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ANALYSIS AND BEHAVIOUR OF SANDWICH PANELS WITH PROFILED METAL FACINGS UNDER TRANSVERSE LOAD

BY

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Sandwich panels with thin steel facings and polyurethane core combine the load-carrying capacity of metal facings and protection functions with core properties. The core separates the two facings and keeps them in a stable condition, transmits shear between external layers, provides most of the shear rigidity and occasionally makes of useful contribution to the bending stiffness of the sandwich construction as a whole [1]. An experimental programme on sandwich panels has been organized to prove that the mechanical properties of core and interface satisfy the load-carrying requirements for structural sandwich panels. The analysis of sandwich panels with deep profiles facings for cladding elements, respectively the roof constructions, has been carried out according to the European design norms [1], [5].

1. Introduction

Sandwich constructions are known to provide high stiffness with lightweight. Lower modulus cores, such as characteristic of the expanded polymers used in commercial applications, require a more refined analytical treatment. A high order theory for sandwich panels is investigated in the present paper. The applications of this theory reveal certain features of the solution process that must be addressed. The obtained results show significant stress (concentrations) in both the core and the faces at load application and support points. The behaviour of sandwich panel in the static and strength context and design methods has been well documented in the papers published by P l a n t e m a [2], A l l e n [3], V i n s o n [4], D a v i e s [5] and Ţ ă r a n u and I s o p e s c u [6].

Sandwich panels, consisting of two stiff, strong face sheets and a lightweight core, can be designed to possess a high bending stiffness and strength at low weight. For a good approximation, the facings carry the bending and in-plane loads, whilst the core carries transverse shear. At the interface, the adhesive bond between the facings and the core must to transfer the contact shear stresses, to hold the facings against buckling away from the core and to carry loads applied normal to the sandwich panel [2]. Commonly used materials for sandwich panels are metallic or composites faces, with a foam core made from a polymeric or metallic, or wood.

An important loading configuration for sandwich panels is transverse load: this is the subject of the present paper. When the sandwich panel is subjected to bend loading. failure may occur by facing yield, fracture of the core in shear. facings wrinkling or face/core interface damage and upper face slipping relative to the lower face.

2. Analytical Solutions

The simplest structural sandwich is three–layered "stressed skin" construction by bonding two thin layers (facings) to each thickness of a thick layer (core). The essential principle is much the same of I beam, which is efficient structural shape, when it is subjected to bending. In structural sandwich, the facings take the place of the flanges. and the core takes the place of the web. The difference is that the core material of sandwich is different from the facings, and it is spread out instead of concentrated in a narrow web. The outside layer act together to form an efficient internal stress couple or resisting moment which balances the external imposed bending moment. Three main types of bent sandwich element are utilized in civil engineering structures: narrow beams, wide beams and panels. In this paper the analysis and behaviour of sandwich panels with one deeply profiled face and the other lightly profiled face are presented.

2.1. The Characteristic of the Sandwich Panels

The cross-section of the two-span roof panel is shown in Fig. 1. The sandwich panel has been considered simply supported on each span and act by the uniformly distributed loads on the all spans (L), which have the intensity q.

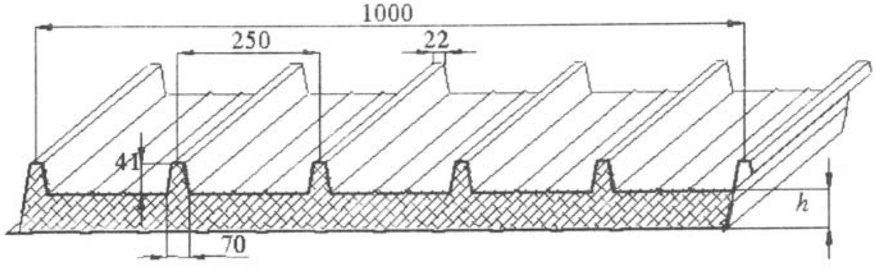

Fig. 1. The sandwich panel with profiled metal facings.

