geq \beta h_w\,, \tag{5}$$

where limiting web slenderness for plastic design has the value

$$\frac{1}{\beta} = 83\varepsilon\,, \tag{6}$$

$\varepsilon=\sqrt{235/f_y}$ .

For the compression flange the limiting plate slenderness for plastic design is

$$\frac{1}{\delta}=10\varepsilon\ . \tag{7}$$

For the shear resistance check, the inequality

$$V_{Ed} \le V_{c,Rd} \tag{8}$$

must be satisfied, in which $V_{Ed}$ is the maximum design shear force, and $V_{c,Rd}$ – the design shear resistance. For a stocky web with $h/t_w \le 72\varepsilon$ and for which the elastic shear stress distribution is approximately uniform (as in the case of an equal flanged **I**-section), the uniform shear resistance, $V_{c,Rd}$, is usually given by

$$V_{c,\,Rd} = A_v \frac{f_y}{\sqrt{3}\gamma_{M0}}, \tag{9}$$

where $f_y/\sqrt{3}$ is the shear yield stress and $A_v$ – the shear area of the web defined in Clause 6.2.6(3) of EN 1993-1-1.

### 3.5. Flexural Strength Constraints

The composite section is assumed to be of Class 1 or 2, so that the whole of the design load can be assumed to be resisted by the composite member, whether the construction was propped or unpropped.

The moment capacity of the composite section must be checked to make sure that the composite section can support all dead and live loads, as defined by the constraint

$$M_{Ed} \le M_{pl,\,Rd}\ , \tag{10}$$

where: $M_{Ed}$ is the maximum design moment, $M_{pl,Rd}$ – the plastic moment resistance of the composite beam.

The calculation method for $M_{pl,\,Rd}$ depends on the location of the plastic neutral axis (EN 1994-1-1). The plastic moment resistance, assuming full shear connection and a symmetric steel section, is expressed in terms of the resistance of various elements of the beam.

Three cases to be considered for **I** sections are presented in what follows, the neutral axis (*PNA*) being situated in the steel beam:

a) in the flange of the steel section: $N_a > N_c > N_w$ (Fig. 2 *a*);

b) in the web of the steel section: $N_a > N_c < N_w$ (Fig. 2 *b*);
c) in the concrete flange: $N_a < N_c$ (Fig. 2 *c*).

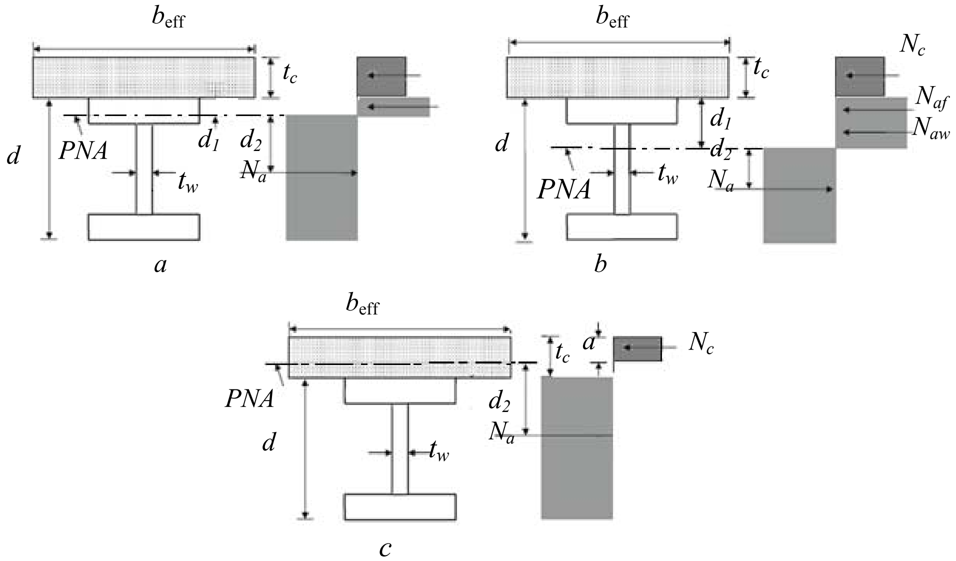


Fig. 2 – Plastic design of composite beam: *a* – when *PNA* lies in steel flange; *b* – when PNA lies in steel web; *c* – when *PNA* lies in concrete slab.

