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INFLUENCE OF OPEN CROSS-SECTION SHAPE ON COMPRESSION MEMBERS BUCKLING RESISTANCE

BY

PETRU MOGA, ŞTEFAN GUŢIU and ALEXANDRA-DENISA DANCIU*

Technical University of Cluj-Napoca
Faculty of Civil Engineering

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Abstract. The methodology designs of the uniform compression member with open cross-section, according to EN 1993-1-1/2006. Eurocode 3: Design of steel structures, is briefly presented.

The influence of the open cross-section shape on the compression member buckling resistance for six constructive cross-section shapes is also numerically analysed.

Key words: buckling resistance; compression members; comparative analysis; Euronorm EN 1993-1-1: Design of steel structures.

1. Introduction

The imperfections have an important influence on the phenomenon of buckling, the behaviour of real steel structures being dependent of

a) geometrical imperfections, due to defects causing lack of straightness, unparallel flanges, asymmetry of cross-section, etc;

b) material imperfections, due to residual stresses (caused by the rolling or fabrication process) or material inelasticity;

c) deviation of applied load from idealized position due to imperfect connections, erection tolerances or lack of verticality of the member.

*Corresponding author: *e-mail*: alexandra.stan@cfdp.utcluj.ro

European buckling curves include a generalized imperfection factor.

For hot-rolled steel members, with the type of cross-sections commonly used for compression members, the relevant buckling mode is generally flexural buckling; however, in some cases, torsional or flexural–torsional modes may govern and they must be investigated for all sections with small torsional resistance.

Concentrically loaded columns can buckle by flexure around one of the principal axes (classical buckling), twisting about the shear centre (torsional buckling) or a combination of both flexure and twisting (flexural–torsional buckling).

Torsional buckling can only occur if the shear centre and centroid coincide and the cross-section can rotate; this leads to a twisting of the member. Z-sections and I-sections with broad flanges can be subject to torsional buckling.

Symmetrical sections with axial load not in the plane of symmetry, and non-symmetrical sections such as C-sections, hats, equal-leg angles, T-sections and singly symmetrical I-sections, *i.e.* sections where the shear centre and the centroid do not coincide, must be checked for flexural–torsional buckling.

The analysis of torsional bucking is quite complex; the critical stress depends on the boundary conditions and it is very important to evaluate precisely the possibilities of rotation at the ends. The critical stress also depends on the torsional stiffness of the member and on the resistance to warping deformations provided by the member itself and by the restraints at its ends.

Where a Class 4 cross-section is subjected to an axial force, the buckling resistance evaluation of the compression member according to EC3 takes into account the effective cross-section area. In a more accurate calculation the additional bending moment determined by the possible shift of the centroid of the effective area relative to the centre of gravity of the gross section is also taken into consideration (SR EN 1993-1-1/2006; SR EN 1993-1-5/2006; ESDEP, 1994).

2. Buckling Resistance of Member

The design buckling resistance of a compression member should be taken as

$$N_{b.Rd} = \begin{cases} \dfrac{\chi A f_y}{\gamma_{M1}}, \text{for Class 1, 2 and 3 cross-sections;} \\ \dfrac{\chi A_{\text{eff}} f_y}{\gamma_{M1}}, \text{ for Class 4 cross-sections.} \end{cases} \tag{1}$$

For uniform members in compression, the value of the reduction factor, χ, for the appropriate non-dimensional slenderness, $\bar{\lambda}$ should be determined from the relevant buckling curve according to relation

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}; \ \chi \le 1, \tag{2}$$

where: $\varphi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$; α – is an imperfection factor;

$$\bar{\lambda} = \begin{cases} \sqrt{\dfrac{Af_y}{N_{\text{cr}}}}, & \text{for Class 1, 2 and 3 cross-section;} \\ \sqrt{\dfrac{A_{\text{eff}} f_y}{N_{\text{cr}}}}, & \text{for Class 4 cross-sections.} \end{cases} \tag{3}$$

In the Tables 1,…,3, the buckling mode and the buckling forces function of the open cross-section shape are presented.

Table 1
Double-Symmetrical Cross-Section

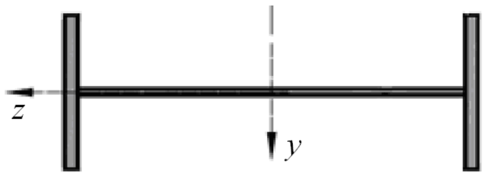
Double-symmetrical cross-section	
Buckling mode	Critical buckling force
Flexural buckling	$N_{\text{cr}} = \min\left[N_{\text{cr}.y} = \dfrac{\pi^2 EI_y}{L_{\text{cr}.y}^2};\ N_{\text{cr}.z} = \dfrac{\pi^2 EI_z}{L_{\text{cr}.z}^2}\right]$
Torsional buckling	$N_{\text{cr}.T} = N_{\omega} = \dfrac{A}{I_0}\left(GI_t + \dfrac{\pi^2 EI_{\omega}}{L_{\text{cr}.T}^2}\right)$

