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SLENDERNESS PARAMETERS OF STEEL ELEMENTS

BY

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Abstract. A modality to establish the plate slenderness parameters for plates which are used in compression and bending elements is presented. The reserve of post-buckling resistance of the thin plates is taken into account in the establishing of the steel plate slenderness.

Key words: slender plates; compression and bending elements; post - buckling resistance.

1. Introduction

Thin plates with a high slenderness ratio are frequently used in the construction of compression and bending elements because they are economically and structurally efficient.

It is generally accepted that some constitutive plates of the cross-section are temporarily in a buckled shape without a negative influence as regards the safety; this assumption is justified by the fact that, after local buckling, the slender plates have a significant resistance reserve owed to post-critical behaviour.

This paper presents a modality to establish the plate slenderness parameters for plates which are used in compression and bending elements, taking into account the reserve of post-critical resistance of the slender plates.

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Starting from the norm EN 1993 -2: *Design of steel structures*. Part 2: *Steel bridges*, according to §7.4: *Limitation of web breathing*, the slenderness of web plates should be limited to avoid excessive breathing; web breathing may be neglected for certain web panels, if the following criteria are met:

$$\frac{b}{t} \leq 30 + 4.0L \leq 300 \text{, for road bridges} \tag{1a}$$

$$\frac{b}{t} \leq 55 + 3.3L \leq 250 \text{, for railway bridges,} \tag{1b}$$

where L is the span length, [m], but not less than 20 m.

If the provisions in relations (1 *a*) and (1 *b*) are not satisfied, web breathing should be checked as follows:

$$\sqrt{\left(\frac{\sigma_{x,Ed,\text{ser}}}{k_\sigma\ \sigma_E}\right)^2 + \left(\frac{1.1\ \tau_{x,Ed,\text{ser}}}{k_\tau\ \sigma_E}\right)^2} \leq 1.1\,, \tag{2}$$

where: $\sigma_{x,Ed,\text{ser}}$, $\tau_{Ed,\text{ser}}$ are the stresses for the frequent load combination; k_σ, k_τ – the linear elastic buckling coefficients, assuming hinged edges of the panel;

$$\sigma_E = \frac{\pi^2 E}{12(1-\mu^2)}\left(\frac{t}{b_p}\right)^2 = 190{,}000\left(\frac{t}{b_p}\right)^2 \text{, [N/mm²];}$$

b_p – is the smaller of a and b.

2. Plate Slenderness Parameters of Steel Compression and Bending Elements

2.1. Plate Slenderness Parameters of Steel Elements Subjected to Compression and Shear

The plate subjected to combined compression and shear can be analysed in a simplified manner by the component method (Johansson *et al*., 2007), which requires to approximate the behaviour of a part of the full member by the behaviour of a set of basic component fields, each loaded by either σ or τ (Fig. 1).

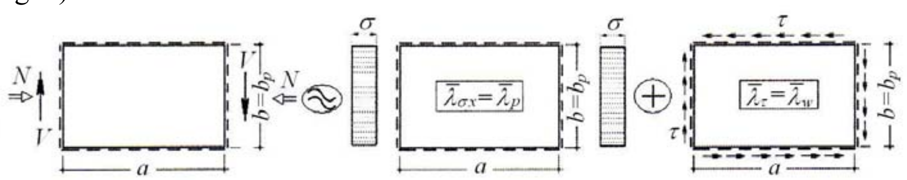
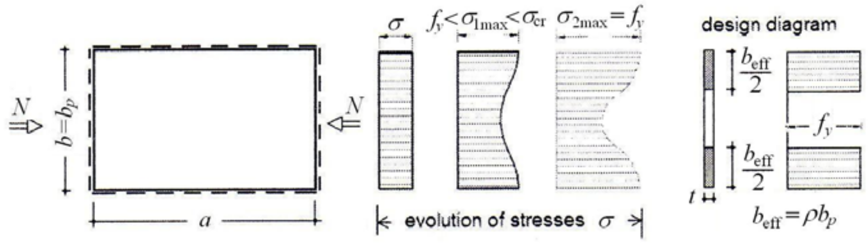


Fig. 1 – Breakdown of full stress fields to basic stress components (Johansson *et al*., 2007).