Using the notation from Fig. 2, the moment capacity of the composite beam when the plastic neutral axis (PNA) lies within the beam flange is given by

$$M_{pl,Rd} = N_c\left(d_1 + d_2 + \frac{t_c}{2}\right) + N_{af}\left(d_2 + \frac{d_1}{2}\right). \tag{11}$$

The distance between the bottom of the concrete slab and the PNA, $d_1$, is found by equating the tension force which the compression force

$$N_{af} = \frac{N_a - N_c}{2}. \tag{12}$$

Hence, the neutral axis depth is

$$d_1 = \frac{N_a - N_c}{2b_1\left(f_y / \gamma_{M0}\right)}. \tag{13}$$

The distance, $d_2$, between the PNA of the composite section and plastic centroid is

$$d_2 = \frac{A_a - b_1 d_1}{2t_w} - \frac{b_1(t_{f1} - d_1)}{t_w} + d_1 - t_{f1}. \tag{14}$$

When the PNA lies within the beam web, as shown in Fig. 2 *b*, the moment capacity of the composite beam is given by

$$M_{pl,Rd} = N_c\left(d_1 + d_2 + t_c\right) + N_{af}\left(d_2 + \frac{d_1}{2}\right) + N_{aw}\left(\frac{d_1 - t_f}{2} + d_2\right). \tag{15}$$

Similarly, $d_1$ and $d_2$ are determined as follows:

$$d_1 = t_{f1} + \frac{N_a - N_c - 2N_{af}}{2t_w\left(f_y / \gamma_{M0}\right)}, \tag{16}$$

$$d_2 = \frac{A_a - b_1 d_1}{2t_w} - \frac{d_1 - t_{f1}}{2}. \tag{17}$$

If the neutral axis is situated in the concrete flange, as shown in Fig. 2 *c*, $N_a < N_c$ and the bending moment of resistance is

$$M_{pl,Rd} = N_c\left(d_2 + \frac{a}{2}\right), \tag{18}$$

where *a* is the depth of the concrete equivalent rectangular stress block, which is given by

$$a = \frac{N_c}{0.85\left(f_{ck} / \gamma_c\right)b_{\text{eff}}}, \tag{19}$$

with $\gamma_c = 1.5$.

The distance, $d_2$, between the PNA of the composite section and plastic centroid is

$$d_2 = \frac{d}{2} + t_c - \frac{a}{2}. \tag{20}$$

## 4. Numerical Implementation

The advantages of computer technology have been incorporated in the optimization of concrete design and mixture proportions; most notably is the use of spreadsheets. Spreadsheets are user friendly and exceedingly powerful but are not being exploited as much as they could be in structural engineering design.

Closed analytical solutions for practical optimization problems are difficult to obtain if the number of variables is more than two and/or the constraints expressions (eqs. (5), (7), (8) and (10)) are complex.

In this section, use of a widely available spreadsheet program like Microsoft Excell and its Solver tool (Fig. 3) is demonstrated for solution of composite steel–concrete girder design problem.

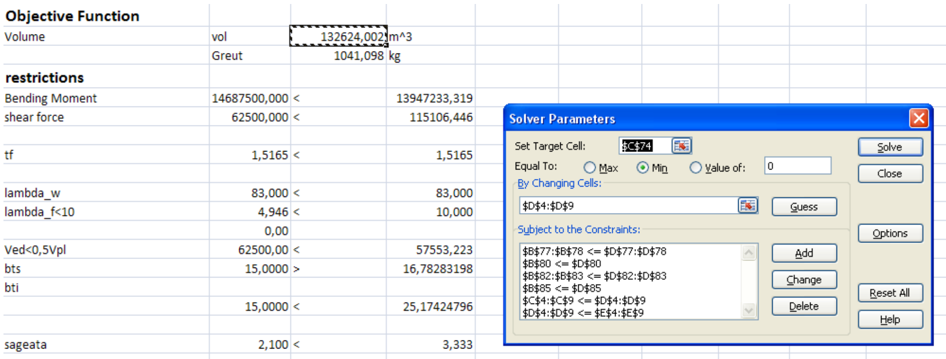


Fig. 3 – Screen dump of Solve dialogue box for composite *I* beam floor system design.

The implementation of design variables and design procedures and eqs. respect Eurocodes. Altough material selection may be treated as a design variable; this aspect is not studied here.

Various sections of the spreadsheet contains the designer parameters, including their names, symbols, value and units. These values are typically helded fixed during the solution process, but can be changed from one solution to the next to investigate "What If" scenarios. Other sections contains variables of the problems, *i.e.* quantities that depend on those defined in the previous sections.

Once the spreadsheet has been created, the next step is to define the optimizazion problem in the solver Add In. The solver is an "Add In" to Microsoft Excel, which is an optional module for adding optimization capabilities to the spreadsheet program. The optimal solutions are obtained in a matter of minutes on a PC computer. Main requirements are that one cell contain the objective function formulae. Fig. 3 shows a screen dump of the solver dialogue box. The objective function cell is entered as the "Target cell" that is to be minimized. The independent design variables are identified next under the "By changing cells" heading.The constraints are entered under the "Subject to the constraints" heading.