Table 2
Mono-Symmetrical Cross-Section

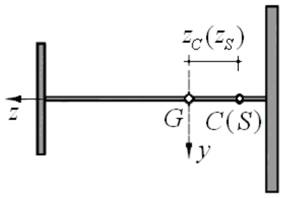
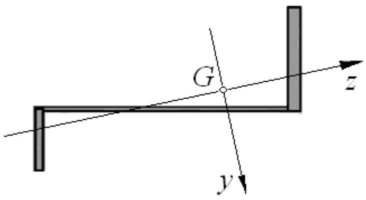
Mono-symmetrical cross-section	
Buckling mode	Critical buckling force
Flexural buckling	$N_{cr} = \min\left[N_{cr.y} = \frac{\pi^2 EI_y}{L_{cr.y}^2};\ N_{cr.z} = \frac{\pi^2 EI_z}{L_{cr.z}^2}\right]$
Torsional buckling	$N_{cr.T} = N_\omega = \frac{A}{I_0}\left(GI_t + \frac{\pi^2 EI_\omega}{L_{cr.T}^2}\right)$
Flexural–torsional buckling	$N_{cr.TF} = \frac{I_0}{2(I_y + I_z)}\left[(N_{cr.z} + N_{cr.T}) - \sqrt{(N_{cr.z} + N_{cr.T})^2 - 4\frac{I_y + I_z}{I_0}N_{cr.z}N_{cr.T}}\right]$

Table 3
Non-Symmetrical Cross-Section

Non-simmetrical cross-section	
Buckling mode	Critical buckling force
Flexural–torsional buckling	$f(N) = i_0^2(N - N_{cr.y})(N - N_{cr.z})(N - N_\omega) - N^2 z_c^2(N - N_{cr.y}) -$ $-N^2 y_c^2(N - N_{cr.z}) = 0,$ where $N_{cr} = \min[N_1; N_2; N_3]$

3. Numerical Analysis

The influence of the open cross-section shape on the values of the critical buckling force and of the buckling resistance of a compression member is analysed (Moga *et al.*, 2011).

The following constructive solutions are discussed:

Case 1: Non-symmetrical cross-section, with the flanges in opposite position regarding the web.

Case 2: Non-symmetrical U shape open cross-section.

Case 3: Mono-symmetrical U shape open cross-section.

Case 4: Mono-symmetrical double T shape cross-section.

Case 5: Double-symmetrical double T shape cross-section.

Case 6: Cross shape mono-symmetrical cross-section.

In all cases the cross-section area has the same value and also the dimensions of the web are preserved, respectively

$$A = 140\ \text{cm}^2;\ \ A_w = 500 \times 10\ \text{mm}^2 = 50\ \text{cm}^2.$$

The following design data are presumed as being known:

a) member cross-section;

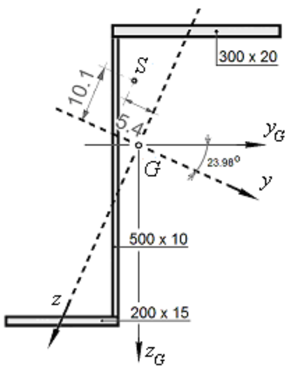
b) material: Steel S 355: $f_y = 355\ \text{N/mm}^2; \varepsilon = 0.81;$

c) buckling lengths:

$$L = 8.00\ \text{m};\ \ \mu_y = 1;\ \ \mu_z = \mu_\omega = 0.5;\ L_{\text{cr}.y} = \mu_y L = 8.0\ \text{m};$$
$$L_{\text{cr}.z} = L_{\text{cr}.\omega} = \mu_z(\mu_\omega)L = 4.0\ \text{m}.$$

3.1. Case 1: Non-Symmetrical Cross-Section, with the Flanges in Opposite Position Regarding the Web

The cross-section and the corresponding characteristics are presented in Fig. 1.



Characteristics

$A = 140\ \text{cm}^2;$
$I_y = 7.833 \times 10^4\ \text{cm}^4;$
$I_z = 6{,}518\ \text{cm}^4;$
$I_\omega = 4.552 \times 10^6\ \text{cm}^6;$
$I_t = 119\ \text{cm}^4;$
$i_y = 23.7\ \text{cm};$
$i_z = 6.8\ \text{cm};$
$y_S = 5.4\ \text{cm};$
$z_S = 10.1\ \text{cm}.$

Fig. 1 – Non-symmetrical cross-section, with the flanges in opposite position regarding the web.

a) *Critical buckling force*

$$N_{\mathrm{cr}}^{\mathrm{theor.}} = \begin{cases} N_{\mathrm{cr}.y} = \dfrac{\pi^2 EI_y}{L_{\mathrm{cr}.y}^2} = \dfrac{\pi^2 2.1\times10^6\times7.833\times10^4}{800^2}10^{-2} = 25,341\,\mathrm{kN}; \\ N_{\mathrm{cr}.z} = \dfrac{\pi^2 EI_z}{L_{cr.z}^2} = \dfrac{\pi^2 2.1\times10^6\times6,518}{400^2}10^{-2} = 8,435\,\mathrm{kN}; \\ N_\omega = \dfrac{1}{i_0^2}\left(\dfrac{\pi^2 EI_\omega}{L_{\mathrm{cr}.\omega}^2} + GI_t\right) = \\ = \dfrac{1}{737.23}\left(\dfrac{\pi^2 2.1\times10^6\times4.552\times10^6}{400^2} + 0.807\times10^6\times119\right)\times10^{-2} = 9,293\,\mathrm{kN}, \end{cases}$$

where:

$$i_0^2 = \frac{I_y + I_z + A\left(y_s^2 + z_s^2\right)}{A} = \frac{\left(7.833 + 0.6518\right)\times10^4 + 140\left(5.4^2 + 10.1^2\right)}{140} = 737.23\,\mathrm{cm}^2.$$

For non-symmetrical cross-section, N_{cr} is obtained from eq.