The distribution of the σ-stresses becomes non-uniform when the compression is increasing due to the tendency of buckling of the central plate zone; the consequence is that the stresses on edges increase and the stresses on the central zone decrease (Fig. 2 *b*). The non-uniform distribution of the compression stresses can be replaced by a uniform stress distribution on a so-called *effective section* (Fig. 2 *c*).



a *b* *c*

Fig. 2 – Distribution of stress σ caused by local buckling.

It is admitted that the ultimate plate resistance is obtained when $\sigma_{\max}$ reaches f_y .

To determine the effective width of the plate, b_{eff} , the von Karman hypothesis is utilized

$$\sigma_{\max} = (\sigma_{\text{cr}})_{\text{eff}} \,. \tag{3}$$

The critical stress is given by the relation

$$\sigma_{\text{cr}} = k_\sigma \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b_p}\right)^2 . \tag{4}$$

It results

$$\sigma_{\max} = k_\sigma \frac{\pi^2 E}{12(1-\mu^2)} \left(\frac{t}{b_p}\right)^2 \left(\frac{b_p}{b_{\text{eff}}}\right)^2 = \sigma_{\text{cr}} \left(\frac{b_p}{b_{\text{eff}}}\right)^2 . \tag{5}$$

In the case of the limit state we have

$$\sigma_{\max} = (\sigma_{\text{cr}})_{\text{eff}} = \sigma_{\text{cr}} \left(\frac{b_p}{b_{\text{eff}}}\right)^2 = f_y \tag{6}$$

and consequently

$$\frac{b_{\text{eff}}}{b_p} = \rho = \sqrt{\frac{\sigma_{\text{cr}}}{f_y}} \le 1 \,. \tag{7}$$

The coefficient ρ is the plate buckling reduction factor, depending on the plate slenderness, $\overline{\lambda}_p$

$$\rho = \sqrt{\frac{\sigma_{\text{cr}}}{f_y}} = \frac{1}{\overline{\lambda}_p}, \tag{8}$$

$$\overline{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{\text{cr}}}}. \tag{9}$$

For a compression plate Fig. 3 presents the graph $\overline{\lambda}_p - \rho = b_{\text{eff}} / b$.

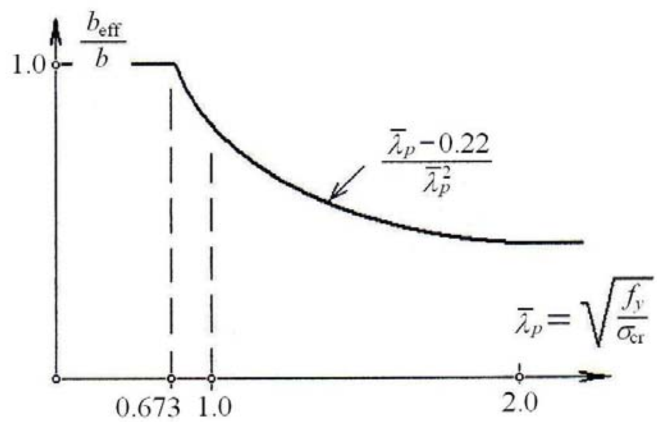


Fig. 3 – Graphic $\overline{\lambda}_p - \rho$.

When the plate slenderness is $\overline{\lambda}_p = 1$, the corresponding ratio $s = b_p / t$ can be evaluated as follows:

$$\overline{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{\text{cr}}}} = 1, \ \sigma_{\text{cr}} = k_\sigma \frac{190\,000}{(b_p / t)^2} = f_y, \ k_\sigma = 4.0$$

and consequently

$$s = \frac{b_p}{t} = \sqrt{\frac{4 \times 190{,}000}{f_y}}.$$

Table 1 presents the values of the s function of the steel grade.

