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APPLYING PARAMETER IDENTIFICATION OF STRUCTURAL MODELS ASSISTED BY MATLAB

BY

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Abstract. The major goal of this paper is to study the fundamental concepts of *structural dynamics* and of *system identification* methods, respectively, applied in the field of civil engineering with the aid of MATLAB programming environment. In the process of dynamic modeling and analysis of constructions, the notion of system identification has been introduced to come to our aid. These processes are based on simplified or equivalent models of the real structure, and in order to obtain the best results that are closest to reality various methods for estimating the model's parameters can be used.

Key words: modeling; simulation; parametric identification; MATLAB applications.

1. Introduction

Full-scale measurements of buildings show that the mode shapes and frequencies from the analytical models analysis are not in accordance with the experimental results. This is the consequence of improper modeling of some of the structural elements. The mathematical model needs to be improved, such that the difference between the modal response of the analytical and

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experimental analysis is minimized. In order to close the gap between analytical and experimental mode shapes and frequencies, the buildings stiffness and mass matrices may be updated. The computation of the mass matrix is based on the mass of dead weight of the elements, components and systems and is accurate than the stiffness matrix, therefore, the stiffness matrix of the building should be revised (Natke, 1988).

In most cases of System-Identification (SI) applications on constructed civil structures, the analytical degrees of freedom defining finite element models are much greater than the measurement degrees of freedom. This incompatibility creates a number of difficulties in correlating the analytical and experimental results. To solve this problem, model expansion or reduction must be used to either expanding the measured mode shapes to the size of their analytical counterparts or reduce the predicted mode shapes to the size of measured ones.

Finite Element Method (FEM) is a powerful tool to simulate and predict the dynamic properties of various systems. As a result of the discretization and idealization processes, errors will appear in the presentation of continuous systems by finite element (FE) models. It is important that in the process of SI the causes of observed structural behaviors must be understood in order to eliminate modeling errors that can appear in the initial FE model.

Some of the errors come from uncertainty associated with experimentation, and some can be attributed to modeling error inherent in finite element models of the structure. Recognizing the sources and locations of the modeling errors in preliminary models can help analysts to implement experimental validation and assess the reliability of structural predictions from refined models (Quin Pan, 2007).

Commonly encountered modeling errors are: discretization errors, parameter errors and conceptualization errors.

A series of parameter error localization methods have been proposed over the years such as

a) the best subspace method (Lallement & Piranda, 1990); Link, 1991); Maia *et al*., 1994);

b) force balance method (Fissette&Ibrahim, 1988); Lallement&Piranda, 1990; Baker &Marsh 1996);

c) substructure energy function method (Link, 1991; Fritzen & Kiefer, 1992).

2. System Identification Applied in Civil Engineering

A system can be considered to be a collection of objects, parts and components, which interact with each other, within a notional boundary to produce a particular pattern of behavior (Gordon, 1969; Shearer *et al*., 1967).

For educational purposes the problem of system and model system definition begins with physical modeling in order to realize a smooth transition from earlier concept of modeling to system simulation concept.

For simulation purposes the abstract modeling is important and it takes one of the two forms:

1. *Continuous models*, when the model behavior characteristics change continuously and are accessible at every moment of time.

2. *Discrete models*, for which the changes of state occur only at set instants of time.

The concept of dynamic model means a mathematical model consisting in a set of differential eqs. that described the dynamic behavior of the system. The used models are *linear and time-invariant models* (LTI).

The representative models used in structural dynamics with application to earthquake engineering are the following:

a) *Equivalent models*, which are linear time-independent static models still common for in the codes for seismic design (Reinhorn *et al.*, 2000).

The static equilibrium equation is

$$F_{\max} = kS_d = mS_a, \tag{1}$$

where: k, m represent the stiffness and, respectively, the mass of a single degree of freedom (SDOF); S_d, S_a mean the displacement spectrum and, respectively, the acceleration spectrum.

b) *Lumped linear models*, in which the dynamic model is expressed using Newton's law and the free body diagram. The governing eq. is a second order differential eq., which has the following form for LTI models (Reinhorn *et al.*, 2000):

$$\mathbf{M}\ddot{x}(t) + \mathbf{C}\dot{x}(t) + \mathbf{K}x(t) = \mathbf{F}(t), \tag{2}$$

where: **M** is the $(n \times n)$ mass matrix; **C** – the damping matrix; **K** – the $(n \times n)$ stiffness matrix.

