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INFLUENCE OF LOADING PARAMETERS ON LATERAL TORSIONAL BUCKLING RESISTANCE OF STEEL PLATE GIRDERS

BY

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Abstract. The results of an analysis regarding the influence that the load location relative to shear centre of the section and the cross-section shape have on the bending resistance of the mono-symmetrical steel plate girders are presented. The girder bending resistance is evaluated according to the Euro norms EN 1993-1-1: *Design of Steel Structures. General Rules and Rules for Buildings* and EN 1993-1-5: *Plated Structural Elements.*The obtained results can be useful in the optimal design of steel plate girders.

Key words: steel plate girders; lateral-torsional buckling; bending resistance; influence parameters; Euronorms EN 1993-1-1: *Design of Steel Structures. General rules and rules for buildings*; EN 1993-1-5: *Plated Structural Elements.*

1. Introduction

The steel girders subjected to bending generally have a very different rigidity relative to the main axes ($I_y >> I_z$) so, they should be verified against lateral-torsional buckling.

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In the most practical cases of steel buildings and bridges, girders are built up of hot-rolled I and H sections or welded sections.

The evaluation of the actual critical bending moment for each loading pattern leads to solving a complex of differential eqs. so, in the design activity we use conservative and approximate eqs., useful in the most practical cases.

This paper presents an analysis regarding the influence that the load location relative to shear centre of the section and the cross-section shape have on the lateral torsional buckling resistance of the mono-symmetrical steel plate girders. The girder bending resistance is evaluated according to the Euronorms EN 1993-1-1: *Design of Steel Structures. General Rules and Rules for Buildings* and EN 1993-1-5: *Plated Structural Elements*.

2. Lateral-Torsional Buckling Resistance

2.1. Buckling Resistance

According to EN 1993-1-1:2006 - §6.3.2, the design buckling resistance moment of a laterally unrestrained beam should be taken as:

$$M_{b.Rd} = \chi_{\mathrm{LT}} W_y \frac{f_y}{\gamma_{M1}}, \tag{1}$$

where:

$$W_y = \begin{cases} W_{pl.y}, \text{ for class 1 or 2 cross-sections;} \\ W_{el.y}, \text{ for class 3 cross-sections;} \\ W_{eff.y}, \text{ for class 4 cross-sections;} \end{cases} \tag{2}$$

X_{LT} is the reduction factor for lateral-torsional buckling.

For bending moments of constant cross-sections, the value X_{LT} for the appropriate non dimensional slenderness, $\overline{\lambda}_{\mathrm{LT}}$, should be determined from:

$$\chi_{\mathrm{LT}} = \frac{1}{\Phi_{\mathrm{LT}} + \sqrt{\Phi_{\mathrm{LT}}^2 - \overline{\lambda}_{\mathrm{LT}}^2}}, \quad \text{but} \quad \chi_{\mathrm{LT}} \le 1, \tag{3}$$

where: $\Phi_{\mathrm{LT}} = 0.5\left[1 + \alpha_{\mathrm{LT}}\left(\overline{\lambda}_{\mathrm{LT}} - 0.2\right) + \overline{\lambda}_{\mathrm{LT}}^2\right]$; α_{LT} is an imperfection factor; $\overline{\lambda}_{\mathrm{LT}} = \sqrt{W_y f_y / M_{\mathrm{cr}}}$; M_{cr} is the elastic critical moment for lateral-torsional buckling.

The imperfection factor, α_{LT}, corresponding to the appropriate buckling curve may be obtained from Table 1.

Table 1
Imperfection Factors

Buckling curve	*a*	*b*	*c*	*d*
Imperfection factor, α_{LT}	0.21	0.34	0.49	0.76

The recommendations for buckling curves are given in Table 2.