Table 1
Values of the Parameter s Function of the Steel Grade

	S235	S275	S355
$s = b_p / t$	56.7	52.6	46.3

When the buckling reduction factor is $\rho = 1$, the corresponding plate slenderness, $\overline{\lambda}_p$, can be obtained as follows:

$$\rho = \frac{b_{\text{eff}}}{b_p} = \frac{\overline{\lambda}_p - 0.22}{\overline{\lambda}_p^2} = 1 \text{ and then } \overline{\lambda}_p = 0.673 .$$

Taking into account the things mentioned above and the condition (2), given by SR EN 1993-2/2007, the following conditions for a plate subjected to compression and shear (Johansson *et al*., 2007) result:

a) SLS:

$$\frac{\sigma_{x.Ed.\text{ser}}}{\sigma_{x.\text{cr}}} \le 1.1 , \quad \frac{1.1\,\tau_{Ed.\text{ser}}}{\tau_{\text{cr}}} \le 1.1 ; \tag{10 a,b}$$

b) ULS:

$$\frac{\sigma_{x.Ed}}{\rho f_y / \gamma_{M1}} \le 1 , \quad \frac{\tau_{Ed}}{\chi_w f_y / \left(\sqrt{3}\gamma_{M1}\right)} \le 1 . \tag{11 a,b}$$

The working stresses $\sigma_{x,Ed,\text{ser}}$ and $\tau_{Ed,\text{ser}}$ may be taken as

$$\sigma_{x,Ed,\text{ser}} = K\sigma_{x,Ed} \quad \text{and} \quad \tau_{Ed,\text{ser}} = K\tau_{Ed} , \tag{12 a,b}$$

where

$$K = \frac{\nu}{\gamma_G} + \frac{\psi_1}{\gamma_Q}(1-\nu) , \tag{13}$$

in which: ψ_1 is the combination factor for frequent loads; γ_G, γ_Q – partial factors for permanent and variable loads; $\nu = G/(G + Q)$; $\gamma_{M1} = 1.10$.

Table 2 presents parameter values to be taken into account for a global analysis.

Table 2
Parameter Values for a Global Analysis

Factor	Road bridges		Railway bridges	
	Small spans	Large spans	Small spans	Large spans
ψ_1	0.75	0.40	0.75	0.40
γ_G	1.35	1.35	1.35	1.35
γ_Q	1.35	1.35	1.45	1.45
v	0.50	0.40	0.30	0.20
K	0.65	0.47	0.58	0.37

The reduction factor for a uniform compressed plate has the value

$$\rho = \frac{\overline{\lambda}_p - 0.22}{\overline{\lambda}_p^2}\,. \tag{14}$$

The reduction factor, χ_w, for a plate subjected to shear for usual slenderness, $\overline{\lambda}_w$, has the value

$$\chi_w = \frac{0.83}{\overline{\lambda}_w}\,. \tag{15}$$

Taking these data into consideration, we can express relations (10 *a*,*b*) and (11 *a*,*b*) as follows:

a) SLS:

$$\frac{\sigma_{x,Ed,\mathrm{ser}}}{1.1\sigma_{x,\mathrm{cr}}} \le 1, \quad \frac{\tau_{Ed,\mathrm{ser}}}{\tau_{\mathrm{cr}}} \le 1; \tag{16 a,b}$$

b) ULS:

$$\frac{\sigma_{x,Ed,\mathrm{ser}}}{K\rho f_y / \gamma_{M1}} \le 1, \quad \frac{\tau_{Ed,\mathrm{ser}}}{K\chi_w f_y / (\sqrt{3}\gamma_{M1})} \le 1; \tag{17 a,b}$$