c) *Distributed parameter models*. The equation of motion, in the case of free vibrations, is a fourth-order differential eq. with partial derivatives (Reinhorn *et al.*, 2000)

$$EI\frac{\partial^4 w}{\partial x^4} + \rho\frac{\partial^2 w}{\partial t^2} = 0, \tag{3}$$

where: E is Young's modulus; I – the cross-sectional moment of inertia; ρ – the density; w – the deflection.

d) *The state-variable form for models* is an alternative way of expressing the second-order differential eq. that characterizes the model, is given by the vector

$$\dot{\mathbf{x}} = f(\mathbf{x}, u), \tag{4}$$

$$\mathbf{y} = h(\mathbf{x}, u), \tag{5}$$

where: u is the input and the vector $\mathbf{x}$ is called *the state of the system*.

The dynamic modeling based on state variable form model has been used in identification and structural control by computer-aided control system design (CACSD) software packages, including MATLAB with its toolboxes. The model can be expressed in the following forms: continuous-time (analog) and discrete-time (digital) forms (Atanasiu, 2007).

For the continuous-time models we have:

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u, \tag{6}$$

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}u. \tag{7}$$

where: for the *n-th* order system, $\mathbf{A}$ is a $(2n \times 2n)$-system matrix; $\mathbf{B}$ is a $(2n \times \times 1)$-input matrix; $\mathbf{C}$ is a $(1 \times 2n)$-output matrix; $\mathbf{D}$ a (1×1) matrix called *direct transmission term*.

For the discrete- time models we have:

$$x(k+1) = \mathbf{A}x(k) + \mathbf{B}u(k), \tag{8}$$

$$y(k) = \mathbf{C}x(k) + \mathbf{D}u(k). \tag{9}$$

The process of identification, which catches some of the most important properties of the process behavior, is based on step response analysis. System identification can be achieved when the inputs as well as the output signals are available as measured quantities.

There are two kinds of models including parametric model and non-parametric model.

a) The *parametric model* (*white box models*) is the model in which the transmission of the signal through the object is supposed to be known and can be described by differential equations.

b) In *non-parametric model*, modeling the geometrical and the physical structure of a system are established by means of regression or correlation analysis (behavior model). Non-parametric models are called *black box models* because system identification is based not only on measurements but also on the mechanical model.

It is necessary to identify such a system at first, and this means to get its statistically adequate mathematical model. This is possible by developing a fast and correct control system and by using suitable software to forecast the behavior of the system in the near future. This way we have the possibility to make corrections before the system reaches an unstable region (Leitner, 2011).

3. Examples

To illustrate the principles of parametric identification, a series of algorithms implemented in the MATLAB environment have been proposed and updated. The computations have been performed on an equivalent single degree of freedom model (EMSDOF) system of a real structure with the following material and dynamic characteristics:

$E = 3\text{e}7\ \text{kN/m}^2$,

$\zeta = 0.2$,

$m = 960.37$ t,

$k = 2.86\text{e}6$ kN/m,

$\omega = 54.569$ Hz,

$T = 0.115$ s,

$c = 2.096\text{e}4$.

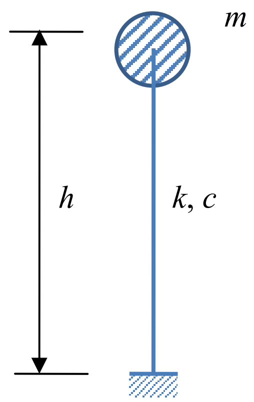

Fig. 1 – SDOF model.

3.1. Example 1

This example is aiming to solve a single degree of freedom (SDOF) system, given the mass, damping and stiffness terms in dimensionless units (Figs. 2 and 3).

a) *Partial-fraction expansion* (*residues*).

[R,P,K] = res (B,A) finds the residues, poles and direct term of a partial fraction expansion of the ratio of two polynomials, $B(s)/A(s)$. If there are no multiple roots,

$$\frac{B(s)}{A(s)} = \frac{R(1)}{s-P(1)} + \frac{R(2)}{s-P(2)} + \ldots + \frac{R(n)}{s-P(n)} + K(s)\,. \tag{10}$$