Table 2.
Buckling Curves

Cross-section	Limits	Buckling curve
Rolled I - section	$h/b \le 2$	*a*
	$h/b > 2$	*b*
Welded I - section	$h/b \le 2$	*c*
	$h/b > 2$	*d*
Other cross - section	–	*d*

2.2. Elastic Critical Bending Moment

For a double T cross-section (I-section), the elastic critical moment for lateral torsional buckling is given by the expression derived from the buckling theory

$$M_{\mathrm{cr}} = C_1 \frac{\pi^2 EI_z}{(kL)^2 g}\left[\sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z} + \left(C_2 z_g - C_3 z_j\right)^2} - \left(C_2 z_g - C_3 z_j\right)\right]. \quad (4)$$

For doubly-symmetric I-section, $z_j = 0$, and the expression (4) becomes

$$M_{\mathrm{cr}} = C_1 \frac{\pi^2 EI_z}{(kL)^2 g}\left[\sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z} + \left(C_2 z_g\right)^2} - \left(C_2 z_g\right)\right]. \quad (5)$$

where: C_1, C_2 and C_3 are coefficients depending on the loading (moment diagram), section properties and support conditions,

$$g = \sqrt{\left(1 - \frac{I_z}{I_y}\right)} \approx 1.0.$$

If there are no destabilizing loads, $z_g = 0$ and it follows:

$$M_{\mathrm{cr}} = C_1 \frac{\pi^2 EI_z}{(kL)^2}\left[\sqrt{\left(\frac{k}{k_w}\right)^2 \frac{I_w}{I_z} + \frac{(kL)^2 GI_t}{\pi^2 EI_z}}\right]. \tag{6}$$

3. Analysis of Loading Parameters on Buckling Resistance

In what follows next, we analyse the influence of the loading parameters and of the cross-section shape on the lateral torsional buckling resistance of a girder with a mono-symmetric section.

The following analytical data are known:

a) static scheme, loading and the cross-section, Fig. 1;

b) material: steel S 355 – f_y = 355 N/mm^2;

c) the girder is laterally restrained at the end supports.

The following cases are analyzed:

a) the compression flange is larger in comparison with the tension flange (I_z is greater) ;

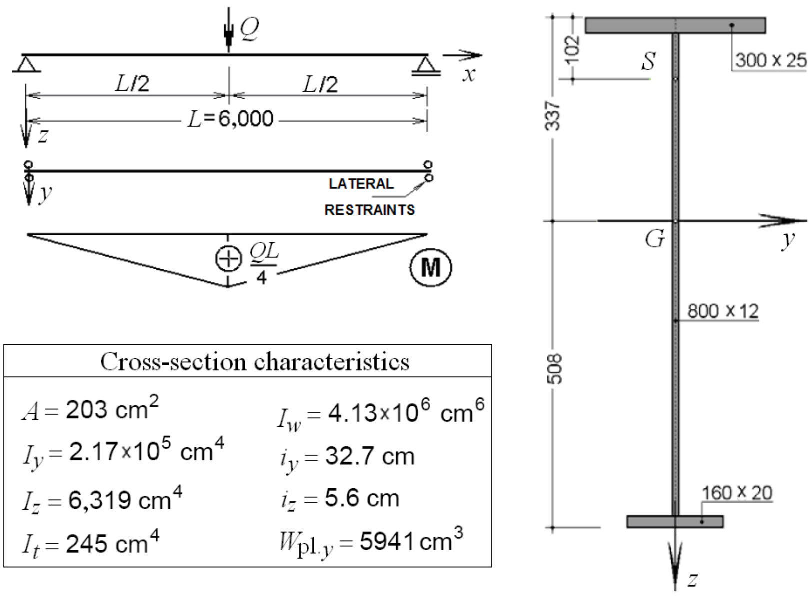
b) the compression flange is smaller in comparison with the tension flange (I_z is smaller);

The force acts in the following points of the cross-section:

a) at the compression flange (with a destabilizing effect);

b) in the shear centre;

c) at the tension flange.