$$f\left(N\right) = i_0^2\left(N - N_{\mathrm{cr}.y}\right)\left(N - N_{\mathrm{cr}.z}\right)\left(N - N_\omega\right) - N^2 z_s^2\left(N - N_{\mathrm{cr}.y}\right) - \\ - N^2 y_s^2\left(N - N_{\mathrm{cr}.z}\right) = 0. \qquad (4)$$

Using numerical values it results

$$737(N - 25,341)(N - 8,435)(N - 9,293) - N210.12(N - 25,341) - \\ - N25.42(N - 8,435) = 0$$

and consequently

$$N_{\mathrm{cr}} = \min\left[N_1; N_2; N_3\right] = \min\left[6,407;\ 13,619;\ 27,695\right] = 6,407\ \mathrm{kN}..$$

b) *Effective cross-section*

TOP FLANGE EFFECTIVE AREA

Outstand flanges are subjected to uniform compression (Fig. 2 *a*)

$$\frac{c}{t} = 14.5 > 14\varepsilon = 11.34 \Rightarrow \text{Class } 4;\quad \sigma_2 = \sigma_1 \Rightarrow \psi = \frac{\sigma_2}{\sigma_1} = +1.$$

The buckling coefficient for the flange is $k_\sigma = 0.43$;

$$\overline{\lambda}_p = \frac{b_p / t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{290/20}{28.4 \times 0.81 \times \sqrt{0.43}} = 0.96 > 0.748\,.$$

It results

$$\rho = \frac{\overline{\lambda}_p - 0.188}{\overline{\lambda}_p^2} = 0.84\,.$$

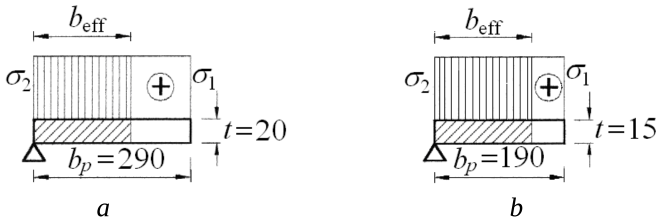


Fig. 2 – Compression in outstand flanges.

The flange effective width is

$$b_{\text{eff}} = \rho b_p = 0.84 \times 290 = 244 \text{ mm}\,.$$

BOTTOM FLANGE

Outstand flanges are subjected to uniform compression (Fig. 2 *b*)

$$\frac{c}{t} = 12.67 > 14\varepsilon = 11.34 \Rightarrow \text{Class 4}\,; \quad \sigma_2 = \sigma_1 \Rightarrow \psi = \frac{\sigma_2}{\sigma_1} = +1\,.$$

The buckling coefficient for the flange is $k_\sigma = 0.43$;

$$\overline{\lambda}_p = \frac{b_p / t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{190/15}{28.4 \times 0.81 \times \sqrt{0.43}} = 0.84 > 0.748\,.$$

It results

$$\rho = \frac{\overline{\lambda}_p - 0.188}{\overline{\lambda}_p^2} = 0.92\,.$$

The flange effective width is

$$b_{\text{eff}} = \rho b_p = 0.92 \times 190 = 175 \text{ mm}\,.$$

WEB EFFECTIVE AREA

The web is a doubly supported element under uniform compression (Fig. 3)

$$\frac{c}{t} = 50 > 42\varepsilon \Rightarrow \text{Class } 4\,;\ \sigma_2 = \sigma_1 \Rightarrow \psi = \frac{\sigma_2}{\sigma_1} = +1\,;\ k_\sigma = 4\,;$$

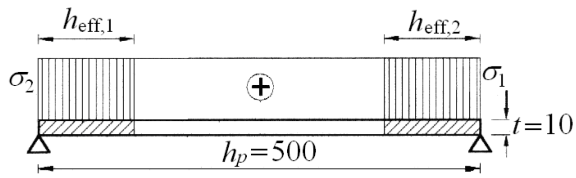


Fig. 3 – The web.

$$\overline{\lambda}_p = \frac{b_p / t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{500/10}{28.4 \times 0.81 \times \sqrt{4}} = 1.09 > 0.673\,.$$

For $\psi = 1$ it results

$$\rho = \frac{\overline{\lambda}_p - 0.22}{\overline{\lambda}_p^2} = 0.73\,;\ h_{\text{eff}} = 0.73 \times 500 = 365 \text{ mm};\ \frac{h_{\text{eff}}}{2} = 182 \text{ mm}.$$

In Fig. 4 the effective cross-section of the member is presented.

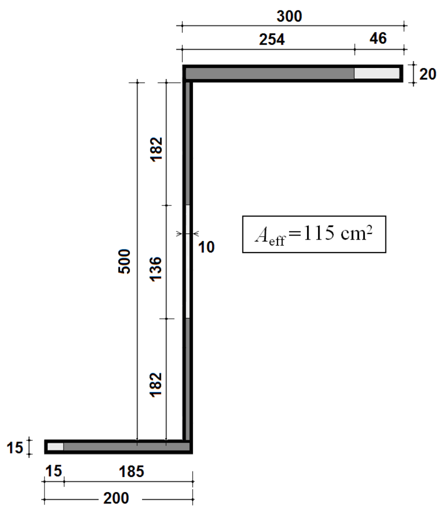


Fig. 4 – Effective cross-section.

c) *Buckling resistance of member*
The non-dimensional slenderness coefficient is

$$\bar{\lambda} = \bar{\lambda}_{FT} = \sqrt{\frac{A_{\text{eff}} f_y}{N_{\text{cr}}}} = \sqrt{\frac{115 \times 3,550}{6,407 \times 10^2}} = 0.798.$$

The reduction coefficient: the appropriate buckling curve is $d \Rightarrow \chi = 0.58$.
The buckling resistance will be

$$N_{b.Rd} = \chi \frac{A_{\text{eff}} f_y}{\gamma_{M1}} = 0.58 \frac{115 \times 3,550}{1.1} 10^{-2} = 2,153 \text{ kN}.$$

3.2. Case 2: Non-Symmetrical U-Shape Open Cross-Section

The cross-section and the corresponding characteristics are presented in Fig. 5.