Relations (17 *a*,*b*) can be written in the form:

$$\frac{\sigma_{x,Ed,\mathrm{ser}}}{K\rho f_y / \gamma_{M1}} \cdot \frac{1.1\sigma_{x,\mathrm{cr}}}{1.1\sigma_{x,\mathrm{cr}}} = \left[\frac{1.1\sigma_{x,\mathrm{cr}}\gamma_{M1}}{K\rho f_y}\right]\left[\frac{\sigma_{x,Ed,\mathrm{ser}}}{1.1\sigma_{x,\mathrm{cr}}}\right] = E_\sigma\left[\frac{\sigma_{x,Ed,\mathrm{ser}}}{1.1\sigma_{x,\mathrm{cr}}}\right] \le 1, \tag{18 a}$$

$$\frac{\tau_{Ed,\mathrm{ser}}}{K\chi_w f_y / (\sqrt{3}\gamma_{M1})} \cdot \frac{\tau_{\mathrm{cr}}}{\tau_{\mathrm{cr}}} = \left[\frac{\tau_{\mathrm{cr}}\sqrt{3}\gamma_{M1}}{K\chi_w f_y}\right]\left[\frac{\tau_{Ed,\mathrm{ser}}}{\tau_{\mathrm{cr}}}\right] = E_\tau\left[\frac{\tau_{Ed,\mathrm{ser}}}{\tau_{\mathrm{cr}}}\right] \le 1, \tag{18 b}$$

where

$$E_\sigma = \frac{1.1\sigma_{x,\text{cr}}\gamma_{M1}}{K\rho f_y}; \quad E_\tau = \frac{\tau_{\text{cr}}\sqrt{3}\gamma_{M1}}{K\chi_w f_y}. \qquad (19\ a,b)$$

With the relations

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{x,\text{cr}}}} \quad \text{and then } \sigma_{x,\text{cr}} = \frac{f_y}{\bar{\lambda}_p^2}$$

$$\bar{\lambda}_w = 0.76\sqrt{\frac{f_y}{\tau_{\text{cr}}}} \quad \text{that is } \tau_{\text{cr}} = \frac{0.76^2 f_y}{\bar{\lambda}_w^2} = \frac{0.58 f_y}{\bar{\lambda}_w^2}$$

it results

$$E_\sigma = \frac{1.21}{K(\bar{\lambda}_p - 0.22)}, \quad E_\tau = \frac{1.33}{K\bar{\lambda}_w}. \qquad (20\ a,b)$$

To fulfil the simultaneous inequalities (19), (20) and (21), the following conditions have to be satisfied:

$$E_\sigma = \frac{1.21}{K(\bar{\lambda}_p - 0.22)} \geq 1 \text{ that is } \bar{\lambda}_p \leq \frac{1.21}{K} + 0.22, \qquad (21\ a)$$

$$E_\tau = \frac{1.33}{K\bar{\lambda}_w} \geq 1 \text{ that is } \bar{\lambda}_w \leq \frac{1.33}{K}. \qquad (21\ b)$$

Table 3 presents the slenderness obtained by using the relations (21 *a*, *b*).

Table 3

Slenderness Values

Factor	Road bridges		Railway bridges	
	Small spans	Large spans	Small spans	Large spans
K	0.65	0.47	0.58	0.37
$\bar{\lambda}_p$	2.08	2.79	2.30	3.49
$\bar{\lambda}_w$	2.05	2.83	2.29	3.59

Using the relations

$$\overline{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{x.cr}}} = \frac{b_p / t}{28.4\varepsilon\sqrt{k_\sigma}} \;;\; k_\sigma = 4\,, \tag{22 a}$$

$$\overline{\lambda}_w = 0.76\sqrt{\frac{f_y}{\tau_{\mathrm{cr}}}} = \frac{b}{86.4t\varepsilon} \text{(transverse stiffeners on supports only)} \tag{22 b}$$

it results the following slenderness limits

$$\frac{b_p}{t} = 56.8\varepsilon\overline{\lambda}_p \;; \qquad \frac{b}{t} = 86.4\varepsilon\overline{\lambda}_w\,. \tag{23 a,b}$$

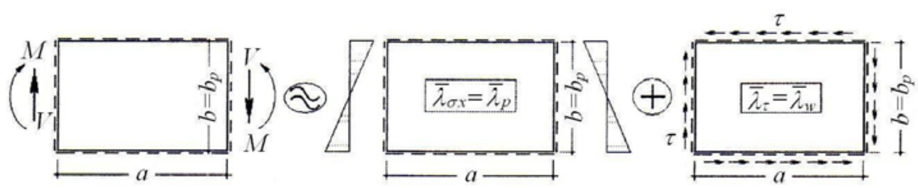
The value $\varepsilon = 1$ (Steel S235) leads to the slenderness limits presented in Table 4.