The geometric characteristics of the sandwich cross-section are shown in Fig. 2 and the relevant dimensions are as follow:

$$t_1 = 0.5 \text{ mm}; \quad t_{d_1} = 0.46 \text{ mm}; \quad A_{f_1} = 627.61 \text{ mm}^2;$$
$$d_{11} = 30.0 \text{ mm}; \quad d_{12} = 11.0 \text{ mm}; \quad I_{f_1} = 138,852 \text{ mm}^4;$$
$$t_2 = 0.5 \text{ mm}; \quad t_{d_2} = 0.46 \text{ mm}; \quad A_{f_2} = 540.95 \text{ mm}^2;$$
$$d_{21} = 1.5 \text{ mm}; \quad d_{22} = 0.5 \text{ mm}; \quad I_{f_2} = 1,586 \text{ mm}^4.$$

Also, $h = 40.0$ mm – the nominal depth of the tests sandwich panels and the distance between the outside layers gravity centres results:

$$d = h - \frac{t_1 + t_2}{2} + d_{12} - d_{22}. \tag{1}$$

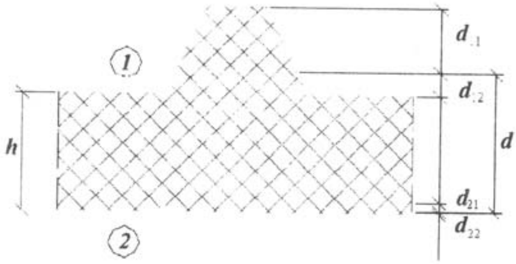

Fig. 2.– Cross-section of the sandwich panel.

The following additional values are required for the design:

a) the elastic modulus of steel, $E_s = E_{f_1} = E_{f_2} = 2.1 \times 10^6$ daN/cm^2;
b) the shear modulus of the core, $G = 30.2$ daN/cm^2;
c) the design depth of the sandwich panel, $b = 1.00$ m;
d) the total length of panel, $l = 3.00$ m.

2.2. The Design Parameters (Rigidities)

The bending stiffness of sandwich panel may be computed with the relation:

$$D_s = \frac{E_{f_1} A_{f_1} E_{f_2} A_{f_2} d^2}{E_{f_1} A_{f_1} + E_{f_2} A_{f_2}}. \tag{2}$$

The bending stiffness of the both facings is, in this case,

$$D_f = E_{f_1} I_{f_1} + E_{f_2} I_{f_2}. \tag{3}$$

The shear and β factors are given by the expressions:

$$k = \frac{D_s}{A_{\text{eff}} G_{\text{eff}} L^2}, \tag{4}$$ $$\beta = \frac{D_f}{D_f + D_s/(1+k)}, \tag{5}$$

where: $A_{\text{eff}} = bd$ is the effective area of the foam core, $G_{\text{eff}} = Gd/c$ – the effective shear modulus of the core and c – the core thickness [5].

2.3. Analysis of the Stresses and the Deflections

The relationships between the stress resultants and deformations are:

$$M_s = D_s \gamma_2' = D_s(\gamma' - w''), \tag{6}$$ $$Q_s = A_{\text{eff}} G_{\text{eff}} \gamma. \tag{7}$$

where, in addition to the quantities defined in Fig. 3, a prime denotes differentiation with respect to x, which is measured along the length of the panel; D_s, A_{eff}, G_{eff} have been calculated above; γ – the strain in the core (divergence of the normal from the horizontal axis of the section); w – the total deflection of the sandwich panel; q – distributed transverse load per unit length; M_s – the bending moment of sandwich panel; Q_s – the shear force; N – the axial force in facings.

Further relationships are added as follow:

$$M_{f_1} = -D_{f_1} w'', \quad M_{f_2} = -D_{f_2} w'', \tag{8}$$

$$Q_{f_1} = -D_{f_1} w''', \quad Q_{f_2} = -D_{f_2} w''', \tag{9}$$

where in addition to the quantities defined in Fig. 3 and below (Eqs. (6) and (7)), $D_{f_1} = E_{f_1} I_{f_1}$ is the bending stiffness of the upper face; $D_{f_2} = E_{f_2} I_{f_2}$ – the bending stiffness of the lower face.

