

BULETINUL INSTITUTULUI POLITEHNIC DIN IAȘI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iași
Tomul LVIII (LXII), Fasc. 4, 2012
Secția
CONSTRUCȚII. ARHITECTURĂ

NUMERICAL EXPERIMENTS ON DYNAMIC RESPONSE EVALUATION OF SOIL STRUCTURE INTERACTION EFFECTS

BY

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Received: September 5, 2012

Accepted for publication: September 29, 2012

Abstract. Soil–Structure Interaction is an important issue in the dynamics of structures, being not taken into consideration, in many cases, due to complex volume of computations. The interaction between soil and foundation becomes meaningful especially in the case of rigid structures and foundations, being object of intensive investigations in latest 30 years. The present study is aiming to evaluate the dynamic response of a foundation structure, interacting with the soil using the computational environment offered by EDT software (version 2.2, 2010).

Soil–structure interaction of rigid foundations is characterized by impedance functions. They are determined in this paper for a soil which may be arbitrarily layered and is underlain by a rigid or an elastic half space. Vertical and rocking motions are considered.

Two methods for the computation of the impedance functions are applied. The first approach is the so-called *Thin Layer Method* (TLM) following a theory developed by Waas-Riggs-Werkle (1985). It has been implemented by Horst Werkle (1985) in a MathCAD environment. In addition a solution in the wave number domain using the *Elasto-Dynamic Toolbox* developed by Schevenels

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(Katholieke Universiteit Leuven, Belgium – 2008) is implemented in MATLAB. The results of dynamic analysis obtained with both theories are compared with the analytical solution for an elastic half space obtained by Gazetas (1983).

Key words: Soil Structure Interaction; Impedance functions computation; ESD; Elasto-Dynamic Toolbox; MATLAB; MathCAD.

1. Introduction

In case of seismic or dynamic actions as explosions for example, the soil is modelled as a visco-elastic medium. The nonlinear behaviour of the soil may be approximated by its equivalent linear stiffness and damping. The time dependent displacements in the soil caused by a dynamic loading of the foundation can be considered as a superposition of three-dimensional waves, travelling through a continuum as *P*- or *S*-waves and surface waves as Rayleigh or Love waves. The *P*-waves are compression waves that appear in case of sudden growth of the compressive stress in the underground, in which the particles move in the direction of wave propagation. For *S*-waves, transversal or shear waves, the soil particles move perpendicular to the direction of wave propagation. The *S*-waves can be further divided into components in a horizontal plane, named as *SH*-waves, and in a vertical plane, called *SV*-waves, respectively. Other types of waves are surface waves which propagate along the soil surface. Their amplitudes decrease rapidly with depth. There are two types namely Rayleigh waves and Love waves (Fig. 1). A Rayleigh wave is a surface wave causing the ground to shake in an elliptical motion with no transverse displacements. A Love wave is a surface wave having a horizontal displacement that is transverse to the direction of wave propagation.

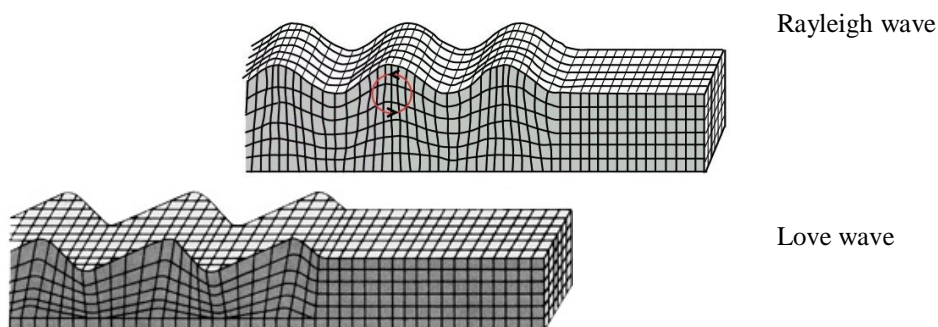


Fig. 1– Surface waves (<http://earthquake.usgs.gov/learn/glossary>).

In this study one considers a structure having finite dimensions, lying over a deformable soil of infinite dimensions. If dynamic loads are applied the

structure with its foundation starts to move. The motion of the foundation placed on a half space causes time-dependent soil deformations which can be described by seismic waves. Due to the propagation of waves energy is dissipated from the structural system. This effect is denoted as *geometric damping*.

The relationships between the time dependent forces and displacements of a rigid foundation on a homogeneous or layered half-space representing the elastic properties and the wave propagation in soil can be reduced to a single mass oscillator system for each degree of freedom (Fig. 1) for vertical motion. For a rigid foundation the impedance functions, *i.e.* the relationship between its forces and the displacements can be represented by springs and dampers. Fig. 1 illustrates a spring-damper system in which the spring resumes the elastic behaviour of the soil, while the damper describes the geometrical energy radiation (reproduced after Studer *et al.*, 2007). However the springs and dampers are frequency dependent, *i.e.* the analysis is done in frequency domain. An overview on soil–structure–interaction is also given by Wolf (1985, 1988).