All the solutions shown in Tables 1 and 2 for continuous variable formulations were also obtained with Excell.

## 5. Illustrative Example

An example taken from the literature (Pacurar & Aribert, 1997) were analysed and presented in order to illustrate the use of the proposed model, to demonstrate its capabilities, and to validate its results.

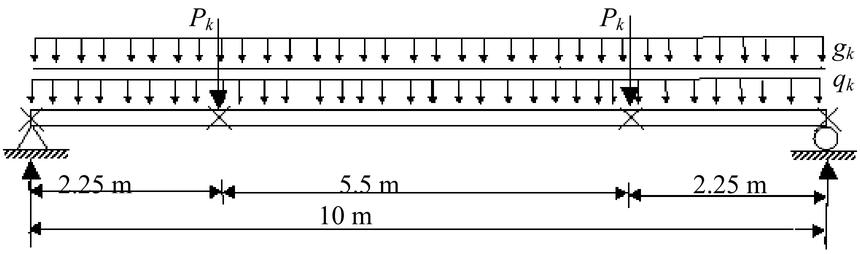


Fig. 4 – Variable imposed load and the span of the composite **I** beam floor system.

**Table 1**
*Result Summary*

| Cases | Concrete classes | Steel grade | Steel section, [cm] | Area cm$^2$ | Position PNA | % cost saving |
|---|---|---|---|---|---|---|
| 1 | C20/25 | S235 | $h_w$ = 80.15<br>$b_1$ = 15.00<br>$b_2$ = 15.00<br>$t_{f1}$ = 1.45<br>$t_{f2}$ =1.80<br>$t_w$= 0.97 | 126 | in the steel flange | 14.6 |
| 2 | C35/45 | S235 | $h_w$ = 80.15<br>$b_1$ = 15.00<br>$b_2$ = 15.00<br>$t_{f1}$ = 1.45<br>$t_{f2}$ = 1.73<br>$t_w$= 0.97 | 125.3 | in the concrete slab | 15.3 |
| 3 | C20/25 | S355 | $h_w$ = 65.21<br>$b_1$ = 15.00<br>$b_2$ = 15.00<br>$t_{f1}$ = 1.18<br>$t_{f2}$ = 1.86<br>$t_w$= 0.79 | 96.75 | in the steel flange | 34.5 |
| Example* | C20/25 | S235 | $h_w$ = 72.00<br>$b_1$ = 25.00<br>$b_2$ = 35.00<br>$t_{f1}$ = 1.5<br>$t_{f2}$ = 1.5<br>$t_w$= 0.8 | 147.60 | in the steel web | |

* Pacurar & Aribert, 1997

The considered composite **I** beam floor system is 10 m long, and beam spacing is of 1.2 m subjected to combined effects of the self-weights, $g_k =$ $= 5$ kN/m and the variable imposed load of $q_k = 16$ kN/m and two concentrated forces $P_k = 250$ kN (Fig. 4).

Table 1 summarizes the results obtained in the examples. The results show that the proposed model was able to achieve significant cost savings in both examples as it is shown that the cost savings in examples reach the values up to 14.6% for the same steel grade and concrete class and 34.5% for a higher steel grade S355, respectively. Note that not significant cost saving are obtained when uses a higher concrete class.

## 6. Parametric Study

A parametric study was also presented to investigate the effects of beam spans and loadings on the cost optimization of composite beams. In this study, S235 steel grade, concrete class C20/25 and concrete slab thickness of 100 mm were keeping fixed.

Fig. 5 shows the curves representing the variations of the total costs *vs*. the beam spans for two different loadings. Designers can use this figure to establish the optimum beam cross section that can achieve the minimum cost for each beam span.

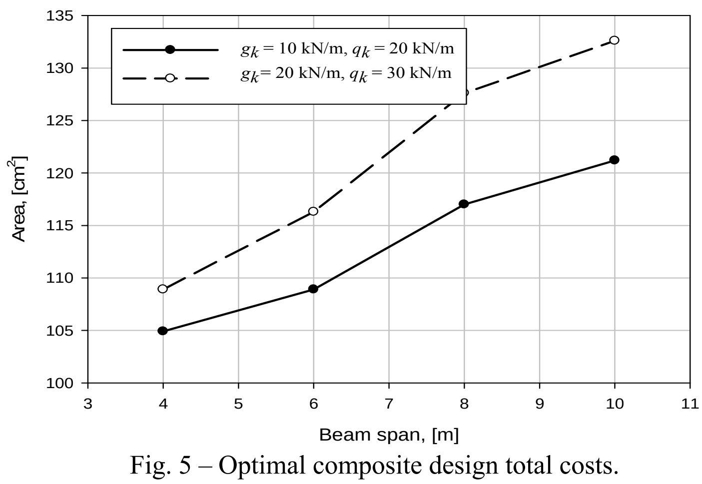

Fig. 5 – Optimal composite design total costs.