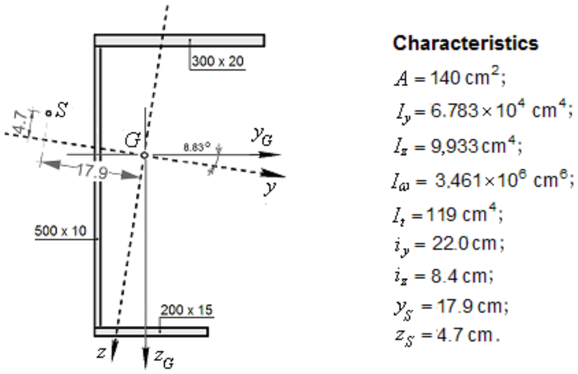


Fig. 5 – Non-symmetrical U-shape cross-section.

a) *Critical buckling force*

$$N_{\text{cr}}^{\text{theor.}} = \begin{cases} N_{\text{cr}.y} = \dfrac{\pi^2 EI_y}{L_{\text{cr}.y}^2} = \dfrac{\pi^2 2.1 \times 10^6 \times 6.783 \times 10^4}{800^2} 10^{-2} = 21,944 \text{ kN}; \\ N_{\text{cr}.z} = \dfrac{\pi^2 EI_z}{L_{\text{cr}.z}^2} = \dfrac{\pi^2 2.1 \times 10^6 \times 9\,933}{400^2} 10^{-2} = 12,854 \text{ kN}; \\ N_\omega = \dfrac{1}{i_0^2}\left(\dfrac{\pi^2 EI_\omega}{L_{\text{cr}.\omega}^2} + GI_t\right) = \\ = \dfrac{1}{898}\left(\dfrac{\pi^2 2.1 \times 10^6 \times 3.461 \times 10^6}{400^2} + 0.807 \times 10^6 \times 119\right) 10^{-2} = 6,057 \text{ kN}, \end{cases}$$

where:

$$i_0^2 = \frac{I_y + I_z + A\left(y_s^2 + z_s^2\right)}{A} = \frac{\left(6.783 + 0.9933\right)10^4 + 140\left(17.9^2 + 4.7^2\right)}{140} = 898\ \text{cm}^2.$$

N_{cr} is obtained from the eq.

$$f\left(N\right) = i_0^2\left(N - N_{\text{cr}.y}\right)\left(N - N_{\text{cr}.z}\right)\left(N - N_{\omega}\right) - N^2 z_s^2\left(N - N_{\text{cr}.y}\right) -$$
$$- N^2 y_s^2\left(N - N_{\text{cr}.z}\right) = 0.$$

Introducing numerical values it results

$$898(N - 21{,}944)\ (N - 12{,}854)\ (N - 6{,}057) - N24.72(N - 21{,}944) -$$
$$- N217.92(N - 12{,}854) = 0.$$

It follows that

$$N_{\text{cr}} = \min\left[N_1;\ N_2;\ N_3\right] = \min\left[5{,}348;\ 13{,}155;\ 39{,}254\right] = 5{,}348\ \text{kN}.$$

In Fig. 6 the effective cross-section of the member is presented.

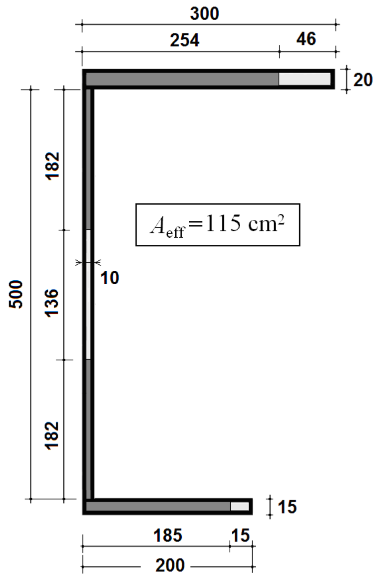


Fig. 6 – Effective cross-section

b) *Buckling resistance of member*
The non-dimensional slenderness coefficient is

$$\bar{\lambda} = \bar{\lambda}_{FT} = \sqrt{\frac{A_{\text{eff}}\, f_y}{N_{\text{cr}}}} = \sqrt{\frac{115 \times 3{,}550}{5{,}348 \times 10^2}} = 0.874.$$

The reduction coefficient: the appropriate buckling curve is $d \Rightarrow \chi = 0.53$. The buckling resistance will be

$$N_{b.Rd} = \chi \frac{A_{\text{eff}} f_y}{\gamma_{M1}} = 0.53 \frac{115 \times 3,550}{1.1} 10^{-2} = 1,967 \text{ kN}.$$

3.3. Case 3: Mono-Symmetrical U-Shape Open Cross-Section

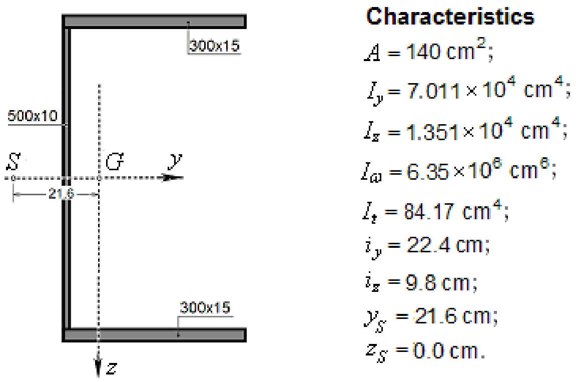


Fig. 7 – Mono-symmetrical U-shape open cross-section.

