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EQUIVALENT VISCOUS DAMPING FOR THE ELASTO-PLASTIC HYSTERETIC MODEL

BY
RAUL ZAHARIA

The paper proposes some formulae to determine the equivalent linear parameters for spectral earthquake response of SDOF non-linear systems. The proposed formulae for the equivalent viscous damping and equivalent period are valid for the elasto-plastic hysteretic model and for earthquakes compatible with Eurocode 8 response spectra. This study is part of a research aimed to determine the equivalent linear parameters in order to predict the maximum displacement response for earthquakes compatible with given response spectra, for different hysteretic models.

1. Introduction

In a previous paper [11] some equivalent viscous damping models used in the displacement based seismic design were reviewed. Based on comparisons from the literature, the author highlighted the limits of application of the existing models and concluded that up to present time, no direct displacement-based design method, based on equivalent linearization, is able to provide satisfactory results in the attempt to determine the maximum displacement response from real earthquake records. Most of the methods, like the ones proposed by Rosenblueth and Herrera [10] or Kowalsky [8], [9], use a theoretical approach to compute the equivalent viscous damping. These methods are assuming a sinusoidal steady-state response and are based on the arbitrary choice of the one cycle criterion to estimate the equivalent viscous damping. This means that the energy dissipation of the system will be approximated from the cycle corresponding to the maximum level of deformation, cycle supposed to be symmetric and with the same shape for the entire excitation time. For an earthquake loading, the response is most of the time smaller than the response amplitude, so the use of this criterion will overestimate the equivalent damping. In the same time, the equivalent stiffness is approximated also function of the maximum level of deformation. This leads, for a particular earthquake, to a wide dispersion in the estimation of the maximum displacement, from conservative to unconservative range.

Methods combining the theoretical approach with testing using recorded accelerograms, like Gulkan and Sozen [3] proposal, or empirical determination of equivalent parameters based on numerical simulations of recorded accelerograms, like Iwan and Gates [5]. [6] would seem to be more reliable when applied to real earthquake records. However, this kind of approach is generally suitable only for earthquake records having similar spectral response with the one considered in determining the equivalent linear characteristics.

The paper presents the procedure developed by the author for determining the equivalent parameters for the elasto-plastic hysteretic model, considering Eurocode 8 [1] compatible response spectra.

The methodology follows the empirical approach of Iwan and Gates (*op. cit.*), considering a more refined range of structural vibration periods. For the family of generated earthquake records compatible with Eurocode 8 response spectra, and considering the elasto-plastic hysteretic model, the inelastic displacement response is calculated by numerical integration for different ductility ratios. After determining the elastic displacement response spectrum, for different values of damping and period shift, the optimal pair of these equivalent linear parameters is determined. The optimal equivalent parameters correspond to the elastic displacement spectrum that gives the lowest averaged error in comparison with the "exact" inelastic displacement spectrum. By representing the optimal period shift *versus* the ductility, and the optimal equivalent damping *versus* the optimal period shift, empirical formulas for equivalent period and damping function of the ductility are obtained.

Artificially generated time-history motions compatible with Type 1 response spectra of Eurocode 8, for different types of soil, were considered for this study.

2. Generation of Response Spectrum Compatible Accelerograms

The Eurocode 8 compatible response spectra accelerograms were generated using the procedure described by Clough and Penzien [2]. For Type 1 spectrum, used to generate the artificial ground motions, three different magnitudes were considered, of 6.5, 7 and 7.5. For each of the five soil types (A,...,E) given in Eurocode 8 and for the selected magnitudes, two accelerograms were generated. Thus, a total of 30 accelerograms compatible with Type 1 elastic response spectrum from Eurocode 8 were generated to determine the parameters of the linear equivalent system for the elasto-plastic hysteretic model.