Cross-section characteristics	
A = 203 cm^2	$I_w = 4.13\times10^6$ cm^6
$I_y = 2.17\times10^5$ cm^4	i_y = 32.7 cm
I_z = 6,319 cm^4	i_z = 5.6 cm
I_t = 245 cm^4	$W_{\mathrm{pl},y}$ = 5941 cm^3

Fig. 1 – Design data.

Solution:

In Tables 3 *a* and 3 *b* we present the analyzed cases (a total of 6 cases) and the geometric design parameters used to evaluate the lateral buckling resistance.

Table 3 *a*
Loading Parameters

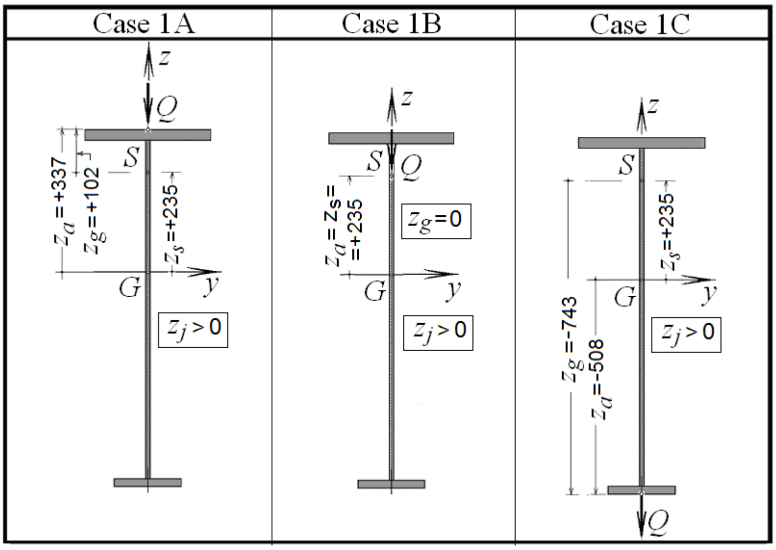
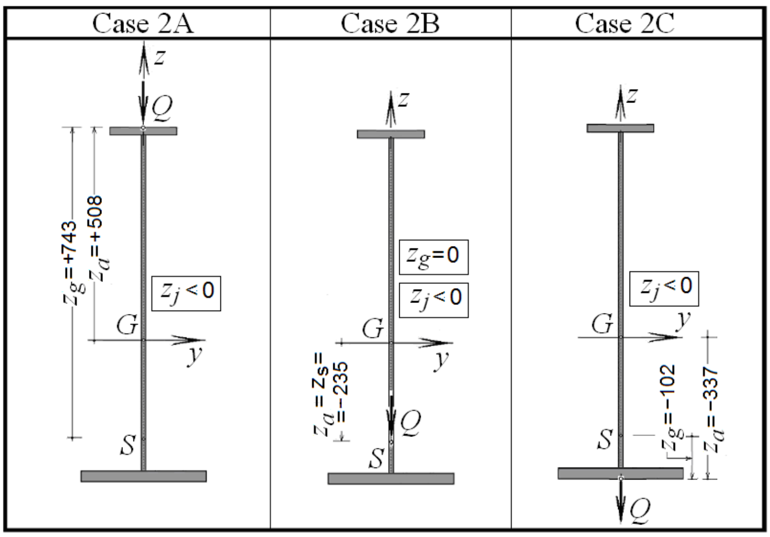


Table 3 *b*
Loading Parameters



3.1. Cross-Section Class

C a s e s 1: The compression flange is greater.
Compression flange: $\psi = 1$, $k_\sigma = 0.43$,

$$\frac{c}{t} = \frac{(300-12)/2}{25} = 5.76 < 9\varepsilon = 7.29 \Rightarrow \text{class } 1.$$

The web is an internal, partially compressed plate, Fig. 2 *a*.

$$\sigma_2 = -\frac{z_2}{z_1}\sigma_1 = -\frac{488}{312}\sigma_1 = -1.56 \cdot \sigma_1 \Rightarrow \Psi = -1.56 < -1, , \quad \alpha = \frac{312}{800} = 0.39 < 0.5,$$

$$\frac{c}{t} = \frac{800}{12} = 66.67 < 36\varepsilon / \alpha = 74.77 \Rightarrow \text{class } 1.$$

It results: that cross section belongs to class 1.