Table 2 summarizes the design results obtained in the study case using the present model. As expected, the steel section size increases with both the beam span and the acting loads as to satisfy the strength and the deflection constraints.

**Table 2**
*Parametric Study*

| Dead loads | Live loads | Beam spacing, [m] | Beam span, [m] | Steel section, [cm] | Area $cm^2$ |
|---|---|---|---|---|---|
| 10 | 20 | 3 | 4 | $h$ = 73.0; $t_w$ = 0.9<br>$b_s$ =15.0; $t_{fs}$ = 1.3<br>$b_i$ =15.0; $t_{fi}$ = 1.4 | 104.9 |
| | | | 6 | $h$ = 75.4; $t_w$ = 0.9<br>$b_s$ = 15.0; $t_{fs}$ = 1.4<br>$b_i$ = 15.0; $t_{fi}$ = 1.3 | 108.9 |
| | | | 8 | $h$ = 77.8; $t_w$ = 0.9<br>$b_s$ = 15.0; $t_{fs}$ = 1.4<br>$b_i$ = 15.0; $t_{fi}$ = 1.5 | 117 |
| | | | 10 | $h$ = 80.1; $t_w$ = 1.0<br>$b_s$ = 15.0; $t_{fs}$ = 1.4<br>$b_i$ = 15.0; $t_{fi}$ = 1.5 | 121.2 |
| 20 | 40 | 3 | 4 | $h$ = 76.2; $t_w$ = 0.9<br>$b_s$ = 15.0; $t_{fs}$ = 1.4<br>$b_i$ = 15.0; $t_{fi}$ = 1.2 | 108.9 |
| | | | 6 | $h$ = 80.1; $t_w$ = 1.0<br>$b_s$ = 15.0; $t_{fs}$ = 1.4<br>$b_i$ = 15.0; $t_{fi}$ = 1.1 | 116.3 |
| | | | 8 | $h$ = 83.9; $t_w$ = 1.0<br>$b_s$ = 15.0; $t_{fs}$ = 1.5<br>$b_i$ = 15.0; $t_{fi}$ = 1.3 | 127.6 |
| | | | 10 | $h$ = 83.9; $t_w$ = 1.0<br>$b_s$ = 15.0; $t_{fs}$ = 1.5<br>$b_i$ = 15.0; $t_{fi}$ = 1.7 | 132.6 |

## 7. Conclusions

The paper proposes the cost optimization of the composite **I** beam floor system. This system consists of a reinforced concrete slab of constant depth and non-symmetrical welded steel **I** beams. The optimization was performed by the nonlinear programming approach (NLP). An NLP optimization model for composite **I** beam floor system was thus developed. The objective function of the structure's manufacturing costs was subjected to a rigorous system of design, load, resistance and deflections inequality constraints, defined in accordance with Eurocode 4 to satisfy both the ultimate and the serviceability limit states.

To accomplish the above vonditions, the model incorporates: (i) a design module that performs the design of composite beams; (ii) a cost module that computes the total cost of composite beams; and (iii) an optimization module that searches for and identifies optimal/near – optimal design alternatives.

Two examples and a case study were used to illustrate the capabilities of the developed model in generating all optimal design solutions that achieve minimum total costs. Substantial cost savings were achieved by using the present model. This new capability should prove useful to structural designers and is expected to advance existing design practices of composite beams.

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## OPTIMIZAREA GRINZILOR MIXTE OȚEL–BETON

(Rezumat)

În domeniul construcțiilor, prin utilizarea de materiale diferite, cum sunt structurile mixte din oțel–beton, se pot obține sisteme constructive cu eficiență sporită şi cost minim. Pentru aceste tipuri de structuri pot fi obținute economii importante de material prin utilizarea tehnicilor de optimizare în procesul de proiectare. Se propune un algoritm pentru optimizarea greutății grinzilor mixte din oțel–beton, conform cerințelor impuse de standardul SR EN 1994-1-1: *Proiectarea structurilor compozite de oțel şi beton*. În scopul validării rezultatelor numerice şi pentru a demonstra capacitatea de optimizare a grinzilor compozite, aplicațiile numerice au fost comparate cu rezultate obținute din literatura de specialitate. Rezultatele obținute arată că prin metodologia propusă se pot realiza substanțiale economii de cost, prin minimizarea greutății. Studiul evidențiază importanța practică a implementării optimizării în cadrul proiectării structurilor.