The elastic displacement spectrum, $S_{De}(T)$, in Eurocode 8, may be obtained by direct transformation of the elastic acceleration response spectrum, $S_e(T)$, from the following expression

$$S_{De}(T) = S_e(T)\left(\frac{T}{2\pi}\right)^2 . \tag{1}$$

Eurocode 8 recommends to use this expression for vibration periods, not exceeding 4.0 s. For structures with vibration periods longer than 4.0 s, an expression for deriving the Type 1 elastic displacement response spectrum is presented in Informative Annex A of Eurocode 8. Up to a period of vibration, T_E, between 4.5 s and 6.0 s (depending on the soil type) the spectral ordinates of the displacement spectrum are obtained from $S_e(T)$ considering equation (1). For vibration periods beyond T_E, the ordinates of the elastic displacement response spectrum are obtained from the following equations

$$T_E \leq T \leq T_F : S_{De}(T) = 0.025 a_g S T_C T_D \left[2.5\eta + \left(\frac{T - T_E}{T_F - T_E} \right) (1 - 2.5\eta) \right], \tag{2}$$

$$T \geq T_F : S_{De}(T) = 0.025 a_g S T_C T_D, \tag{3}$$

where: a_g is the design ground acceleration, S – the soil factor, η – the damping correction factor (equal to 1.00 for 5% viscous damping), T_C and T_D – reference vibration periods given by Eurocode 8 function of the soil type and $T_F = 10.0$ s is the vibration period from which the elastic structural displacement equals the ground displacement.

The procedure proposed in this study considers a shift in the vibration period of the SDOF oscillator. The present study is limited to structures with periods up to 4.0 s. Thus, considering the period shift, it is expected that for high ductility ratios the equivalent period may reach values up to around 9.0 s. For this reason it is important that the response spectrum can give a correct estimation of elastic displacements at large periods of vibration, exceeding the values commonly used in design based on reduction of the acceleration response.

3. Description of the Procedure Utilized and Obtained Results

The methodology for determining the equivalent period and damping follows the empirical approach of Iwan and Gates (*op. cit.*), but instead of analysing a limited number of vibration periods, a more refined procedure was set-up using the "SPON" procedure in Cast3m computer code. This procedure calculates the inelastic displacement spectrum of a SDOF oscillator for a given hysteretic model and ductility ratio. Using this approach, a large set of models can be considered for a wide range of vibration periods and ductilities. For this study, vibration period between 0 and 4.0 s at 0.02 s increments and six ductility ratios (1.5, 2, 3, 4, 5 and 6) were considered.

Iwan and Gates (*op. cit.*) observed that the overall shape of the inelastic PSV response spectrum of a SDOF oscillator for a given ductility ratio, resembles to the shape of an elastic PSV spectrum shifted in period by a certain factor. In the present study, the comparison was made at the level of elastic and inelastic displacement spectra. Thus, by making an estimation of the damping and period shift of the elastic spectra, for which the resulted displacements approximate with the smallest error the inelastic displacements, a set of "optimal" equivalent damping and equivalent period values may be obtained, for each earthquake, function of the ductility ratio. The procedure for determining the equivalent linearized properties of the elasto-plastic hysteretic model to earthquake excitation, compatible with Eurocode 8 Type 1 spectra, is described in the following.

First, for a given earthquake and ductility ratio, the inelastic displacement response spectrum, S_{Di}/a_g, is determined. Fig. 1 shows the inelastic spectrum of a 7.5 magnitude ground motion, considering type A soil conditions, and a ductility ratio of 1.5.

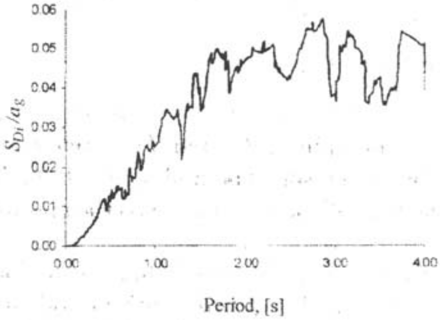

Fig. 1.– Inelastic displacement spectra.

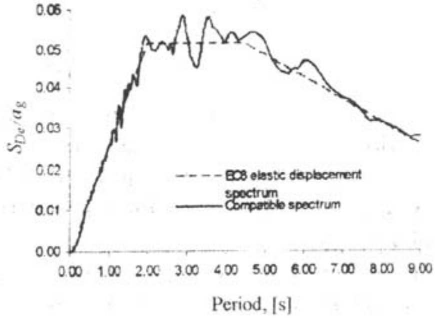

Fig. 2.– Elastic displacement spectra.