Because the stress resultants in the two faces are proportional to the same deformations, it is convenient to treat them together, so that:

$$M_f = M_{f_1} + M_{f_2}, \quad M = M_f + M_s; \tag{10}$$

$$Q_f = Q_{f_1} + Q_{f_2}, \quad Q = Q_f + Q_s; \tag{11}$$

$$D_f = D_{f_1} + D_{f_2}, \quad D = D_f + D_s. \tag{12}$$

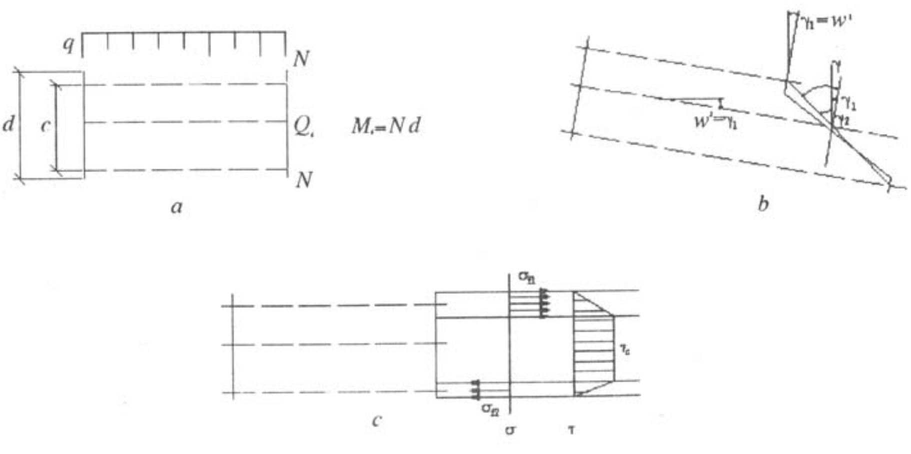

Fig. 3.- Forces, deformations, stress and shear stress distributions: a – dimensions and stress resultants; b – deformed infinitesimal element; c – stress and shear stress distributions.

The above equations imply the separation of the stress resultants into a sandwich part and a flange part as shown in Fig. 4. This separation is fundamental to understand the behaviour of sandwich panels with stiff faces.

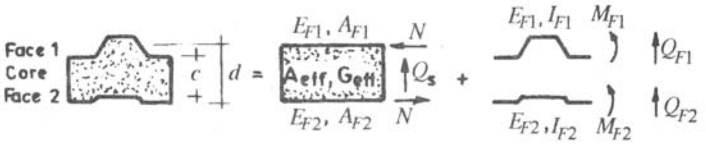

Fig. 4.- Separation of stress resultants into sandwich part and flange part.

The equilibrium equations are those from the engineer's theory of bending:

$$(13) \qquad \frac{\mathrm{d}M_s}{\mathrm{d}x} - Q_s = 0, \qquad (14) \qquad \frac{\mathrm{d}Q_s}{\mathrm{d}x} + q = 0$$

and substituting the stress resultants - deflections relationships into these ones it results:

$$(15) \qquad D_s(\gamma'' - w''') - A_{\text{eff}}G_{\text{eff}}\gamma = 0, \qquad (16) \qquad A_{\text{eff}}G_{\text{eff}}\gamma' + q = 0.$$

From equations (6),....,(9) and together (10),...,(12), two differential equations result namely:

$$(17) \qquad M = D_s\gamma' - Dw'', \qquad (18) \qquad Q = A_{\text{eff}}G_{\text{eff}}\gamma - D_f w'''.$$

Eliminating γ and denoting $Q' = -q$, a fourth order differential equation in w is obtained:

$$(19) \qquad w^{IV} - \left(\frac{p}{L}\right)^2 w'' = \left(\frac{p}{L}\right)^2 \frac{M}{D} + \frac{1+m}{m} \cdot \frac{q}{D},$$

where

$$(20) \qquad m = \frac{D_f}{D_s}, \;\; n = \frac{D_s}{A_{\text{eff}}G_{\text{eff}}L^2}, \;\; p^2 = \frac{1+m}{mn}.$$

