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SOME CONSIDERATIONS CONCERNING THE ENTROPY CONCEPT IN THE EVALUATION OF MATERIAL PROPERTIES

BY

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Homogeneity is a general concept often used in science and mainly engineering as a work hypothesis with essential implications in the expression of some rules. From a mechanical viewpoint, the features of a material are expressed by the characteristics that define them, the latter being determined by the chemical and physical features of the constituent material. In this paper is used the entropy concept defined in the information theory, on which basis are defined the notions of *degree of homogeneity* and *homogeneity factor*. With means of these sizes different statistics populations can be analysed. Particularly it refers to the elasticity moduli of the steel. For example have been considered the two statistics series representing the elasticity moduli obtained by testing some steel square tubular bars.

1. Introduction

The homogeneity of a material possesses a particular significance in what concerns the construction structures realization as being one of the main factors determining the safety which those structures are realized at. The uncertainty at which a structure is designed and realized, owing to the variability of the factors intervening in design (variability with a random character) renders a correct design itself to be based on probabilistic methods. The random character both of the physical-mechanical properties of materials and of the loads renders even the so-called deterministic design to possess, in a certain degree, a probabilistic character. Homogeneity is a general concept often used in science and mainly engineering as a work hypothesis with essential implications in the expression of some rules [1]. It follows that the care of knowing materials as constitutive elements of structures presents a main interest. The homogeneity of the material implies homogeneity of its features and also of its characteristics. Thus, we can talk about modulus of elasticity, elastic limit, yield strength, ultimate strength, failure limit, fatigue strength, critical strength, coefficient of thermal expansion, coefficient of contraction, etc., as parameters that define a certain material. From these natural sizes one passes to the conventional ones such as the admissible strengths, the normed strengths, the calculus strengths, etc. The setting up of the basic, natural, above mentioned sizes, is made experimentally by testing on standard samples. The obtained results constitute statistics series by means of which the properties of materials used in structure carrying out be characterized. These experimental series would be characterized by a restricted number of sizes with well known statistical significations as: the average, the standard deviation, the variation factor, etc.; a statistics series may be segmented in subintervals, each subinterval being characterized at least by its central value and by the frequency at which the results are inscribing inside it. By means of the mentioned statistic sizes, one can also characterize the homogeneity of a series of results, in several manners. Intuitively, the homogeneity is given by the degree of scattering the results about the average. The nearest to the mean the results are grouped, the more homogeneous would the material be and it would participate in realizing the structural safety with a less degree of uncertainty.

2. Homogeneity and Entropy Concept

We proposed to use for homogeneity the concept of entropy, defined in the information theory [3]. In statistics the entropy is used for the measuring uniformity/non-uniformity, concentration/variation, organization/disorganization degree. Among the meanings of entropy concept there is also the one that measures the variety. The variety notion is correlated with the homogeneity notion. For a statistics series given in the following table

$$\begin{bmatrix} E_1 & E_2 & \dots & E_n \\ p_1 & p_2 & \dots & p_n \end{bmatrix}, \tag{1}$$

in the first line being the events, the interval values of the statistics series and in the second line the probabilities of appearance or frequencies of realization, the entropy, H, as a measure redundancy of that field of probability, is

$$H = \sum_{i=1}^{n} p_i \log_2 \frac{1}{p_i}. \tag{2}$$

It was defined the *homogeneity degree*, h, as being

$$h = 1 - \frac{H}{H_{\max}} = 1 - \frac{H}{\log_2 n}, \tag{3}$$

where: n is representative intervals (equivalent to n events); $H_{\max}$ – the maximum entropy which corresponds to the uniform distribution with the largest disorganisation degree.

The homogeneity degree varies between zero for a totally non-homogeneous material (with equal interval frequencies so that $H = \log_2 n$) and the unit for a perfectly homogeneous material (with all the events grouped in a single interval, the frequencies for the others ones being null, so that $H = 0$). In Fig. 1 is graphically represented the relation between h and H from which the above mentionned remarks are resulting. The variation low of degree of homogeneity, h *vs.* the entropy of the experiment, is linear. The degree of homogeneity, h, represents an indicator for the information quantity refering to the experimentally performed test.