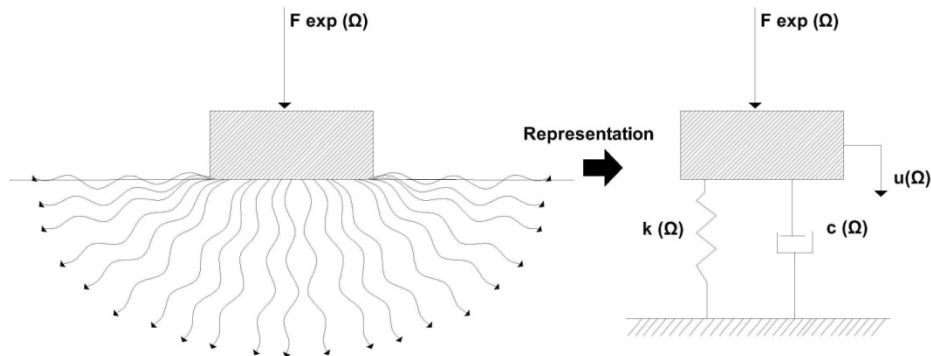


Fig. 2 – Model for a mass less rigid foundation on a half-space (Studer *et al.*, 2007).

2. State of the Art for Computation of Impedance Functions

The impedance functions give the harmonic reaction forces or moments caused by a harmonic displacement or rotation, respectively, which has amplitude of one. In frequency domain complex symbolic representation has been used to describe the amplitude and phase of the motion. A force can be described by

$$P = \Re e(\tilde{P}e^{i\Omega t}) \quad (1)$$

and a displacement by

$$u = \Re e(\tilde{u}e^{i\Omega t}). \quad (2)$$

(see Fig. 1). Herein Ω denotes the circular frequency of the vibration, t – the time variable and \tilde{F} , \tilde{u} – the complex force and displacement, respectively. The complex spring, \tilde{K} , is now given as

$$\tilde{F} = \tilde{K}\tilde{u}, \quad (3)$$

with

$$\tilde{K} = k_1 + ik_2. \quad (4)$$

The real part of the complex spring denotes the static part of the spring whereas the imaginary part is related to damping. In general real and imaginary part of a complex spring may depend on frequency, Ω . Hysteretic material damping may be conveniently taken into account by a complex Young's modulus or shear modulus as

$$\tilde{E} = E(1 + 2i\xi), \quad (5a)$$

$$\tilde{G} = G(1 + 2i\xi), \quad (5b)$$

where: E and G are the Young's modulus and shear modulus, respectively, and ξ – the hysteretic damping ratio.

In order to describe the impedance functions of rigid circular foundations on a homogeneous soil the following notation is often used in literature:

$$\tilde{K}(\Omega) = K_{\text{stat}} [k(\Omega) + ia_0c(\Omega)](1 + 2i\xi). \quad (6)$$

Herein K_{stat} denotes the static stiffness and $k(\Omega)$, $c(\Omega)$ are dimensionless but frequency dependent impedance functions. The dimensionless frequency, a_0 , is defined as

$$a_0 = \frac{\Omega r_0}{v_s}, \quad (7)$$

where: r_0 is the radius of the circular foundation, $v_s = \sqrt{G/\rho}$ – the shear wave velocity, ρ – the density and ξ – the material damping of the soil.

Impedance functions of rigid foundations have been derived for different geometries of soil and foundation by many authors (see Studer (2007)). Fig. 2 gives the dimensionless impedance functions according to eq. (6) of vertical and rocking motion of a rigid circular foundation on elastic-half space with $\zeta = 0$ after Gazetas (1983). The corresponding static stiffness constants, K_{stat} , are for vertical motion

$$K_{z, \text{stat}} = \frac{4Gr_0}{1-\nu} \quad (8 a)$$

and for rocking

$$K_{\eta, \text{stat}} = \frac{8Gr_0^3}{3(1-\nu)}. \quad (8 b)$$

G again denotes the shear modulus and ν – the Poisson ratio of the soil.