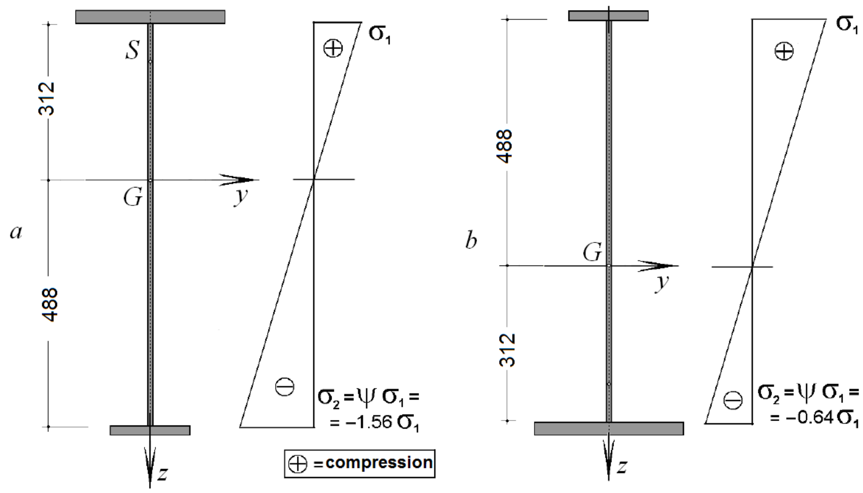


Fig. 2 – Evaluation of Cross – section class.

C a s e s 2: The compression flange is smaller

Compression flange:

$$\frac{c}{t} = \frac{(160-12)/2}{20} = 3.7 < 9\varepsilon = 7.29 \Rightarrow \text{class } 1$$

Web (see Fig. 2 *b*):

$$\sigma_2 = -\frac{z_2}{z_1}\sigma_1 = -\frac{312}{488}\sigma_1 = -0.64\sigma_1 \Rightarrow \Psi = -0.64 > -1\,,$$

$$\alpha = \frac{488}{800} = 0.61 > 0.5\,,$$

$$\frac{c}{t} = \frac{800}{12} = 66.67$$

$$\frac{c}{t} > \frac{456\varepsilon}{13\alpha - 1} = 53.29;\ \frac{c}{t} < \frac{42\varepsilon}{0.67 + 0.33\psi} = 74.15 \Rightarrow \text{class 2}\,.$$

It results: that cross section belongs to class 2

Parameters valid for all cases: α_{LT} = 0.76 – curve d for h/b > 2 (Table 1 and Table 2);

$$N_{\text{cr.}z} = \frac{\pi^2 EI_z}{(kL)^2} = \frac{\pi^2 2.1\times10^6 \times 6{,}319}{600^2}10^{-2} = 3{,}634.33\ \text{kN}\,,$$

$$\lambda_z = \frac{kL}{i_z} = \frac{600}{5.6} = 107.14\,,\ \lambda_1 = \pi\sqrt{\frac{E}{f_y}} = 93.9\varepsilon = 93.9\times0.81 = 76.1,$$

$$\beta_w = \frac{W_y}{W_{\text{pl.}y}} = 1.0\,,$$

$$C_1 = 1.365,\quad C_2 = 0.553,\quad C_3 = 1.73\,,$$

$$C_1 N_{\text{cr.}z} = 1.365\times3{,}634.33 = 4{,}961\ \text{kN}\,,$$

$$\frac{I_w}{I_z} = \frac{4.13\times10^6}{6{,}319} = 653.58;\quad GI_t = 0.81\times10^6\times245 = 198.45\times10^6\,,$$

$$W_{\text{pl.}y} = 5{,}941\ \text{cm}^3\,.$$

The coefficient z_j

$$z_j = \begin{cases} 0.4h_s\left(2\beta_f - 1\right), & \text{for } \beta_f > 0.5, \\ 0.5h_s\left(2\beta_f - 1\right), & \text{for } \beta_f < 0.5, \end{cases} \qquad \text{where: } \beta_f = \frac{b_{fc}^3 t_{fc}}{b_{fc}^3 t_{fc} + b_{ft}^3 t_{ft}}\,;$$

z_j is positive when the flange with greater I_z is in the compression zone.