Similarly, eliminating w from Eqs. (17) and (18) one obtain:

$$(21) \qquad \gamma'' - \left(\frac{p}{L}\right)^2 \gamma = -\frac{mp^2 Q}{D}.$$

The equations in the above form are particularly useful when the distributions of the total bending moment, M, and shear forces, Q, are known. For such cases, the general solutions of equations (19) and (21) are:

$$(22) \qquad w = C_1 \cosh\frac{px}{L} + C_2 \sinh\frac{px}{L} + C_3 + C_4 x + w_0.$$

and, respectively

$$\gamma = C_5 \cosh \frac{px}{L} + C_6 \sinh \frac{px}{L} + \gamma_0, \tag{23}$$

where w_0 and γ_0 are the particular integrals which depend primary on the loading. As these solutions must also satisfy equations (15), it result that:

$$C_5 = (1+m)\frac{p}{L}C_2, \quad C_6 = (1+m)\frac{p}{L}C_1. \tag{24}$$

Thus the number of constants of integration reduces to four and these can be determined from the boundary conditions, in particular, for a simply supported panel these ones are:

$$w(0) = 0, \quad w''(0) = 0; \quad w(L) = 0, \quad w''(L) = 0. \tag{25}$$

When the simply supported panel is loaded with the uniformly distributed load, q, per unit length, the stress resultants are:

$$M = \frac{q}{2}(Lx - x^2), \quad Q = \frac{q}{2}(L - 2x), \quad x \in [0, L]. \tag{26}$$

The particular integrals in equations (22), (23) are:

$$w_0 = \frac{q}{24D}\left(x^4 - 2Lx^3 - \frac{12L^2}{mp^2}x^2\right), \tag{27}$$ $$\gamma_0 = \frac{nqL^2}{2L}(L - 2x). \tag{28}$$

It result that the constants of integration are:

$$C_1 = \frac{qL^4}{mp^4D}, \quad C_2 = -\frac{qL^4}{mp^4D}\cdot\frac{\cosh p - 1}{\sinh p}, \quad C_3 = -\frac{qL^4}{mp^4D}, \tag{29}$$
$$C_4 = -\frac{qL^3}{D}\left(\frac{1}{24} + \frac{1}{2mp^2}\right),$$

so that the general solutions are:

$$w = \frac{q}{D}\left\{\frac{x}{24}(L^3 - 2Lx^2 + x^3) + \frac{L^2x(L-x)}{2mp^2} - \frac{L^4\left[\cosh\frac{p}{2} - \cosh\frac{p(L-2x)}{2L}\right]}{mp^4\cosh\frac{p}{2}}\right\}, \tag{30}$$

respectively,

$$\gamma = \frac{nq}{D}\left[\frac{L^3}{2} - L^2x - \frac{L^3\sinh\frac{p(L-2x)}{2L}}{p\cosh\frac{p}{2}}\right]. \tag{31}$$

Performing the calculi, the stress resultants are obtained namely:

$$M_s = \frac{q}{1+m}\left\{\frac{Lx}{2} - \frac{x^2}{2} - \frac{L^2\left[\cosh\frac{p}{2} - \cosh\frac{p(L-2x)}{2L}\right]}{p^2\cosh\frac{p}{2}}\right\}, \tag{32}$$

$$M_f = \frac{mq}{1+m}\left\{\frac{Lx}{2} - \frac{x^2}{2} - \frac{L^2\left[\cosh\frac{p}{2} - \cosh\frac{p(L-2x)}{2L}\right]}{mp^2\cosh\frac{p}{2}}\right\}, \tag{33}$$

$$Q_s = \frac{q}{1+m}\left[\frac{L}{2} - x + \frac{L\sinh\frac{p(L-2x)}{2L}}{p\cosh\frac{p}{2}}\right], \tag{34}$$

$$Q_f = \frac{mq}{1+m}\left[\frac{L}{2} - x + \frac{L\sinh\frac{p(L-2x)}{2L}}{mp\cosh\frac{p}{2}}\right]. \tag{35}$$