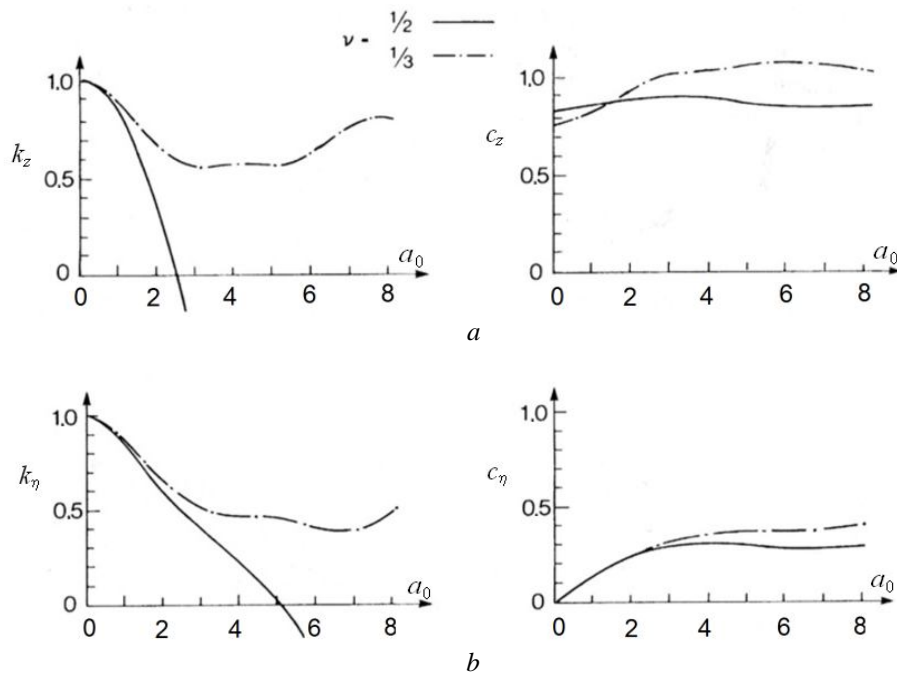


Fig. 3 – Dimensionless impedance functions of a rigid circular foundation on a half-space (Gazetas, 1983): a – vertical; b – rocking.

For an elastic layer over a rigid base solutions have been derived by different authors, *e.g.* by Gazetas (1983). In case of a soil with a shear modulus

linearly increasing with depth, impedance functions of a circular foundation have been given by Werkle (1988). Some solutions are also available for embedded foundations (see Studer, 2007).

3. Greens Functions for a Layered Soil

3.1. Thin Layer Method

The computation of dynamic impedance functions is based on the Green's functions of soil. A Green's function is a fundamental solution which describes in the case of elasto-dynamics, the dynamic displacements in an elastic medium due to a unit point load.

Classical solutions have been given for a point load on the surface of an elastic half-space in frequency domain. The Thin Layer Method (TLM), as developed by Lysmer and Waas (1972), was one of the first solutions for layered soils. In this semi-analytical technique the displacement functions are expressed by their analytical solution in the horizontal direction whereas in vertical direction piecewise linear shape functions as in the finite element method are used. It is well suited to model horizontally layered media where each layer may possess individual elastic parameters. Kausel and Peek (1982), as well as Waas *et al.* (1985) applied the method to obtain Green's functions for point and distributed loads acting within (or on) a layered horizontally unbounded medium, with homogeneous or orthotropic material, respectively. These Green's functions were later extended by Seale and Kausel (1989) for the modeling of layered media over elastic half spaces. Whereas the classical method is in frequency domain, Kausel (1992) also gave a solution in time domain.

In the following the formulation according to Waas *et al.* (1985) is studied. Starting with the eqs. of motion of the three-dimensional visco-elastic continuum, first a solution of the homogeneous boundary value problem of the layered soil with a rigid base is investigated. This leads to two eigenvalue problems, one for Rayleigh waves and another for Love waves (not shown here).

According to Waas (1972) the eigenvalue problem for Rayleigh waves in a layered visco-elastic continuum can be written

$$\left([A_R]k_R^2 + [B_R]k_R + [C_R] \right) \{v\} = \{0\}. \quad (9)$$

The matrices $[A_R]$, $[B_R]$ and $[C_R] = [G_R] - \Omega^2 [M_R]$ represent the system's stiffness and mass and are assembled by the corresponding 4×4 -matrices of each layer, $[A_{el}]$, $[B_{el}]$, $[G_{el}]$ and $[M_{el}]$ (for details see Waas,

1972, 1985). Hence, for a system with n_{lay} thin layers, the system matrixes have a size of $(2n_{lay}) \times (2n_{lay})$. The eigenvalues, k_R , are the complex wave numbers, $\{v\}$ is the shape of the Rayleigh wave. Only those Rayleigh waves, which decay or propagate energy from the origin, are selected in order to satisfy the radiation condition at infinity.

The solution of the inhomogeneous boundary value problem with prescribed loads at the surface or within the layered continuum is now obtained as superposition of the Rayleigh and Love waves multiplied by participation factors. In the horizontal direction the analytical solution in cylindrical coordinates is represented by Hankel functions. For a vertical distributed disk load, p_z , with radius r_0 on the soil surface (Fig. 4) the vector of vertical displacements at the layer interfaces at distance r from the center of the disk load is given by Waas *et al.* (1985) as

$$\{w\} = -\frac{i\pi}{2} p_z e^{i\Omega t} \sum_{j=1}^{2n_{lay}} \{z_j\} Z_{1j} \begin{cases} k_{R_j} r_0 H_0^{(2)}(k_{R_j} r) J_1(k_{R_j} r_0), & \text{for } r \geq r_0, \\ k_{R_j} r_0 J_0(k_{R_j} r) H_1^{(2)}(k_{R_j} r_0) + \frac{2}{i\pi}, & \text{for } r \leq r_0 \end{cases}, \quad (10)$$

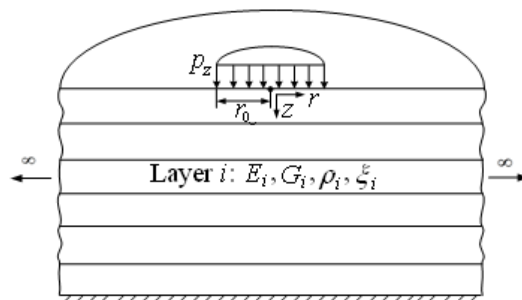


Fig. 4 – Disk load on a layered continuum with a rigid base.

where k_{R_j} and $\{z_j\}$ are the wave number k_R and the vector of the vertical displacement components of the eigenvector $\{v\}$ in eq. (9), respectively, in the j -the mode. $\{v\}$ has to be normalized according to eq. (A 12). Z_{1j} is the first element of $\{z_j\}$ which relates to the vertical displacement at the soil surface. J_0, J_1 are Bessel functions and $H_0^{(2)}, H_1^{(2)}$ – Hankel functions of second kind. Solutions for horizontal displacements and other types of loading are given by Waas *et al.*, (1985).