The elastic displacement response spectrum, S_{De}/a_g, is then determined, for the same earthquake, for an initial damping ratio of 5%. This spectrum is compatible with EC8 Type 1 response spectrum for soil A, as shown in Fig. 2. The elastic displacement response spectrum of the selected earthquake is calculated for damping ratios varying between 5 and 25% at 0.01% increments and with shifts of the initial period between 1 and 3 s at 0.01 s increments. The upper bound values in period shift and damping ratio were established after preliminary simulations performed with the largest value of ductility. For a given combination of damping ratio ξ_{eq} (equivalent damping), and shifted period, T_{eq} (equivalent period), the shifted "equivalent" elastic spectral displacement is computed, for vibration periods of a SDOF oscillator between 0.02 and 4 s, at 0.02 s increments. The ratio between the inelastic "exact" displacement and the equivalent elastic displacement is calculated with the error estimation, ε_i, of the difference between these two displacements expressed as

$$\varepsilon_i(\xi_{\text{eq}}; T_{\text{eq}}) = \frac{S_{De}}{S_{Di}} - 1. \tag{4}$$

The measure of the overall error for the entire range of vibration periods between 0.02...4.0 s is given by the average error

$$\varepsilon(\xi_{\text{eq}}; T_{\text{eq}}) = \sqrt{\sum_{i=1}^{N} \frac{\varepsilon_i^2}{N}}, \tag{5}$$

where $N = 200$ is the number of SDOF oscillators with vibration periods up to 4.0 s for which the inelastic and elastic shifted displacements were computed.

For a given ductility, the average error is calculated for all the combinations of ξ_{eq} and T_{eq}. The pair $(\xi_{\text{eq}}; T_{\text{eq}})$ corresponding to the minimum value, $\varepsilon_{\min}(\xi_{\text{eq}}; T_{\text{eq}})$, is retained, and defines the parameters of the equivalent linear system that givens the best approximation of the inelastic displacement for the considered range of periods.

For the 7.5 magnitude earthquake compatible with the Type 1 soil A spectrum, the following optimal parameters were determined, for the ductility ratio of 1.5:

$$\xi_{\text{eq}} = 5.95\%, \quad T_{eq} = 1.1T_0,$$

where T_0 is the initial period of the SDOF oscillator considered. For this pair of optimal values, the corresponding average error is $\varepsilon_{\min} = 12.6\%$. Fig. 3 shows the inelastic "exact" and the elastic equivalent displacement spectra calculated considering these optimal equivalent damping and period shift values. Fig. 4 shows the average minimum error in term of the ductility ratio, computed for the 30 earthquakes compatible with Type 1 spectra, considering the elasto-plastic hysteretic model. It can be observed that the errors range between 10 and 37% and increase with ductility.

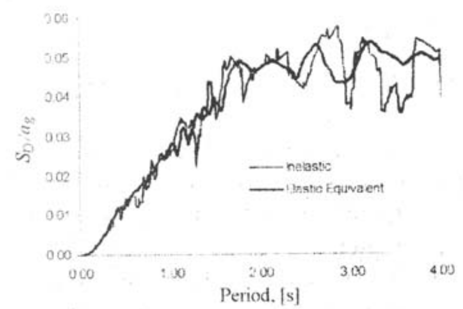

Fig. 3.– Inelastic and equivalent elastic displacement spectrum.

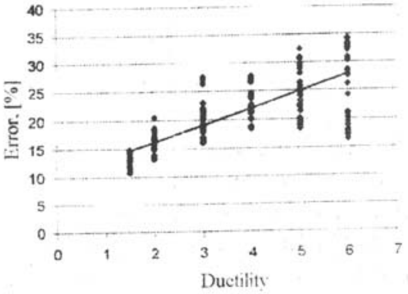

Fig. 4.– Averaged error.

The values of optimal period shift are plotted in Fig. 5 *versus* the ductility ratio for all the 30 earthquakes compatible with Type 1 spectra. The optimal damping ratio *versus* optimal period shift is plotted, for all earthquakes and ductilities, in Fig. 6.

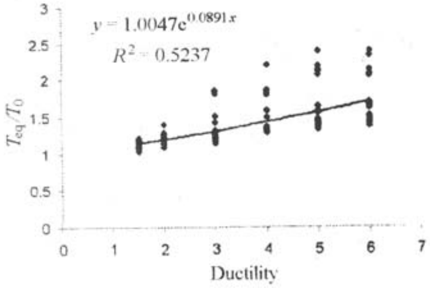

Fig. 5.– Optimal period shift *versus* ductility.