a) *Critical buckling force*

$$N_{\text{cr}}^{\text{theor.}} = \begin{cases} N_{\text{cr}.y} = \dfrac{\pi^2 EI_y}{L_{\text{cr}.y}^2} = \dfrac{\pi^2 2.1\times 10^6 \times 7.011\times 10^4}{800^2} 10^{-2} = 22,682 \text{ kN}; \\ N_{\text{cr}.z} = \dfrac{\pi^2 EI_z}{L_{\text{cr}.z}^2} = \dfrac{\pi^2 2.1\times 10^6 \times 1.351\times 10^4}{400^2} 10^{-2} = 17,483 \text{ kN}; \\ N_{\omega} = \dfrac{1}{i_0^2}\left(\dfrac{\pi^2 EI_{\omega}}{L_{\text{cr}.\omega}^2} + GI_t\right) = \\ = \dfrac{1}{1,064}\left(\dfrac{\pi^2 2.1\times 10^6 \times 6.35\times 10^6}{400^2} + 0.807\times 10^6 \times 84.17\right) 10^{-2} = 8,361 \text{ kN}, \end{cases}$$

where

$$i_0^2 = \frac{I_y + I_z + Ay_s^2}{A} = \frac{(7.011+1.351)10^4 + 140\times 21.6^2}{140} = 1,064 \text{ cm}^2.$$

For mono-symmetrical cross-section (symmetry about y-y axis) N_{cr} is obtained from the eq.

$$N_{\mathrm{cr}.TF} = \frac{I_0}{2\,(I_y + I_z)}\left[N_{\mathrm{cr}.y} + N_{\mathrm{cr}.T} - \sqrt{\left(N_{cr.y} + N_{\mathrm{cr}.T}\right)^2 - 4\frac{I_y + I_z}{I_0} N_{\mathrm{cr}.y} N_{\mathrm{cr}.T}}\right] =$$

$$= \frac{14.896}{2 \times 8.362}\left[(22,682 + 8,361) - \sqrt{(22,682 + 8,361)^2 - 4\frac{8.362}{14.896} 22,682 \times 8\,361}\right] =$$

$$= 6,988\ \mathrm{kN},$$

where $N_{\mathrm{cr}.T} = N_\omega = 8,626\ \mathrm{kN}$; $I_0 = i_0^2 A = 14.896 \times 10^4\,\mathrm{cm}^4$.

Taking into account the obtained data it results that the relevant buckling mode is flexural–torsional buckling, where $N_{\mathrm{cr}} = N_{\mathrm{cr.TF}} = 6,988\ \mathrm{kN}$.

b) *Effective cross-section*

The flanges are cantilevers subjected to uniform compression

$$\frac{c}{t} = 19.33 > 14\varepsilon = 11.34 \Rightarrow \text{Class } 4;\quad \sigma_2 = \sigma_1 \Rightarrow \psi = \frac{\sigma_2}{\sigma_1} = +1.$$

The buckling coefficient for the flange is $k_\sigma = 0.43$

$$\bar{\lambda}_p = \frac{b_p / t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{290 / 15}{28.4 \times 0.81 \times \sqrt{0.43}} = 1.28 > 0.748\,.$$

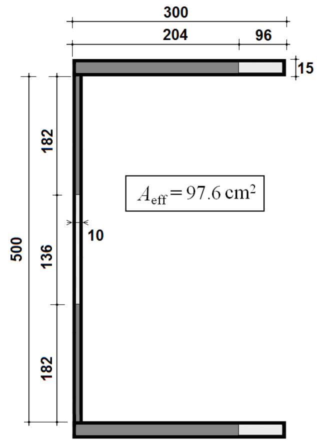


Fig. 8 – Effective cross-section.

It results

$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2} = 0.67\,.$$

The flanges effective width will be $b_{\mathrm{eff}} = \rho b_p = 0.67 \times 290 = 194$ mm .

In Fig. 8 the effective cross-section of the member is presented.

c) *Buckling resistance of member*

The non-dimensional slenderness coefficient is

$$\bar{\lambda} = \bar{\lambda}_{FT} = \sqrt{\frac{A_{\text{eff}} f_y}{N_{\text{cr}}}} = \sqrt{\frac{97.6 \times 3,550}{6,988 \times 10^2}} = 0.70\,.$$

The reduction coefficient: the appropriate buckling curve is $d \Rightarrow \chi = 0.64$.
The buckling resistance will be

$$N_{b.Rd} = \chi \frac{A_{\text{eff}} f_y}{\gamma_{M1}} = 0.64 \frac{97.6 \times 3550}{1.1} 10^{-2} = 2,016\ \text{kN}\,.$$

3.4. Case 4: Mono-Symmetrical Double T-Shape Cross-Section

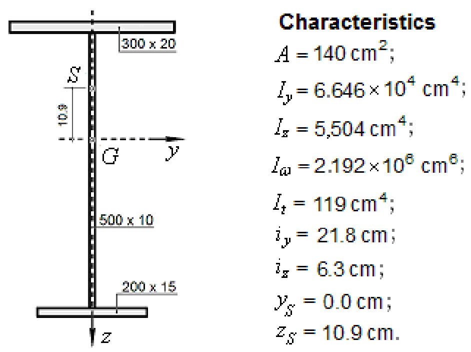


Fig. 9 – Mono-symmetrical double T-shape cross-section.