The TLM has been implemented in several software codes, *e.g.* in SASSI 2000 (Lysmer *et al.*, 2006). An implementation in a MathCAD environment has been provided by Werkle (HTWG, Konstanz).

3.2. Wave Number Domain Solution (EDT)

Now the case of a layered medium underlain by a visco-elastic half-space instead of a rigid base is considered. In this case of a system being unbounded in vertical direction, eigenfrequencies of the layered system and hence definite eigenvalues for the wave numbers as in eq. (9), do not exist. Instead a transformation from the space domain to the wave number domain is performed by means of a Hankel transformation. The algorithm can be improved through the use of a window and a filter to mitigate artifacts caused by the Gibbs phenomenon (Degrande & Geraedts, 2008).

The method has been implemented in the *Elasto Dynamic Toolbox* (EDT) which is a MATLAB toolbox developed by Mattias Schevenels (Katholieke Universiteit Leuven, Belgium) (see Schevenels *et al.*, 2010). The EDT toolbox allows computing the response due to a dynamic harmonic loading of the soil in frequency domain. This necessitates also the evaluation of inverse wave number integrals from the wave number to the spatial domain. An algorithm developed by Talman (1978), based on a logarithmic change of variables, has been implemented in EDT. The EDT toolbox can be used to solve problems of wave propagation as site amplification, surface waves or forced vibration problems.

In order to determine the displacement at the soil surface due to a disk load the function *Disk_rec* of the EDT can be used. EDT provides the function *disk3d_rec* to compute the disk load solutions in Cartesian coordinates. This function computes all three components of the displacement vector and all six components of the stress tensor at any coordinate of the half space.

3.3. Numerical Results

The displacements on the surface of a homogeneous half space due to a time-harmonic disk load, corresponding to a unit force in z -direction, have been computed both in MATLAB (using EDT) and in MathCAD (using the TLM). For the TLM model a large soil layer with 40 thin layers and a depth of 98 m is used in order to simulate a half space.

Following numerical data for very soft clay have been used: modulus of elasticity $E = 5,000 \text{ kN/m}^2$, Poisson ratio $\nu = 0.3$, soil density $\rho = 2.0 \text{ t/m}^3$ and a damping ratio $\xi = 0.05$ in both shear and volumetric deformation. This corresponds to a shear wave velocity $c_s = 31 \text{ m/s}$; compression wave velocity $c_p = 58 \text{ m/s}$. The excitation frequency used was 0, 1.0 and 2.0 Hz. The radius of the disk load is $r_0 = 0.564 \text{ m}$.

A comparison between the numerical results of displacements in z -direction for different frequencies and at different distances from the origin,

obtained in the MATLAB and MathCAD environment, is given in Table 1. The displacements at different distances from the origin for $f = 0, 1.0, 2.0$ Hz are presented in Fig. 5. The values obtained in both computational environments and with both numerical methods are in good agreement for all frequencies.

Table 1
Values of Displacements Obtained within MATLAB and Mathcad for Different Frequencies

Distance r , [m]	Frequency, [Hz]					
	$f = 0$ Hz		$f = 1$ Hz		$f = 2$ Hz	
	MATLAB	MathCAD	MATLAB	MathCAD	MATLAB	MathCAD
0	1.78E-04	1.76E-04	1.74E-04	1.74E-04	1.71E-04	1.69E-04
0.564	7.90E-05	8.09E-05	7.57E-05	7.85E-05	7.14E-05	7.21E-05
1.128	3.65E-05	4.16E-05	3.28E-05	3.81E-05	2.65E-05	2.89E-05
1,692	2.41E-05	3.02E-05	1.98E-05	2.55E-05	1.18E-05	1.41E-05

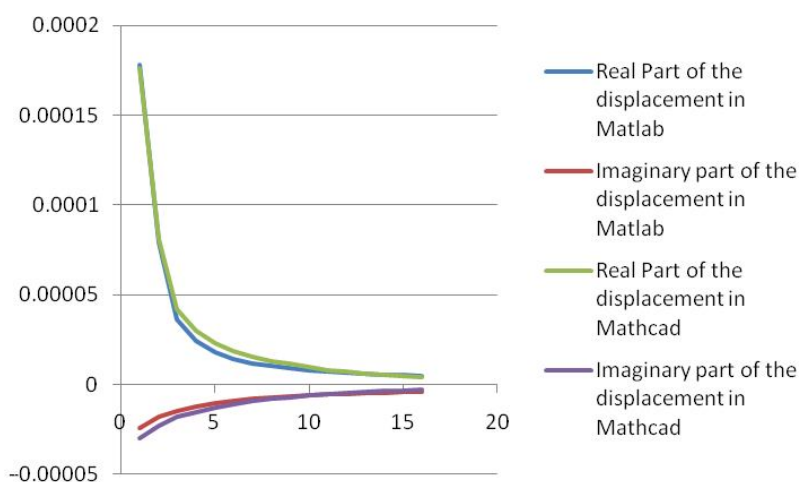


Fig. 5 – Graphical representation of the displacements' results, in MATLAB and MathCAD, for $f = 0$ Hz.