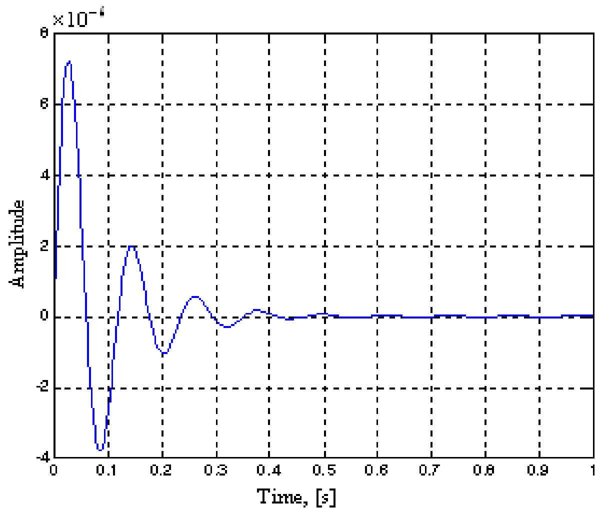

Fig. 2 – Impulse response function: SDOF system.

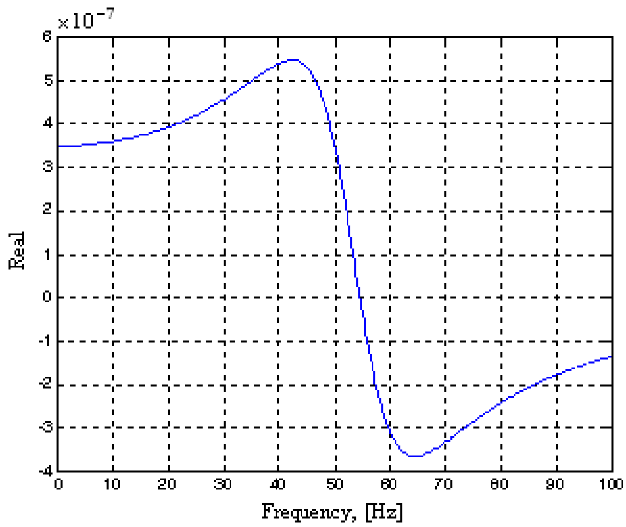

Fig. 3 – Frequency response function (real part): SDOF system.

3.2. Example 2

This example is aiming to solve a single degree of freedom model SDOF, given the mass, damping and stiffness terms in dimensionless units using a Runge-Kutta approach (Fig. 4).

Consider the eq. of motion for a single degree of freedom vibrating system written as a system of first order differential eqs.

$$\dot{V} = \ddot{u}(t) = \frac{1}{m(f - cV - kX)}, \tag{11}$$

where: $A = \ddot{u}(t)$, $V = \dot{u}(t)$, $X = u(t)$.

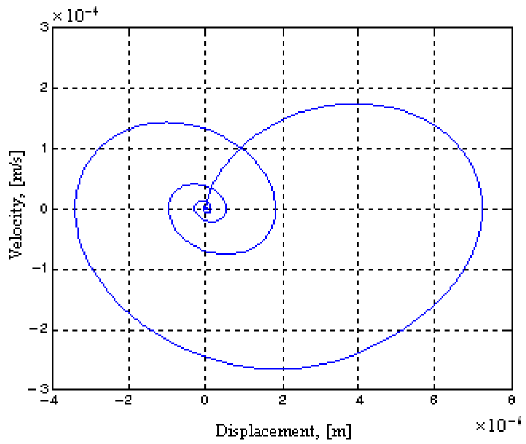

Fig. 4 – Phase plane plot: SDOF model (Runge-Kutta approach).

3.3. Example 3

This example is aiming to plot the displacement transmissibility for several different damping values for a single degree of freedom model SDOF. The plot is made for normalized frequency, being presented in Fig. 5..

Input data: ν = 1; 2; 6; 10; 40; 8; 100%.

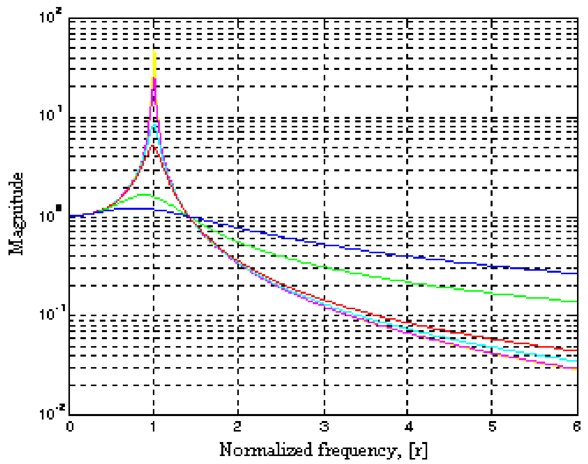

Fig. 5 – Effect on magnitude by changing the damping: SDOF model.