a) *Critical buckling force*

$$N_{\text{cr}}^{\text{theor.}} = \begin{cases} N_{\text{cr}.y} = \dfrac{\pi^2 EI_y}{L_{\text{cr}.y}^2} = \dfrac{\pi^2 2.1\times10^6\times6.646\times10^4}{800^2}10^{-2} = 21,501\ \text{kN}; \\ N_{\text{cr}.z} = \dfrac{\pi^2 EI_z}{L_{\text{cr}.z}^2} = \dfrac{\pi^2 2.1\times10^6\times5,504}{400^2}10^{-2} = 7,123\ \text{kN}; \\ N_\omega = \dfrac{1}{i_0^2}\left(\dfrac{\pi^2 EI_\omega}{L_{\text{cr}.\omega}^2} + GI_t\right) = \\ = \dfrac{1}{633}\left(\dfrac{\pi^2 2.1\times10^6\times2.192\times10^6}{400^2} + 0.807\times10^6\times119\right)10^{-2} = 5,998\ \text{kN}, \end{cases}$$

where:

$$i_0^2 = \frac{I_y + I_z + Az_s^2}{A} = \frac{\left(6.646 + 0.5504\right)10^4 + 140 \times 10.9^2}{140} = 633\,\text{cm}^2.$$

For mono-symmetrical cross-section (symmetry about *z-z* axis), N_{cr} is obtained from the eq.

$$N_{\text{cr}.TF} = \frac{I_0}{2\,(I_y + I_z)}\left[N_{\text{cr}.z} + N_{\text{cr}.T} - \sqrt{\left(N_{\text{cr}.z} + N_{\text{cr}.T}\right)^2 - 4\frac{I_y + I_z}{I_0}N_{\text{cr}.z}N_{\text{cr}.T}}\right] =$$

$$= \frac{8.862}{2 \times 7.20}\left[7{,}123 + 5{,}998 - \sqrt{(7{,}123 + 5{,}998)^2 - 4\frac{7.20}{8.862}7{,}123 \times 5{,}998}\right] = 4{,}520\text{ kN},$$

where: $N_{\text{cr}.T} = N_{\omega} = 5{,}998\text{ kN}$; $I_0 = i_0^2 A = 8.862 \times 10^4\,\text{cm}^4$.

Taking into account the obtained data it results that the relevant buckling mode is flexural-torsional, where $N_{\text{cr}} = N_{\text{cr}.TF} = 4{,}520\text{ kN}$.

In Fig. 10 the effective cross-section of the member is presented.

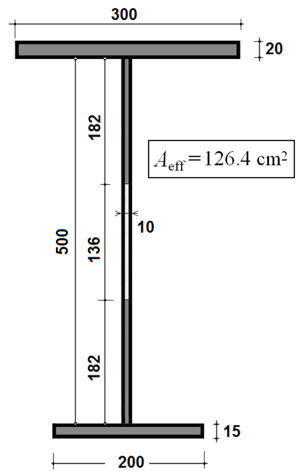


Fig. 10 – Effective cross-section.

b) *Buckling resistance of member*

The non-dimensional slenderness coefficient is

$$\bar{\lambda} = \bar{\lambda}_{FT} = \sqrt{\frac{A_{\text{eff}}\,f_y}{N_{\text{cr}}}} = \sqrt{\frac{126.4 \times 3{,}550}{4{,}520 \times 10^2}} = 0.996.$$

The reduction coefficient: the appropriate buckling curve is *c* (t_f < < 40 mm; axis *z-z*) $\Rightarrow \chi = 0.54$.

The buckling resistance will be

$$N_{b.Rd} = \chi \frac{A_{\text{eff}} f_y}{\gamma_{M1}} = 0.54 \frac{126.4 \times 3{,}550}{1.1} 10^{-2} = 2{,}203\ \text{kN}\,.$$

3.5. Case 5: Double-Symmetrical Double T Shape Cross-Section

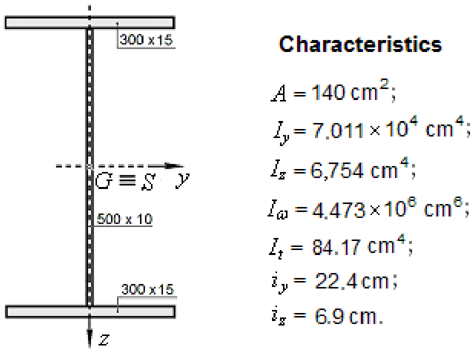


Fig. 11 – Double-symmerical double T shape cross-section.

a) *Critical buckling force*

$$N_{\text{cr}}^{\text{theor.}} = \begin{cases} N_{\text{cr}.y} = \dfrac{\pi^2 EI_y}{L^2_{\text{cr}.y}} = \dfrac{\pi^2 2.1\times10^6\times7.011\times10^4}{800^2}10^{-2} = 22{,}682\ \text{kN}; \\ N_{\text{cr}.z} = \dfrac{\pi^2 EI_z}{L^2_{\text{cr}.z}} = \dfrac{\pi^2 2.1\times10^6\times6{,}754}{400^2}10^{-2} = 8{,}740\ \text{kN}; \\ N_{\omega} = \dfrac{1}{i_0^2}\left(\dfrac{\pi^2 EI_{\omega}}{L^2_{\text{cr}.\omega}} + GI_t\right) = \\ = \dfrac{1}{549}\left(\dfrac{\pi^2 2.1\times10^6\times4.473\times10^6}{400^2} + 0.807\times10^6\times84.17\right)10^{-2} = 11{,}780\ \text{kN}, \end{cases}$$

where:

$$i_0^2 = \frac{I_y + I_z}{A} = \frac{(7.011 + 0.6754)10^4}{140} = 549\ \text{cm}^2.$$

Taking into account the obtained data it results that the relevant buckling mode is flexural buckling about *z-z* axis, where $N_{\text{cr}} = N_{\text{cr}.z} = 8{,}740$ kN.

b) *Effective cross-section*
Flanges: Class 3.
Web: Class 4.