Case 1

$$\beta_f = \frac{b_{fc}^3 t_{fc}}{b_{fc}^3 t_{fc} + b_{ft}^3 t_{ft}} = \frac{30^3 \times 2.5}{30^3 \times 2.5 + 16^3 \times 2} = 0.89 > 0.5,$$

$$z_j = 0.4 \times 82.25\,(2 \times 0.89 - 1) = 25.66.$$

Case 2

$$\beta_f = \frac{b_{fc}^3 t_{fc}}{b_{fc}^3 t_{fc} + b_{ft}^3 t_{ft}} = \frac{16^3 \times 2}{30^3 \times 2.5 + 16^3 \times 2} = 0.108 < 0.5,$$

$$z_j = 0.5 \times 82.25\,(2 \times 0.108 - 1) = -32.24.$$

3.2. Elastic Critical Buckling Moments

By using the expression (4 *a*), the following cases may be considered:
Case 1A

$$M_{\mathrm{cr}} = 4{,}961 \times 10^2 \left[\sqrt{653.58 + \frac{198.45 \times 10^6}{496{,}100} + (0.553 \times 10.2 - 1.73 \times 25.66)^2} - (0.553 \times 10.2 - 1.73 \times 25.66) \right] \times 10^{-4} = 4{,}430 \text{ kN.m.}$$

Case 1B

$$M_{\mathrm{cr}} = 4{,}961 \times 10^2 \left[\sqrt{653.58 + \frac{198.45 \times 10^6}{496{,}100} + (-1.73 \times 25.66)^2} - (-1.73 \times 25.66) \right] \times 10^{-4} = 4{,}930 \text{ kN.m.}$$

Case 1C

$$M_{\mathrm{cr}} = 4{,}961 \times 10^2 \left\{ \sqrt{653.58 + \frac{198.45 \times 10^6}{496{,}100} + \left[0.553 \times (-74.3) - 1.73 \times 25.66\right]^2} - \left[0.553 \times (-74.3) - 1.73 \times 25.66\right] \right\} \times 10^{-4} = 8{,}777 \text{ kN.m.}$$

Case 2A

$$M_{\text{cr}} = 4{,}961\times 10^{2}\left\{\sqrt{653.58+\frac{198.45\times 10^{6}}{496{,}100}+\left[0.553\times 74.3-1.73\times(-32.24)\right]^{2}}-\right.$$
$$\left.-\left[0.553\times 74.3-1.73\times(-32.24)\right]\right\}\times 10^{-4} = 263\ \text{kN.m.}$$

Case 2B

$$M_{\text{cr}} = 4{,}961\times 10^{2}\left\{\sqrt{653.58+\frac{198.45\times 10^{6}}{496{,}100}+\left[-1.73\times(-32.24)\right]^{2}}-\right.$$
$$\left.-\left[-1.73\times(-32.24)\right]\right\}\times 10^{-4} = 263\ \text{kN.m.}$$

Case 2C

$$M_{\text{cr}} = 4{,}961\times 10^{2}\left\{\sqrt{653.58+\frac{198.45\times 10^{6}}{496{,}100}+\left[0.553\times(-10.2)-1.73\times(-32.24)\right]^{2}}-\right.$$
$$\left.-\left[0.553\times(-74.3)-1.73\times 25.66\right]\right\}\times 10^{-4} = 477\ \text{kN.m.}$$