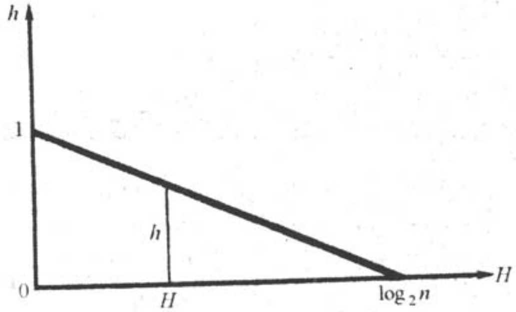

Fig. 1.- h *vs.* H relation.

It results that if the entropy of one statistics series is maximum, thus the homogeneity degree of material, in accordance to this statistical population, is zero. Besides, if through experiment it has been obtained one value for the all specimens, then only one frequency is not null and is equal to unit obtaining $H = 0$ and $h = 1$. This corresponds to the perfect homogeneous material.

As in relation (3) the entropy, H, represents a measure of uncertainty of the probability field which defines the studied characteristic; it is necessary to introduce units of measure for H. If, for defining the characteristics of a material, one uses statistics series of experimental data, then the entropy, H, gets meaning of information. That suggests the possibility to employ the bit as measure unit. In order to the experimental results that are processed, from the statistical view point and to which the homogeneity degree is determined, to be comparable, one introduces the notion of standard interval which may be state precisely by $\log_2 n$. That interval may be taken equal to an integer number of bits so that to cover almost all the experimentally registered values. The length of the total interval against the average may be taken, for example, as large as three times the standard deviation on both parts, for a symmetric distribution of the results. That interval is divided into a number of subintervals that would be a power of two.

Let n be the number of subintervals, then $\log_2 n$ will be the measure, in bits, of the total interval, and the corresponding number of bits will be the measure of the standard interval. It follows that, for a material and for a certain characteristic of material, one fixes the maximum amplitude and n, which determines the standard interval. The experimental testing leads to results that populate differently the standard interval, some of the subintervals remaining unpopulated.

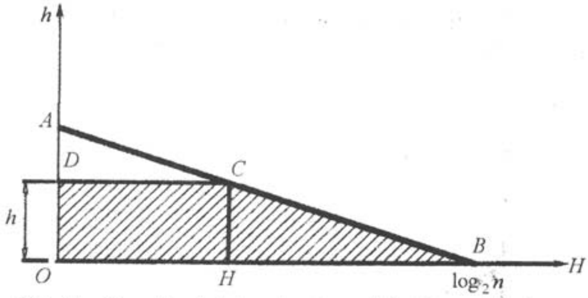

Fig. 2.- Graphical determination of homogeneity factor.

In the graphic representation h (H) the degree of homogeneity determines a point, C, on the line AB. A parallel to OH through that point divide the triangle area OAB into two parts. One founds easily that the hachured area $OBCD$ is proportional to the degree of homogeneity. The relation between that area and the total area OAB (equals to $\frac{1}{2}\log_2 n$) may be also considered as a measure of homogeneity, therefore it is proposed to be named the *homogeneity factor*, k_0 [3]. From geometric elementary considerations it results the relation

(4) $$k_0 = 1 - \frac{H^2}{H_{\max}^2} = 1 - \frac{H^2}{(\log_2 n)^2}.$$

For a given n, k_0 has a parabolic variation. It results that k_0, just like homogeneity degree, h, has a practical significance, only if reported to a standard interval. Particularly, for H equaling half the interval length, the homogeneity factor has the value $k_0 = 0.75$.

The homogeneity factor, k_0, can be expressed as:

(5) $$k_0 \stackrel{\text{..}}{=} h\left(1 + \frac{H}{H_{\max}}\right) = h\left(1 + \frac{H}{\log_2 n}\right).$$

The homogeneity factor, k_0, has a bigger sensitivity at lower variations of H and as a measure of homogeneity has a geometrical image with a suggestive interpretation.

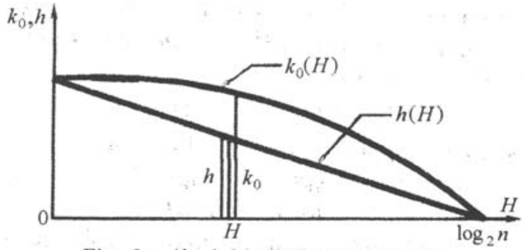

Fig. 3.- (h, k_0) *vs.* H dependences.