The important values of stress resultant and deflection at mid-span follow from the above equations considering $x = L/2$:

$$M_s\left(\frac{L}{2}\right) = \frac{qL^2}{1+m}\left[\frac{1}{8} - \frac{\cosh(p/2)-1}{p^2\cosh(p/2)}\right], \tag{36}$$

$$M_f\left(\frac{L}{2}\right) = \frac{mqL^2}{1+m}\left[\frac{1}{8} + \frac{\cosh(p/2)-1}{mp^2\cosh(p/2)}\right], \tag{37}$$

$$\sigma_{f_1} = -\frac{M_{f_1}d_{11}}{I_{f_1}} - \frac{M_s}{dA_{f_1}}, \quad \sigma_{f_2} = \frac{M_s}{dA_{f_2}}, \quad \tau_c = \frac{Q_s}{bd}, \tag{38}$$

$$w\left(\frac{L}{2}\right) = \frac{qL^4}{D}\left[\frac{5}{384} + \frac{1}{8mp^2} - \frac{\cosh(p/2)-1}{mp^4\cosh(p/2)}\right]. \tag{39}$$

3. Apparatus, Test Sandwich Panels, Experimental Procedure

In an experimental programme are used the sandwich panels with one deeply profiled facing and other lightly profiled facing, realized from galvanized sheet metal.

The core of panels is made from polyurethane foam and the bond between the faces and the core is carried out the same time with the foam formation through injection. The sandwich panels have the following dimensions: 3.00 m length, 1.00 m width, 40.0 mm thick.

The experimental stand shown in Fig. 5 has been utilized for the experimental tests. This stand consists of three reinforced concrete slabs, located at inter-axis distance equal to 1.50 m and fixed at the bottom side with a steel angle section. A cold-rolling steel, $\sum$ shape section, has been fixed on each slab to provide support and fixing conditions for the tested panel, similar with those provided of the ridge purlins.

A special support system for transducers has been designed and manufactured to avoid the errors caused by the distortion of the $\sum$ shape profile. This special device has been suspended in the regions of support and fixing of the sandwich panels on the profiles (Fig. 5). The sandwich panels have been fastened to each of the three $\sum$ profiles with two self-drilling screws placed on the lateral folds of the central profile.

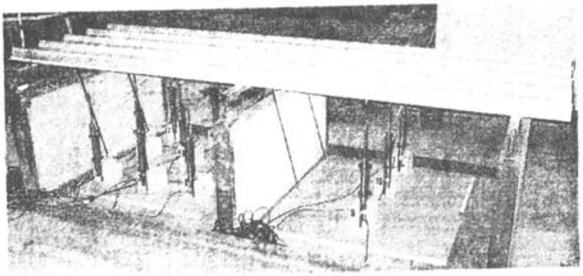

Fig. 5. The experimental stand.

The LVDTs (linear variable displacement transducers), arranged as shown in Fig. 5. have been utilized to measure the transverse deflections. The rods of LVDTs have been fixed by means of steel wires to the studs adhesive bonded to the bottom side of the panels (Fig. 6).

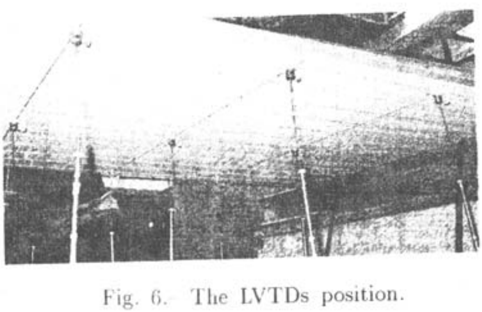

Fig. 6. The LVTDs position.