4. Impedance Functions Computation

The Green's functions as obtained in the MathCAD environment, based on TLM and MATLAB environment using EDT (Talman, 1978) are now used to determine the impedance functions of a square foundation.

In order to compute the flexibility matrix of the soil, the square foundation is divided into n square sub-areas (Fig. 6). The centers of the sub-areas correspond to the nodal points of the flexibility matrix. At each of these points a disk load with radius r_0 is applied. The area of all disk loads is equal to the area of the total square load, *i.e.*

$$a_{pl}a_{pl} = n\pi r_0^2. \quad (11)$$

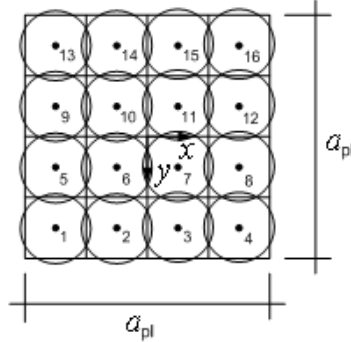


Fig. 6 – Subdivision of the foundation area and application of disk loads, ($n = 4 \times 4 = 16$).

The radius of the disk load hence is obtained as

$$r_0 = \frac{a_{pl}}{\sqrt{n\pi}}. \quad (12)$$

In the study a mesh with $n = 18 \times 18 = 324$ nodal points is used (see Fig. 7).

At each nodal point a load is applied. The resulting complex vertical displacements are computed in frequency domain at each nodal point, applying the Green's functions according to eq. (10) or using the ESD toolbox. This gives the complex flexibility matrix of the soil related to the nodal points of the foundations as

$$[\tilde{N}(\Omega)] = \begin{pmatrix} \tilde{N}_{1,1} & \dots & \tilde{N}_{1,n} \\ \vdots & \ddots & \vdots \\ \tilde{N}_{n,1} & \dots & \tilde{N}_{n,n} \end{pmatrix}. \quad (13)$$

Here $\tilde{N}_{i,j}$ is the vertical complex displacement at node i when a unit (disk) loads with circular frequency Ω is applied at node j . The matrix $[\tilde{N}(\Omega)]$ is symmetric.

The inverse of the flexibility matrix gives the stiffness matrix of the soil namely

$$[\tilde{K}(\Omega)] = [\tilde{N}(\Omega)]^{-1}. \quad (14)$$

The vertical complex spring constant is computed by imposing a displacement equal to one in every node of the model. Multiplying the stiffness matrix with the unity vector, the corresponding nodal forces are obtained namely

$$\begin{bmatrix} \tilde{k}_{1,1} & \dots & \tilde{k}_{1,n} \\ \vdots & \ddots & \vdots \\ \tilde{k}_{n,1} & \dots & \tilde{k}_{n,n} \end{bmatrix} \begin{Bmatrix} 1 \\ \vdots \\ 1 \end{Bmatrix} = \begin{Bmatrix} \tilde{F}_1 \\ \vdots \\ \tilde{F}_n \end{Bmatrix}. \quad (15)$$

Adding all the nodal forces the total force caused by a unit displacement of a rigid foundation is obtained. This is the complex vertical spring constant or the real and imaginary part of the impedance function in frequency domain according to eq. (4)

$$\tilde{k}_k = \tilde{F}_z = \sum_{i=1}^n \tilde{F}_i. \quad (16)$$

The complex rocking spring constant is determined by imposing a rotation equal to one about the y-axis as (for x_i see Fig. 6)

$$\begin{bmatrix} \tilde{k}_{1,1} & \dots & \tilde{k}_{1,n} \\ \vdots & \ddots & \vdots \\ \tilde{k}_{n,1} & \dots & \tilde{k}_{n,n} \end{bmatrix} \begin{Bmatrix} x_1 \\ \vdots \\ x_n \end{Bmatrix} = \begin{Bmatrix} \tilde{F}_1 \\ \vdots \\ \tilde{F}_n \end{Bmatrix}. \quad (17)$$

The vertical nodal forces are multiplied with the corresponding distances, x_i , from the axis of rotation in order to find the rocking moment for a rotation equal to one of the rigid foundation plate. Summing all the bending moments it results

$$\tilde{k}_\eta = \tilde{M}_y = \sum_{i=1}^n (\tilde{F}_i x_i). \quad (18)$$

Eq. (16) describes the complex impedance function for rocking about the y-axis.