Table 4
Factors b/t

Factor	Road bridges		Railway bridges	
	Small spans	Large spans	Small spans	Large spans
b_p/t	118	158	131	198
b/t	177	245	198	310

2.2. Plate Slenderness Parameters of Steel Elements Subjected to Bending and Shear

Following the previous methodology for the case of a plate subjected to bending and shear, the full stress field is broken down to basic stress components (Johansson *et al.*, 2007) (Fig. 4).



a *b* *c*
Fig. 4 – Breakdown of full stress fields to basic stress components.

For bending plate ($\psi = -1$), the buckling reduction factor ρ will be

$$\rho = \frac{\overline{\lambda}_p - 0.055\,(3+\psi)}{\overline{\lambda}_p^2} = \frac{\overline{\lambda}_p - 0.11}{\overline{\lambda}_p^2}\,, \tag{24}$$

where

$$\overline{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{x,\mathrm{cr}}}} = \frac{b_p / t}{28.4\varepsilon\sqrt{k_\sigma}}\,; \quad k_\sigma = 23.9 \tag{25 a}$$

$$\sigma_{x,\mathrm{cr}} = \frac{f_y}{\overline{\lambda}_p^2}\,. \tag{25 b}$$

It results

$$E_\sigma = \frac{1.1\sigma_{x,\mathrm{cr}}\gamma_{M1}}{K\rho f_y} = \frac{1.21}{K(\overline{\lambda}_p - 0.11)}\,. \tag{26}$$

From the condition $E_\sigma \geq 1$ it results: $\overline{\lambda}_p \leq (1.21/k) + 0.11$.

Table 5 presents the slenderness limits for a plate subjected to bending and shear.

Table 5
Slenderness Values

Factor	Road bridges		Railway bridges	
	Small spans	Large spans	Small spans	Large spans
K	0.65	0.47	0.58	0.37
$\overline{\lambda}_p$	1.97	2.68	2.20	3.38
$\overline{\lambda}_w$	2.05	2.83	2.29	3.59

Taking into account the expression of $\overline{\lambda}_p$, we obtain:

$$\frac{b_p}{t} = 138.8\varepsilon\overline{\lambda}_p\,. \tag{27}$$

The value $\varepsilon = 1$ (Steel S235) leads to the slenderness limits presented in Table 6.

Table 6
Factors b/t

Factor	Road bridges		Railway bridges	
	Small spans	Large spans	Small spans	Small spans
b_p/t	273	372	305	469
b/t	177	245	198	310

3. Final Remarks and Conclusions

The thin plates with a high slenderness ratio are frequently used in the construction of compression and bending elements because they are economically and structurally efficient.

The European norms accept the working hypothesis according to which some constitutive plates of the cross-section are temporarily in a buckled shape, which does not negatively affect safety; this assumption is justified by the fact that, after local buckling, the slender plates have a significant resistance reserve owed to post-critical behaviour.

The proposed methodology, establishing the slenderness limits of the steel plates which are components of compression or bending elements, can be useful in the optimal design of steel elements

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PARAMETRII DE ZVELTEȚE A ELEMENTELOR DIN OȚEL

(Rezumat)

Se propune o modalitate de stabilire a svelteții plăcilor care intră în alcătuirea barelor comprimate şi a elementelor încovoiate, prin luarea în considerare a rezervei de rezistență a plăcilor ca urmare a fenomenului de comportare post-critică a acestora.