3.4. Example 4

This example is aiming to solve a SDOF model, given the mass, damping and stiffness terms in dimensionless units. The output is a three dimensional plot in the *s*-domain complex independent variable (Fig. 6).

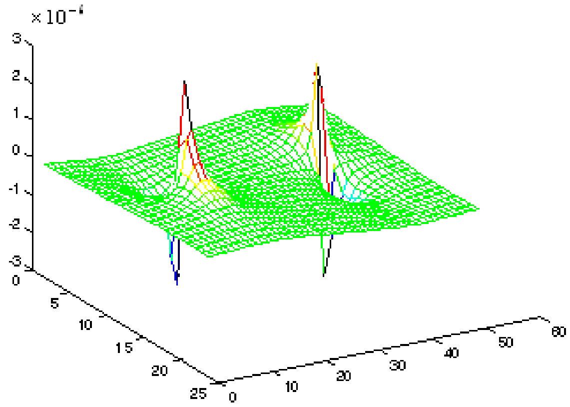

Fig. 6 – Transfer function imaginary part, surface representation: SDOF model.

3.5. Example 5

This example is aiming to solve a single degree of freedom model SDOF, given the mass, damping and stiffness terms in dimensionless units when the stiffness is varied. The output includes poles, residues (modal coefficients) and frequency domain plots of the frequency response functions (Fig. 7).

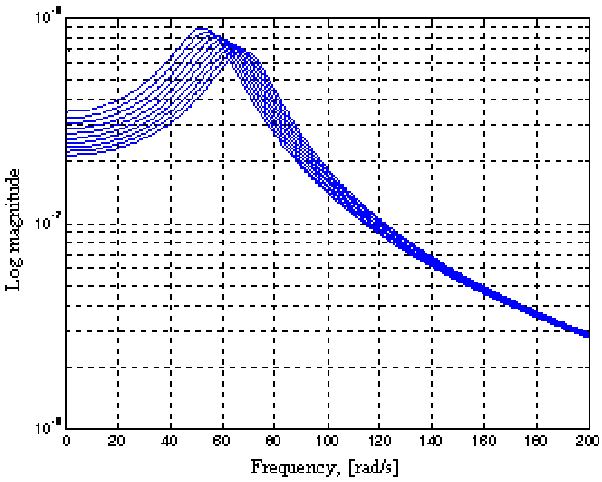

Fig. 7 – Frequency -magnitude response (change of stiffness): SDOF model.

4. Conclusions

System identification can be successfully used for minimizing the errors that appear when comparing analytically predicted dynamic properties with test measurements. The powerful MATLAB software proves to be an useful programming environment that can be employed to make preliminary models, to estimate dynamic properties, to perform simulation analysis thus removing the need for experimental measurements.

The parametric identification methods forecast the behavior of a system and allow us to compute estimations of parameters and to make corrections to the initial computational model.

The examples were carried out in linear domain, and MATLAB has proven to be a very useful tool for computation dynamic properties of a SDOF model, considered as equivalent for the real civil engineering structures.

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APLICAREA IDENTIFICĂRII PARAMETRICE PE MODELE STRUCTURALE ASISTATE DE MATLAB

(Rezumat)

În urma măsuratorilor efectuate pe structuri de clădiri s-a constatat ca modelele analitice nu oferă rezultate care să coincidă cu acurateţe cu cele din măsuratori. Aceasta se poate datora modelării incorecte a elementelor structurale. Ca urmare, modelul matematic utilizat în simulări trebuie îmbunătăţit astfel încât diferenţele dintre răspunsurile modale analitice şi cele experimentale să fie minime. Este important să identificăm sursa şi locaţia erorilor de modelare pentru a putea obţine predicţii de încredere, iar acest lucru se poate realiza cu ajutorul metodelor de identificare parametrică. Lucrarea îşi propune să ilustreze noţiunile care stau la baza identificării sistemelor cu aplicare in domeniul ingineriei civile asistate de metode matematice de calcul prin care se poate atinge acest scop. În finalul lucrării sunt prezentate câteva

exemple care atestă aplicabilitatea programului de calcul MATLAB în studiul caracteristicilor dinamice ale structurilor modelate prin sisteme echivalente cu un singur grad de libertate. Cu ajutorul programului MATLAB se poate evalua comportarea la acţiuni dinamice a structurilor, eliminând necesitatea de a realiza experimente de laborator ce pot fi costisitoare şi pot consuma destul de mult timp pentru pregătire şi interpretarea rezultatelor.