The method for the computation of the impedance functions has been implemented by the first author in MATLAB using the Green's functions provided by ESD and by the second author in MathCAD based on the Green's functions (eq. (10), obtained by using TLM).

5. Numerical Experiments in Computation of Impedance Functions

For verification of the accuracy of the results, a model of a 18×18 m rigid foundation is considered, placed on a half-space. For the soil the same parameters as given in § 3.3 are used. The soil model investigated with the TLM in MathCAD consists of 40 thin layers with a depth of 98 m having the same properties as the half-space within MATLAB environment.

The results obtained for a square foundation with MATLAB and MathCAD are compared with those of the analytical solution for a circular foundation. They are presented with the notation from eq. (6). The radius of an equivalent circular foundation is chosen such that for vertical motion the area and for rocking motion the moment of inertia of the square foundation and the circular foundations are the same. Hence the following equivalent radii are obtained:

$$\text{vertical: } r_{z,\text{eq}} = \frac{a_{\text{pl}}}{\pi} = 10.16 \text{ m,}$$

$$\text{rocking: } r_{\eta,\text{eq}} = \frac{a_{\text{pl}}}{\sqrt[4]{3\pi}} = 10.27 \text{ m.}$$

Table 2 presents the result for the static vertical and rocking stiffness in the MATLAB environment using EDT toolbox, the MathCAD environment using TLM as well as the analytical solution for a half-space according to eqs. (8 a) and (8 b). The results agree very well. Small differences are due to the fact that the MATLAB model as well as the analytical solution refer to a half-space whereas the MathCAD model is a layer model with a rigid base in great depth. There are also differences in the numerical method and the geometry of the foundation. However, there is no big difference in the results.

Table 2
*Comparison for Vertical and Rocking Stiffness Values within
MATLAB & MathCAD*

Stiffness	Relations for static stiffness	Analytical solution for a half-space	Results with Mathcad	Resultswith MATLAB
Vertical	$\frac{4Gr}{1-\nu}$	1.16×10^5	1.24×10^5	1.28×10^5
Rocking	$\frac{8Gr^3}{3(1-\nu)}$	7.94×10^6	8.06×10^6	8.09×10^6

The dimensionless impedance functions for vertical and rocking motion according to eq. (6) are presented in Tables 3 and 4 and in Figs. 7 and 8. They are given for dimensionless frequencies, a_0 , in the range $[0, 1.65]$. Results are shown for the MATLAB computation based on ESD and for an elastic half-space. Both show a good agreement. This is also true for the MathCAD model (not shown here).

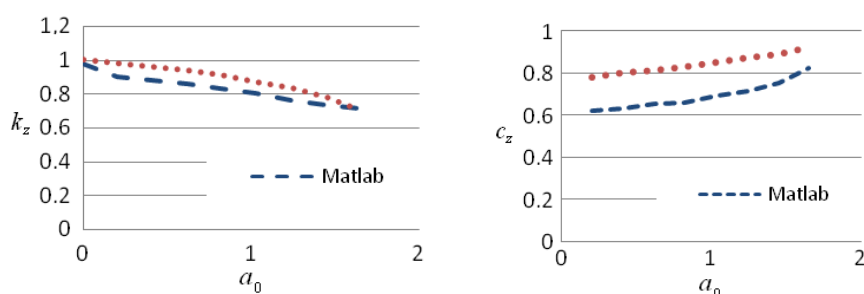


Fig.7 – Dimensionless impedance functions k_z and c_z for vertical motion.

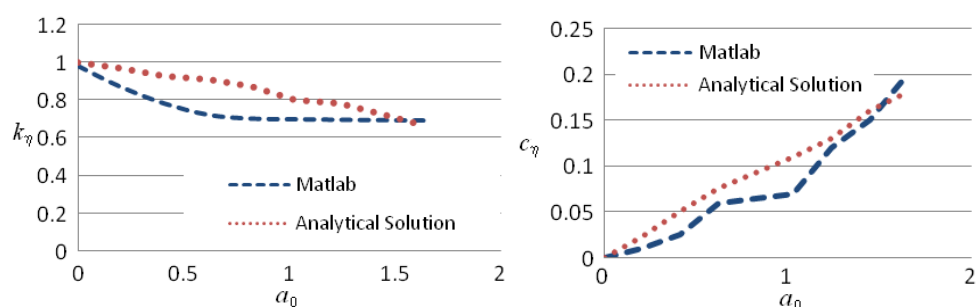


Fig. 8 – Dimensionless impedance functions k_η and c_η for rocking motion.