The loading test has been carried out with successive ballast layers of gravel filled sacks of 50.0 N weight each. In the first stage of loading the ballast sacks have been placed longitudinally between panel profiles and transversely in the second stage. Each ballast layer consists of 24 sacks placed on the two spans (equivalent to a total load equal to 1,200.0 N or a distributed load of 400.0 N/m^2). A complete record of the vertical deflections has been performed in all 12 measuring points for each loading/unloading stage equal to 400.0 N/m^2.

4. Experimental Results

The first panel has been mainly tested to characterize the overall behaviour of the structural system and the experimental work has been developed in two stages:

a) under uniformly distributed load on both spans – experiment *1* – and

b) under uniformly distributed load on one span – experiment *2*.

The load-maximum deflection diagrams for each span are illustrated in Fig. 7 (the average values of 3, 4, 5 and 9, 10, 11 LVDTs, respectively) for the experiment *1*. The panel has been ballast under a total weight equal to 6,000.0 N/m^2. An identical behaviour of the panel in both spans has been noticed up to a uniform transverse load equal to 3,000.0 N/m^2; in case of loads exceeding 5,500.0 N/m^2 a decrease of the panel stiffness has been identified.

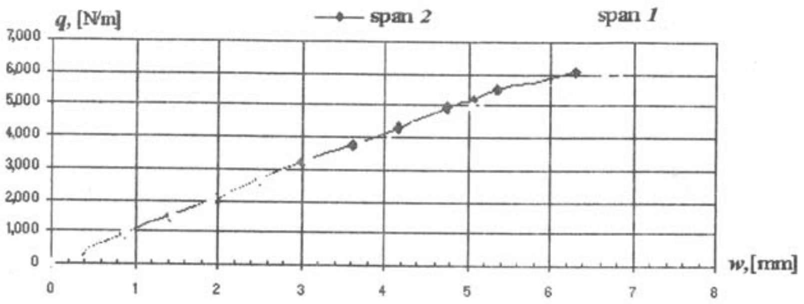

Fig. 7.- The load maximum deflection diagrams.

From the analysis of the two structural responses on the sides of the central zone an insignificant difference between measured deflections has been found out and the ensemble could be considered as a simple supported system on the internal support. This assumption is currently accepted when the ultimate load capacity of the panel is evaluated.

In Fig. 8 the plots of load-average deflection, for span *1* are illustrated (the average values of 3, 4, 5 and 0, 1, 2 and 6, 7, 8 LVDTs, respectively). Performing the analysis of the structural response can be pointed out that the panel behaves like a multi-span beam. The deflection from the central support (right) is less than the deflection from the free end on the whole range of testing load. When the uniformly transverse load was applied on span *1* only (experiment *2*), the test was carried out until the panel failure.

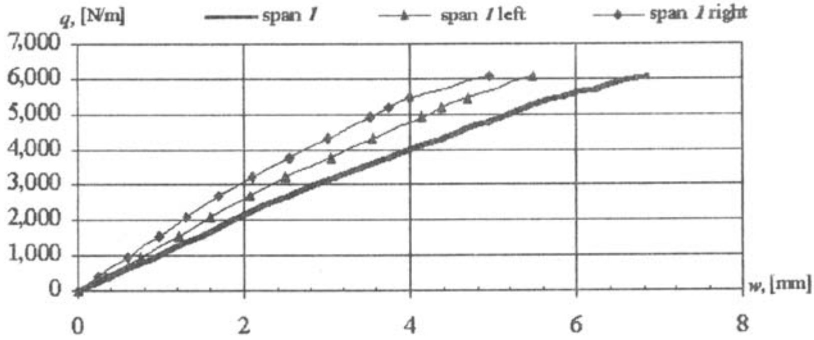

Fig. 8.- The plots of load average deflection, for span *1*.

In Fig. 9 the load–deflection relation in the central zone together the load–deflection relations in the lateral sides of the mid-span are illustrated (the average values recorded by LVDTs 0, 1, 2 and 6, 7, 8, respectively). The sandwich panel with the metallic facings failure has been initiated by the buckling of the compressed (upper) profiled facing, when the uniformly transverse load exceeded 1,100.0 N/m^2.