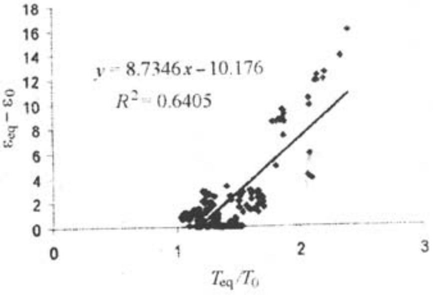

Fig. 6.– Equivalent damping *versus* period shift.

The trend lines of the scatter are given in both figures. The optimal equivalent period may be expressed as a function of the ductility ratio and the initial period by the following exponential equation, which gives an R-squared value of 0.52 for the scatter considered

$$T_{\mathrm{eq}} = 1.005e^{0.089\mu}T_0, \ [\mathrm{sec}]. \tag{6}$$

Considering a linear function for the trend line of the scatter of equivalent damping *versus* period shift, which gives an R-squared value of 0.64, the following equation is determined

$$\xi_{\mathrm{eq}} - \xi_0 = 8.735\frac{T_{\mathrm{eq}}}{T_0} - 10.176, \ [\%]. \tag{7}$$

By replacing expression (6) into equation (7), the equivalent damping ratio is expressed as a function of the ductility ratio

$$\xi_{\mathrm{eq}} = \xi_0 + 8.779e^{0.089\mu} - 10.176, \ [\%]. \tag{8}$$

Equations (6) and (8) give the parameters of the linear equivalent system as functions of the ductility ratio and the initial stiffness.

The ratios of approximate elastic displacements to the exact inelastic displacements were computed, in order to investigate the accuracy of the procedure. The results for all 30 earthquakes were averaged for each ductility ratio for the period range 0.02...4.0 s. Generally, the errors are increasing with the level of deformation (ductility). Fig. 7 shows the obtained ratios by averaging all ductilities, together with the linear trend line of this characteristic (and its formula). It may be observed that the procedure underestimate the exact displacement up to period values around 2.0 s, and overestimates the exact displacement above this period. Because for short periods, below 0.1 s, the methodology produces high errors, the vibration period interval was limited to 0.1...4.0 s.

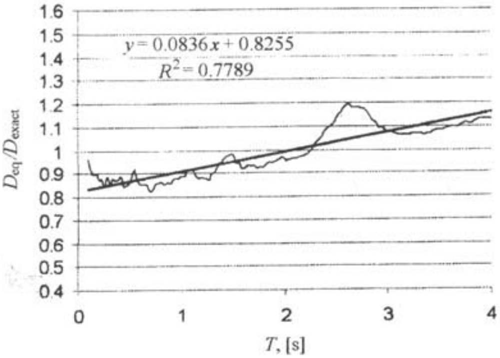

Fig. 7.- Design to exact displacement (averaged ductilities).

4. Conclusions

The equivalent stiffness and equivalent damping for spectral earthquake response of SDOF non-linear systems, subjected to European compatible spectra earthquakes were determined, considering the elasto-plastic hysteretic model.

The empirical procedure tends to underestimate the exact inelastic displacements up to period values around the vibration period, T_D, of Eurocode 8 compatible spectrum, (2 s for Type 1 compatible spectrum), from which the SDOF oscillator is sensitive to the ground displacement. Above this vibration period, the procedure tends to overestimate the exact inelastic displacement.

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„Politehnica" University, Timşoara

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AMORTIZARE VÂSCOASĂ ECHIVALENTĂ PENTRU MODELUL HISTERETIC ELASTO-PLASTIC

(Rezumat)

Se propune un set de formule pentru determinarea parametrilor liniar echivalenţi (amortizare vâscoasă echivalentă şi perioadă echivalentă) care să aproximeze răspunsul seismic al sistemelor

neliniare cu un grad de libertate. Formulele sunt valabile pentru modelul de comportament histeretic elasto-plastic şi pentru seisme compatibile cu spectrele de răspuns date de norma de proiectare antiseismică europeană Eurocode 8. Prezentul studiu face parte dintr-o cercetare in curs de desfăşurare, avand ca obiect determinarea parametrilor modelului elastic echivalent capabil să prezică deplasarea inelastică maximă pentru cutremure compatibile cu un anumit spectru de răspuns, pentru diverse modele histeretice.