In Fig. 12 the effective cross-section of the member is presented.

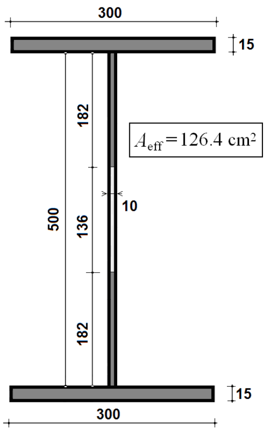


Fig. 12 – Effective cross-section

c) *Buckling resistance of member*

The non-dimensional slenderness coefficient is

$$\bar{\lambda} = \bar{\lambda}_{FT} = \sqrt{\frac{A_{\text{eff}}\, f_y}{N_{\text{cr}}}} = \sqrt{\frac{126.4 \times 3{,}550}{8{,}740 \times 10^2}} = 0.716.$$

The reduction coefficient: the appropriate buckling curve is *c* ($t_f < 40$ mm; axis *z-z*) $\Rightarrow \chi = 0.71$.

The buckling resistance will be

$$N_{b.Rd} = \chi \frac{A_{\text{eff}}\, f_y}{\gamma_{M1}} = 0.71 \frac{126.4 \times 3{,}550}{1.1} 10^{-2} = 2{,}896 \text{ kN}.$$

3.6. Case 6: Cross Shape Mono-Symmetrical Cross-Section

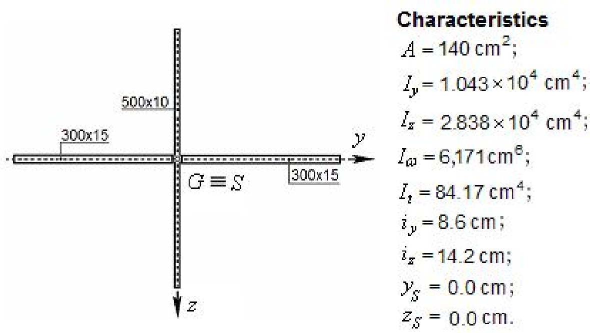


Fig. 13 – Cross shape mono-symmetrical cross-section.

a) *Critical buckling force*

$$N_{\mathrm{cr}}^{\mathrm{theor.}} = \begin{cases} N_{\mathrm{cr}.y} = \dfrac{\pi^2 EI_y}{L_{\mathrm{cr}.y}^2} = \dfrac{\pi^2 2.1\times10^6 \times 1.043\times10^4}{800^2}10^{-2} = 3,374\,\mathrm{kN}; \\ N_{\mathrm{cr}.z} = \dfrac{\pi^2 EI_z}{L_{\mathrm{cr}.z}^2} = \dfrac{\pi^2 2.1\times10^6 \times 2.838\times10^4}{400^2}10^{-2} = 3,673\,\mathrm{kN}; \\ N_\omega = \dfrac{1}{i_0^2}\left(\dfrac{\pi^2 EI_\omega}{L_{\mathrm{cr}.\omega}^2} + GI_t\right) = \\ = \dfrac{1}{277}\left(\dfrac{\pi^2 2.1\times10^6 \times 6,171}{400^2} + 0.807\times10^6\times84.17\right)10^{-2} = 2,481\,\mathrm{kN}, \end{cases}$$

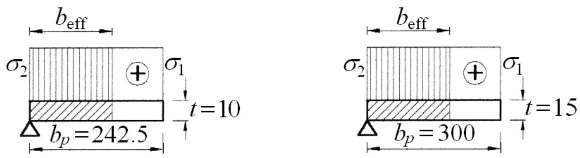
where

$$i_0^2 = \frac{I_y + I_z}{A} = \frac{(1.043 + 2.838)10^4}{140} = 277\ \mathrm{cm}^2.$$

Taking into account the obtained data it results that the relevant buckling mode is torsional: $N_{\mathrm{cr}} = N_{\mathrm{cr}.T} = N_\omega = 2,481\,\mathrm{kN}.$.

b) *Effective cross-section*

V e r t I c a l w a l l. The cantilevers of the vertical wall are subjected to uniform compression (Fig. 14 *a*).



a *b*

Fig. 14 – Uniform compression.

$$\frac{c}{t} = \frac{(500-15)/2}{10} = 24.25 > 14\varepsilon = 11.34 \Rightarrow \text{Class 4;}\ \ \sigma_2 = \sigma_1 \Rightarrow \psi = \frac{\sigma_2}{\sigma_1} = +1.$$

The buckling coefficient for the vertical cantilevers is $k_\sigma = 0.43$

$$\bar{\lambda}_p = \frac{b_p/t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{242.5/10}{28.4\times0.81\times\sqrt{0.43}} = 1.61 > 0.748.$$