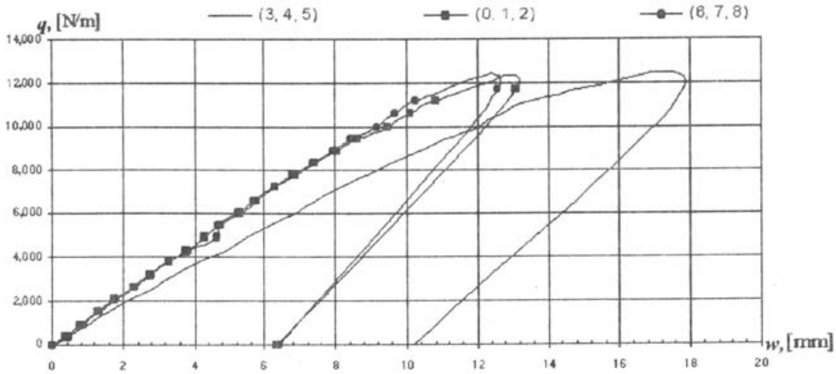

Fig. 9.- The load deflection relation in the central zone.

In Table 1 are presented the theoretical values of the deflections in different points of the sandwich panel span, calculated according to the above relationships and the experimental values measured in the same points.

Table 1
The Theoretical and the Experimental Values of the Deflections

No. sacks	q unit N/m	$x = L/4$		$x = L/2$		$x = 3L/4$	
		w theor. mm	w exp. mm	w theor. mm	w exp. mm	w theor. mm	w exp. mm
24	400	0.299	0.327	0.417	0.382	0.299	0.232
56	933	0.697	0.766	0.972	0.911	0.697	0.598
92	1,533	1.145	1.223	1.597	1.463	1.145	0.976
124	2,067	1.544	1.600	2.154	1.936	1.544	1.311
160	2,667	1.993	2.077	2.779	2.524	1.993	1.701
192	3,200	2.391	2.510	3.335	3.095	2.391	2.089
228	3,800	2.839	3.053	3.960	3.742	2.839	2.536
260	4,333	3.238	3.554	4.515	4.380	3.238	3.000
296	4,933	3.686	4.141	5.140	5.078	3.686	3.522
312	5,200	3.885	4.388	5.419	5.391	3.885	3.733
328	5,467	4.085	4.680	5.697	5.741	4.085	3.996
364	6,067	4.533	5.482	6.322	6.865	4.533	4.942
380	6,333	4.732	5.775	6.599	7.184	4.732	5.194
396	6,600	4.931	6.080	6.878	7.598	4.931	5.494

5. Conclusions

The paper presents the analysis of the sandwich panels with profiled metallic faces. The analysis is carried out regarding the geometrical characteristics and possible loading according with the present European design codes. The experimental work performed on fully sized panels under working loads conditions have shown that:

a) the contribution of the profiled facing is balanced out with regard to the local bending stiffness and global rigidity;

b) both stress distributions and deflections are influenced by the profiled face;

c) the difference in deflection values can be explained by the quality of the face/core interface and the influence of the core rigidity.

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ANALIZA ŞI COMPORTAREA PANOURILOR SANDVIŞ CU FEŢE METALICE PROFILATE SUPUSE LA ÎNCĂRCĂRI TRANSVERSALE

(Rezumat)

Se prezintă calculul panourilor sandviş cu feţe metalice profilate, luând în consideraţie particularităţile de calcul care apar în analiza acestor tipuri de elemente de construcţie. Rezultatele teoretice obţinute prin efectuarea calculului în cazul panourilor sandviş cu feţe cutate sunt comparate cu rezultatele experimentale înregistrate la încercările panourilor la scară naturală. Valorile rezultatelor obţinute pe cale teoretică sunt comparabile cu cele ale datelor înregistrate experimental; astfel se constată că panourile sandviş preiau în condiţii normale încărcările de calcul considerate, concomitent cu satisfacerea cerinţelor de rigiditate şi de deformabilitate locală (voalarea feţei comprimate).