3.3. Non-Dimensional Slenderness, $\overline{\lambda}_{\text{LT}}$, and Reduction Factors, χ_{LT}

The non-dimensional slenderness is evaluated with the expression:

$$\overline{\lambda}_{\text{LT}} = \sqrt{\frac{W_y f_y}{M_{cr}}}\text{, where: } W_y = W_{\text{pl}.y} = 5{,}941\ \text{cm}^3,\ W_{\text{pl}.y} f_y = 2{,}109\ \text{kN.m}\,.$$

Taking into account the values of the non dimensional slenderness, $\overline{\lambda}_{\text{LT}}$, obtained after the calculation of the critical moments, M_{cr}, the reduction factors, χ_{LT}, can be evaluated

$$\chi_{\text{LT}} = \frac{1}{\Phi_{\text{LT}}+\sqrt{\Phi_{\text{LT}}^{2}-\overline{\lambda}_{\text{LT}}^{2}}}\,,$$

where: $\Phi_{\text{LT}} = 0.5\left[1+\alpha_{\text{LT}}\left(\overline{\lambda}_{\text{LT}}-0.2\right)+\overline{\lambda}_{\text{LT}}^{2}\right]$, α_{LT} = 0.76 – curve *d* for *h*/*b* >2 (Tables 1 and 2).

The following values are obtained:

C a s e 1A

$$\bar{\lambda}_{\mathrm{LT}}=\sqrt{\frac{2{,}109}{4{,}430}}=0.69\,;\ \Phi_{\mathrm{LT}}=0.5\left[1+0.76\left(0.69-0.2\right)+0.69^2\right]=0.92\,;$$

$$\chi_{\mathrm{LT}}=\frac{1}{0.92+\sqrt{0.92^2-0.69^2}}=0.65\,.$$

C a s e 1B

$$\bar{\lambda}_{\mathrm{LT}}=\sqrt{\frac{2{,}109}{4{,}930}}=0.65\,;\ \Phi_{\mathrm{LT}}=0.5\left[1+0.76\left(0.65-0.2\right)+0.65^2\right]=0.88\,;$$

$$\chi_{\mathrm{LT}}=\frac{1}{0.88+\sqrt{0.88^2-0.65^2}}=0.68\,.$$

C a s e 1C

$$\bar{\lambda}_{\mathrm{LT}}=\sqrt{\frac{2{,}109}{8{,}777}}=0.49\,;\ \Phi_{\mathrm{LT}}=0.5\left[1+0.76\left(0.49-0.2\right)+0.49^2\right]=0.73\,;$$

$$\chi_{\mathrm{LT}}=\frac{1}{0.73+\sqrt{0.73^2-0.49^2}}=0.77\,.$$

C a s e 2A

$$\bar{\lambda}_{\mathrm{LT}}=\sqrt{\frac{2{,}109}{263}}=2.83\,;\ \Phi_{\mathrm{LT}}=0.5\left[1+0.76\left(2.83-0.2\right)+2.83^2\right]=5.50\,;$$

$$\chi_{\mathrm{LT}}=\frac{1}{5.5+\sqrt{5.5^2-2.83^2}}=0.10\,.$$

C a s e 2B

$$\bar{\lambda}_{\mathrm{LT}}=\sqrt{\frac{2{,}109}{434}}=2.20\,;\ \Phi_{\mathrm{LT}}=0.5\left[1+0.76\left(2.20-0.2\right)+2.20^2\right]=3.68\,;$$

$$\chi_{\mathrm{LT}} = \frac{1}{3.68 + \sqrt{3.68^2 - 2.2^2}} = 0.15$$

C a s e 2C

$$\overline{\lambda}_{\mathrm{LT}} = \sqrt{\frac{2,109}{477}} = 2.10\,;\ \Phi_{\mathrm{LT}} = 0.5\left[1 + 0.76\left(2.10 - 0.2\right) + 2.10^2\right] = 3.43\,;$$

$$\chi_{\mathrm{LT}} = \frac{1}{3.43 + \sqrt{3.43^2 - 2.1^2}} = 0.16\,.$$

3.4. Bending Moment Resistance

According to EN 1993-1-1:2006, §**6.3.2**, the design buckling resistance moment of a beam should be evaluated with the expression (2).