Table 3

Dimensionless Impedance Functions k_z and c_z for Vertical Motion

Dimensionless frequency a_0	Stiffness coefficient k_z		Damping coefficient c_z	
	MATLAB	Analytical solution	MATLAB	Analytical solution
0	0.98	1	–	–
0.2	0.9	0.98	0.62	0.78
0.41	0.88	0.96	0.63	0.8
0.62	0.86	0.94	0.65	0.81
0.82	0.83	0.91	0.66	0.83
1.03	0.8	0.87	0.69	0.85
1.24	0.76	0.83	0.71	0.87
1.45	0.73	0.78	0.75	0.89
1.65	0.71	0.7	0.82	0.92

Table 4
Dimensionless Impedance Functions k_η and c_η for Rocking Motion

Dimensionless frequency a_0	Stiffness coefficient k_η		Damping coefficient c_η	
	MATLAB	Analytical solution	MATLAB	Analytical solution
0	0.98	1.00	0	0
0.2	0.87	0.97	0.01	0.023
0.41	0.78	0.93	0.025	0.05
0.62	0.72	0.91	0.06	0.076
0.82	0.7	0.87	0.065	0.092
1.03	0.697	0.80	0.07	0.11
1.24	0.694	0.78	0.12	0.13
1.45	0.692	0.72	0.15	0.16
1.65	0.69	0.66	0.2	0.18

From the figures shown above we can observe that for dimensionless frequencies situated in $[0, 1.65]$ range, the impedance functions obtained with the numerical method presented here compare well with those given by Gazetas (1983).

6. Conclusions

In this paper the computation of impedance functions for the foundation response in structural dynamics considering the SSI effect has been addressed. Two numerical methods for Green's functions, the Thin Layer Method as developed by Waas *et al.* (1985) and a method based on the transformation in the wave number domain as implemented in the ESD toolbox, have been investigated and implemented in a MathCAD and a MATLAB environment, respectively. In this paper it has been identified that both methods and corresponding tools (TLM/MathCAD and ESD/MATLAB) can be used with an appropriate programming (ESD supplies only Green's functions) to compute impedance functions necessary for SSI effects evaluation. It has been shown, based on a benchmark example, that the results of both methods compared well with the analytical solution for an elastic half space. The potential of the methods presented here, however, goes beyond the simple case of a homogeneous half space since also arbitrarily layered media can be investigated. This allows the analysis of realistic soil models in SSI.

Acknowledgement. This paper represents mainly the results of the ERASMUS mobility at HTWG Konstanz University, being supervised by Professor Horst Werkle. A special thanks to Professor Gabriela M Atanasiu for her help, support and guide lines to complete this paper and Master Thesis.

Appendix

Eigenvalue Problem of the Rayleigh Waves

The eigenvalues and the eigenvectors result from solving the homogeneous eq. of motion (eq. (9))

$$([A_R]k_R^2 + [B_R]k_R + [C_R])\{v\} = \{0\}, \quad (A1)$$

with

$$[C_R] = [G_R] - \Omega^2 [M_R]. \quad (A2)$$

The vector $\{v\}$ contains the complex displacements v_j , ($j = 1, \dots, 2n$). They are related to the n layer interfaces. The odd terms v_1, v_3, v_5 describe the horizontal component and the even terms v_2, v_4, v_6 – the vertical component of the displacements. The $2n \times 2n$ matrices $[A_R]$, $[D_R]$, $[G_R]$ and $[M_R]$ consist of the contributions from the individual layers and can be assembled by addition of layer sub matrices (Fig A1).

The layer sub-matrices of a transversely isotropic medium are (Waas *et al.*, 1985)

$$A_{e,j}(D_{rr}, G_{rz}, h) = \frac{h}{3} \begin{pmatrix} D_{rr} & 0 & \frac{D_{rr}}{2} & 0 \\ 0 & G_{rz} & 0 & \frac{G_{rz}}{2} \\ \frac{D_{rr}}{2} & 0 & D_{rr} & 0 \\ 0 & \frac{G_{rz}}{2} & 0 & G_{rz} \end{pmatrix}, \quad (A3)$$

$$B_{e,j}(D_{rz}, G_{rz}, h) = \frac{1}{2} \begin{pmatrix} 0 & D_{rz} - G_{rz} & 0 & -D_{rz} - G_{rz} \\ D_{rz} - G_{rz} & 0 & D_{rz} + G_{rz} & 0 \\ 0 & D_{rz} + G_{rz} & 0 & -D_{rz} + G_{rz} \\ -D_{rz} - G_{rz} & 0 & -D_{rz} + G_{rz} & 0 \end{pmatrix}, \quad (A4)$$

$$G_{e,j}(G_{rz}, D_{zz}, h) = \frac{1}{h} \begin{pmatrix} G_{rz} & 0 & -G_{rz} & 0 \\ 0 & D_{zz} & 0 & -D_{zz} \\ -G_{rz} & 0 & G_{rz} & 0 \\ 0 & -D_{zz} & 0 & D_{zz} \end{pmatrix}, \quad (A5)$$

$$M_{e,j}(\rho, h) = \frac{h\rho}{6} \begin{pmatrix} 2 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 \\ 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 2 \end{pmatrix}. \quad (\text{A6})$$

For an isotropic material $D_{rr} = D_{zz} = \lambda + 2G$, $D_{rz} = \lambda$, where $\lambda = G(E - 2G)/(3G - E) = 2G\nu/(1 - 2\nu)$ is the Lamé constant. The sublayer matrices are constructed with the material parameters G , ν and thickness h of each layer, i . Using complex elasticity parameters (eq. (5)) the hysteretic material damping of the soil can be included.

The matrices $[A_R]$, $[B_R]$ and $[C_R]$ are symmetric. They are real in the undamped case and complex in the damped case.