It results

$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2} = 0.55\,.$$

Effective width

$$b_{\text{eff}} = \rho b_p = 0.55 \times 242.5 = 133 \text{ mm.}.$$

H o r i z o n t a l w a l l. The cantilevers of the horizontal wall are subjected to uniform compression (Fig. 14 *b*)

$$\frac{c}{t} = \frac{300}{15} = 20 > 14\varepsilon = 11.34 \Rightarrow \text{Class 4;}\;\; \sigma_2 = \sigma_1 \Rightarrow \psi = \frac{\sigma_2}{\sigma_1} = +1.$$

The buckling coefficient for the horizontal cantilevers is $k_\sigma = 0.43$

$$\bar{\lambda}_p = \frac{b_p / t}{28.4\varepsilon\sqrt{k_\sigma}} = \frac{20}{28.4 \times 0.81 \times \sqrt{0.43}} = 1.33 > 0.748.$$

It results

$$\rho = \frac{\bar{\lambda}_p - 0.188}{\bar{\lambda}_p^2} = 0.65\,.$$

Effective width

$$b_{\text{eff}} = \rho b_p = 0.65 \times 300 = 195 \text{ mm.}$$

In Fig. 15 the effective cross-section of the member is presented.

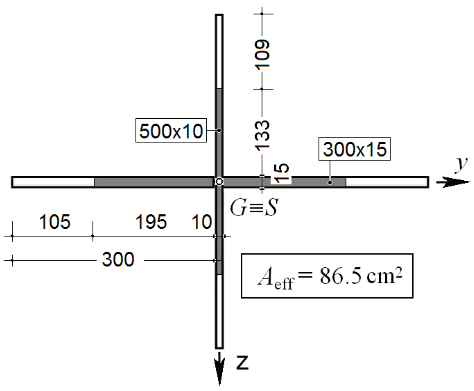


Fig. 15 – Effective cross-section

c) *Buckling resistance of member*
The non-dimensional slenderness coefficient is

$$\bar{\lambda} = \bar{\lambda}_{FT} = \sqrt{\frac{A_{\text{eff}} f_y}{N_{\text{cr}}}} = \sqrt{\frac{86.5 \times 3,550}{2,481 \times 10^2}} = 1.11.$$

The reduction coefficient: the appropriate buckling curve is $d \Rightarrow \chi = 0.41$.
The buckling resistance will be

$$N_{b.Rd} = \chi \frac{A_{\text{eff}} f_y}{\gamma_{M1}} = 0.41 \frac{86.5 \times 3,550}{1.1} 10^{-2} = 1,145 \text{ kN}.$$

4. Conclusions

In Table 4 the numerical analysis results, respectively the values of A_{eff}, N_{cr} and $N_{b.Rd}$ are presented.

Table 4
Numerical Results

Case	A_{eff}, [cm^2]	N_{cr}, [kN]	$N_{b.Rd}$, [kN]
1	115	6,407	2,153
2	115	5,348	1,967
3	97.6	6,988	2,016
4	126.4	4,520	2,203
5	126.4	8,740	2,896
6	86.5	2,481	1,145

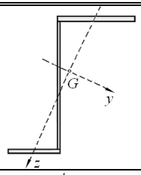
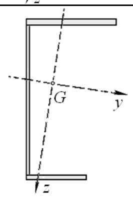
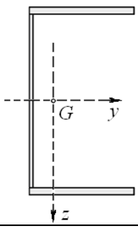
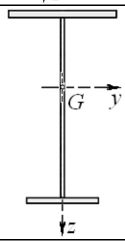
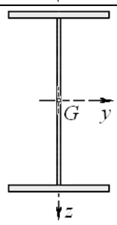
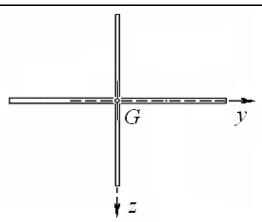
In Table 5 the parameters regarding the cross-section efficiency comparison with cross-section of Case 5 where the maximum efficiency is achieved are presented.

It can be observed that

1° In Cases 1, 2, 3 and 4 the buckling mode is flexural–torsional buckling, in Case 5 the buckling mode is flexural buckling and in Case 6 the buckling mode is torsional buckling;

2° The maximum buckling member resistance is obtained for double T shape, double symmetrical cross-section and the minimum buckling resistance is obtained for a cross shape double symmetrical cross-section (Case 6).

Table 5
Parameters Regarding the Cross-Section Efficiency

Case		$N_{\text{cr}}^{\text{case}[i]}$ / $N_{\text{cr.max}}^{\text{case}[5]}$	$N_{b.Rd}^{\text{case}[i]}$ / $N_{b.Rd.\max}^{\text{case}[5]}$	Buckling mode
1		0.73	0.74	flexural–torsional
2		0.61	0.68	flexural–torsional
3		0.80	0.70	flexural–torsional
4		0.52	0.76	flexural–torsional
5		1	1	flexural
6		0.28	0.40	torsional

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INFLUENȚA FORMEI SECȚIUNII TRANSVERSALE ASUPRA REZISTENȚEI LA FLAMBAJ A BARELOR COMPRIMATE

(Rezumat)

Se prezintă pe scurt metodologia de proiectare a barelor de secțiune transversală deschisă, supuse la compresiune uniformă, în conformitate cu normativul EN 1993-1-1/2006. Eurocod 3: *Proiectarea structurilor de oțel*

Baza de calcul teoretic este însoțită de o analiză comparativă numerică, pentru şase tipuri de secțiuni transversale deschise, în urma căreia au putut fi formulate concluzii şi observații referitoare la influența formei secțiunii transversale asupra rezistenței la flambaj.