In Fig. 3 both the degree of homogeneity, $h(H)$, and the factor of homogeneity, $k_0(H)$, are represented on the same graphic. Obviously k_0 is bigger than h just like the results from relations (4) and (5).

The homogeneity factor can be used for measuring the degree of homogeneity of some physical and chemical properties, of some phenomena concerning biological homogeneity, in the economic problems, for measuring the quality product and services, and also in the marketing research.

3. Case Study

As example, two statistics series, (a) and (b), are presented having their elasticity moduli obtained in testing some steel square tubular bars and the frequencies of appearance of the values of these elasticity moduli (histograms (a) and (b), Fig. 4). For statistics series (a) was used measured section and for statistics series (b) was used standard section [2]. Therefore it was included a dimensional statistic variability. The standard interval has been considered with eight subintervals ($n = 8$) and the calculated entropies are $H = 2.772$ for the histogram (a) and $H = 2.8$ for the histogram (b).

The homogeneity factors calculated by means of the relation (4) have resulted to be $k_0 = 0.692$ for the series of values (a) and $k_0 = 0.686$ for the series (b). The homogeneity factor, k_0, attached to the series of value (a) being higher than that attached to the series of value (b), indicates a better homogeneity of the steel characterized by the series of testing (a).

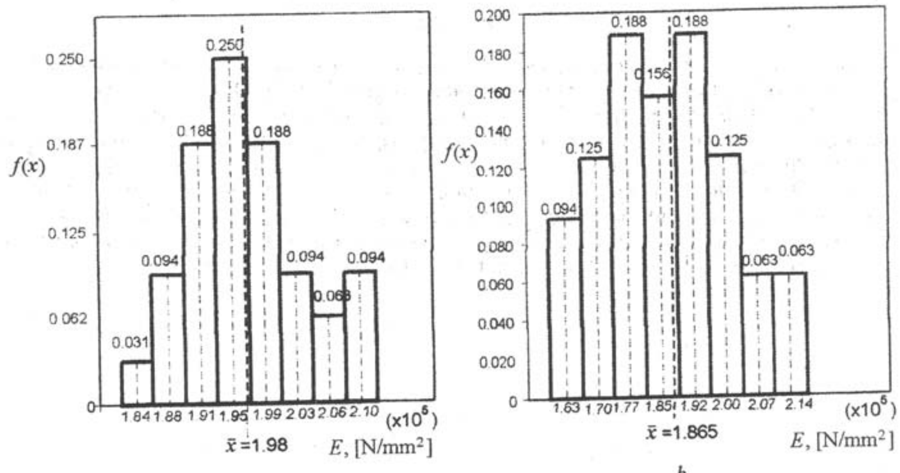

Fig. 4.– Histograms.

4. Conclusions

Introducing the concept of entropy can lead to the measure of homogeneity of some characteristics of materials, by using the notions as degree of homogeneity and homogeneity factor, respectively. These introduced sizes may be at the same time interpreted as quality indicators or as a measure of the degree of reliability in experimental analysis of some characteristics of materials. Along the side other statistic indicators, the homogeneity degree and the homogeneity factor may contribute to a better appreciation of the safety at which the strength structures are realized both in civil and mechanical engineering.

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UNELE CONSIDERAŢII PRIVIND CONCEPTUL DE ENTROPIE ÎN EVALUAREA PROPRIETĂŢILOR MATERIALELOR

(Rezumat)

Omogenitatea este un concept general, folosit adesea în ştiinţă şi îndeosebi în inginerie, ca o ipoteză de lucru, cu implicaţii esenţiale în exprimarea unor legităţi. Din punct de vedere mecanic proprietăţile unui material se exprimă prin caracteristicile care le definesc, determinate la randul lor de proprietăţile chimice şi fizice ale materialului constitutiv. In lucrare, pentru studiul omogenităţii se foloseşte conceptul de entropie utilizat în teoria informaţiei, pe baza căruia s-au definit noţiunile de *grad de omogenitate* şi de *factor de omogenitate*. Cu ajutorul acestor mărimi se pot analiza din punct de vedere al omogenităţii diferite populaţii statistice. In particular se fac referiri la modulele de elasticitate ale oţelului. Ca exemplu s-au considerat două serii statistice reprezentând modulele de elasticitate obţinute la încercarea unor bare din oţel cu secţiune pătrată.