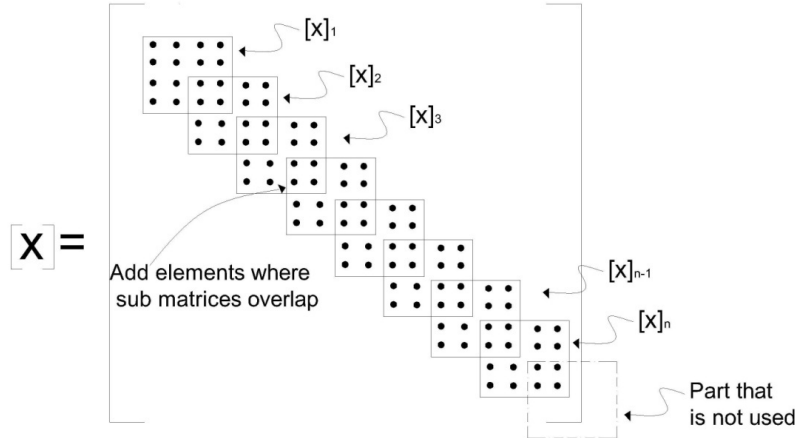


Fig. A1 – Structure of matrices $[A_R]$, $[B_R]$, $[G_R]$ and $[M_R]$.

For any given frequency, Ω , the homogeneous eq. of motion defines the possible wave numbers for generalized Rayleigh waves in the layered region

$$\left[[A_R]k^2 + [B_R]k_R + [C_R] \right] = 0. \quad (\text{A7})$$

The algebraic eigenvalue problem given by eqs. (A1) or (A7) has $4n$ generally complex roots k_s , ($s = 1, 2, \dots, 4n$). The corresponding solution vectors $\{v_s\}$, ($s = 1, 2, \dots, 4n$) are called *eigenvectors* or *mode shapes*. It can be rewritten in the form of a linear eigenvalue problem of double dimension, $2m$ (with $m = 2n$) namely

$$\left([E] - k_r [F] \right) \{w\} = \{0\}, \quad (\text{A8})$$

$$E = \begin{bmatrix} 0 & C_R \\ C_R & B_R \end{bmatrix}, \quad (\text{A9})$$

$$F = \begin{bmatrix} C_R & 0 \\ 0 & -A_R \end{bmatrix}, \quad (\text{A10})$$

and

$$\{w\} = \begin{Bmatrix} v \\ k_R v \end{Bmatrix}. \quad (\text{A11})$$

Waas (1972) presents a numerical method for finding out the eigenvalues and eigenvectors for the eq. (A8).

The quadratic eigenvalue problem (eq. (1)), has $4n$ solutions. If k_R is one eigenvalue, $-k_R$ is also an eigenvalue. Only those eigenvalues and the corresponding eigenvectors which are with negative imaginary parts are used in order to fulfill the radiation condition, *i.e.* the condition that the waves are propagating from the origin to infinity. In case of real eigenvalues those k_R values with positive sign are used.

The eigenvectors fulfill the orthogonality relationship

$$k_s \{v\}_s^T [A] \{v\}_r k_r - \{v\}_s^T [C] \{v\}_r = \begin{cases} 2, & k_r = k_s, \\ 0, & k_r \neq k_s, \end{cases} \quad (\text{A12})$$

where k_r and k_s are two eigenvalues, k_R and $\{v\}_r$ and $\{v\}_s$ are the corresponding eigenvectors. The case k_r is used for the normalization of the eigenvectors, $\{v\}$.

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EVALUAREA RĂSPUNSULUI DINAMIC PENTRU FUNDAȚII RIGIDE CONSIDERÂND EFECTUL INTERACȚIUNE SOL-STRUCTURĂ

(Rezumat)

Interacțiunea dintre sol și structură este foarte importantă în proiectarea unei structuri, dar de foarte multe ori este neglijată. Această interacțiune devine foarte importantă mai ales în cazul fundațiilor amplasate pe un sol cu o capacitate portantă scăzută. Fenomenul de interacțiune sol-structură influențează puternic reacția structurii la acțiunea seismică, datorat în mare parte acțiunii diferitelor tipuri de unde din sol

Se calculează răspunsul dinamic al unei fundații, luând în considerare efectul interacțiunii sol-structură. Două metode au fost folosite pentru calculul funcției Green: metoda Thin Layer (TLM), teorie dezvoltată de Waas *et al.* (1985), și o metodă bazată pe transformarea proprietăților undelor, metoda implementată în toolbox-ul EDT (Elasto Dynamic Toolbox), atât în MATLAB cât și în MathCAD. S-a demonstrat că ambele metode (TLM/MathCAD cât și EDT/MATLAB) pot fi folosite pentru a evalua răspunsul dinamic al unei fundații considerând efectul interacțiunii sol-structură, comparându-se rezultatele din ambele metode cu soluția analitică.

Potențialul metodelor prezentate merge mai departe decât cazul simplu al unui spațiu infinit omogen, deoarece pot fi calculate și diferite spații stratificate. Acest fapt permite un calcul mai realist pentru sol considerând efectul interacțiune sol-structură.