4. Final Remarks and Conclusions

The analysis results are presented in Table 4.

Table 4
Parameters $\overline{\lambda}_{\mathrm{LT}}$ and χ_{LT}

Case	Parmeter	
	$\overline{\lambda}_{\mathrm{LT}}$	χ_{LT}
1A	0.69	0.65
1B	0.65	0.68
1C	0.49	0.77
2A	2.83	0.10
2B	2.20	0.15
2C	2.10	0.16

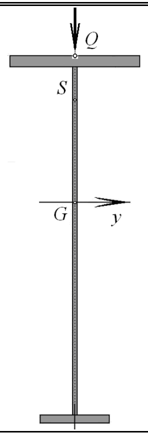
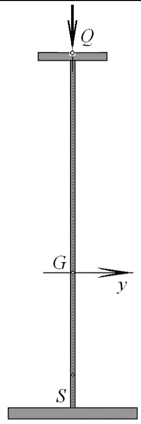
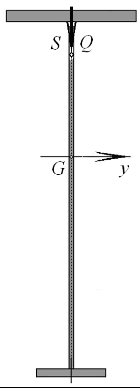
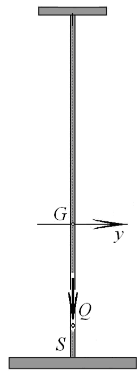
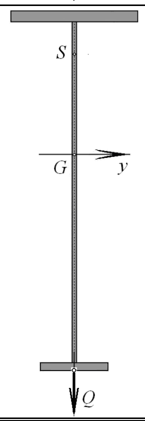
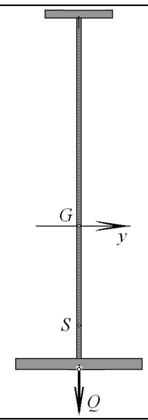
Table 5 synthetically presents the analysis results.

Taking into account that the cross-section class is 1 or 2, the ratio of the bending moment is equal with the ratio of the reduction factors ($W_y = W_{\mathrm{pl}.y}$).

It can be observed that the maximum buckling moment resistance is obtained when the load force has a stabilizing effect and the compression flange is larger in comparison with the tension flange.

Table 5

$M_{b.Rd}/M_{b.Rd}^{\min}$ *Ratio*

Case	Scheme	$M_{b.Rd}/M_{b.Rd}^{\min}$	Case	Scheme	$M_{b.Rd}/M_{b.Rd}^{\min}$
1A		6.5	2A		1.0
1B		6.8	2B		1.5
1C		7.7	2C		1.6

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Moga P., *Grinzi metalice zvelte*. Univ. Tehn., Cluj-Napoca, 2011

INFLUENŢA PARAMETRILOR DE ÎNCĂRCARE ASUPRA REZISTENŢEI LA INCOVOIERE CU FLAMBAJ LATERAL, A GRINZILOR METALICE

(Rezumat)

Se prezintă rezultatele unei analize privind influenţa poziţiei încărcării în raport cu centrul de răsucire – forfecare al secţiunii transversale şi influenţa formei secţiunii asupra rezistenţei la încovoiere cu flambaj lateral a grinzilor metalice cu secţiune monosimetrică.

Calculul momentelor capabile se efectuează în conformitate cu norma SR EN 1993-1-1:2006: *Proiectarea structurilor de oţel*. Partea 1-1: *Reguli generale şi reguli pentru clădiri.*

Rezultatele obţinute pot fi utile în activitatea de proiectare optimală a grinzilor cu inimă plină care sunt susceptibile de a suferi fenomenul de pierdere a stabilităţii generale prin flambaj lateral.