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CONSIDERATIONS REGARDING THE ANALYSIS OF MASONRY WALLS TO HORIZONTAL LOADS USING EQUIVALENT FRAME METHOD BUILDING

BY

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Abstract. The paper presents the calculus of structural masonry walls with average openings to horizontal loads using the equivalent frame method. The equivalent frame has a number of levels equal to the number of floors of the building, the number of columns equal to the number of piers and connecting beams with constant sections. The geometrical characteristics of the equivalent frame are established from the condition of equality between the wall and frame stiffness when they are subjected to horizontal loads. The efforts from the equivalent frame elements are equal to those in the wall members. The advantage of the proposed method lies in the fact that computer software for developing orthogonal frames analysis are used for the statical calculus of the equivalent frame. In the case of 2-D analysis, where every wall is taken individually to simplify the calculus, columns can be chosen as square cross section, while for 3-D analysis the cross section areas of the columns will be rectangular established from the condition of equal stiffness along both building directions.

Key words: stiffness; equivalent frame; distorsion; efforts; masonry.

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1. Introduction

The reinforced concrete structural walls with average-openings arranged as vertical rows on their height, and subjected in their plane by wind or earthquake have a behaviour that resembles that of orthogonal frames. For these types of structures is admitted the use of simplified calculus methods in the elastic domain and the modelling of the structural walls by storeyed frames (CR2-1-1.1/2011 and 2006; Brînzan & Barbaiani, 1976; Agent & Postelnicu, 1983; Marusciac *et al.*, 2000).

The equivalent frame method can also be used for analysis of masonry walls as code CR6-2006, allows their taking as frames in the calculus.

The equivalent frame has the number of levels equal to the number of building levels and the number of columns equal to the number of piers.

The advantage of this method consists in the availability of the software that already exists for the analysis of actual frames. One can define equivalent frames for 2-D software as well as for 3-D software that enable the spatial analysis of the entire structure.

The present paper defines the equivalent frame method which can be used in analysis of coupled structural walls with openings arranged as vertical rows, where the lintel beams present a constant cross-section areas and the equivalent frame can be calculated with any software that is used for common frames.

2. Establishing the Equivalent Frame

For 2-D programmes, the theoretical spans of the equivalent frames that sketches the coupled structural walls are taken between the axis of the piers (L), and the connecting beams are taken as deformable, to bending and shear only along the free span (l), while on sections $L - l$ have infinite stiffness.

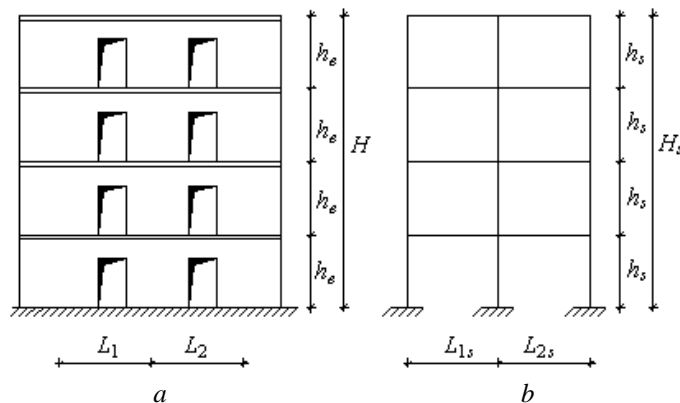


Fig. 1 – Structural wall (a); equivalent frame (b).

The equivalent frame has the number of columns equal to the number of piers of the wall and the number of floors equal to the number of buildings levels. The openings and the height of the storeys can differ as regards the distances between the axes of the piers or the actual height of the floors, but must respect the stiffness condition: the stiffness of the equivalent frame should be equal to the stiffness of the wall (Fig. 1).

The columns of the equivalent frame can have a square or rectangular cross area, and connecting beams can be rectangular in section, their dimensions being established from the stiffness requirement; the distortion of the replacing columns (k_s) and connecting beam (k_{rs}) should be equal to the distortion stiffness of the piers (k_m) and lintelbeams (k_r) of the wall

$$k_s = k_m, \quad (1)$$

$$k_{rs} = k_r. \quad (2)$$

Replacing the stiffness it results:

$$\frac{E_s I_s}{h_s} = \frac{E_m I_m}{h_e}, \quad (3)$$

$$\frac{6E_{rs} I_{rs}}{L_s} = \frac{6\rho_f E_r I_r}{L} \left(\frac{L}{l}\right)^3 \mu, \quad (4)$$

where: E_s , E_{rs} , E_m and E_r are the modulus of elasticity of the columns and connecting beam of the equivalent frame, respectively of the piers and lintel beams; I_r , I_{rs} , I_m and I_r – the inertia moments of the mentioned elements; ρ_f – an under-unit coefficient which introduces the decrease of lintel beam stiffness after their cracking, and μ – an under-unit coefficient that takes into account the effect of the lintel beam shear deformation upon stiffness.

From eqs. (3) and (4) it results the inertia moments of tie-columns and connecting beams

$$I_s = \frac{b_s h_s^3}{12} = I_m \frac{E_m h_s}{E_s h_e}, \quad (5)$$

$$I_{rs} = \frac{b_{rs} h_{rs}^3}{12} = I_r \frac{\rho_f E_r L_s}{E_{rs} L} \left(\frac{L}{l}\right)^3 \mu, \quad (6)$$

In relationships (5) and (6), E_m and E_r are the reinforced concrete modulus of elasticity for the cast-in-place concrete walls or the modulus of

elasticity of the masonry, taking into account the contribution of the concrete in the lintels and reinforced tie-columns in the case of confined masonry walls.

Dimensions L_s and h_s related to the equivalent frame as well as the modulus of elasticity, E_s and E_{rs} , can be equal to those of the wall or can possess other values, established from the condition that the dimension of the columns and lintel beams ranges in the usual limits and the columns and the connecting beams of the equivalent frame also comply with the condition

$$\frac{E_m}{E_s} = \frac{E_r}{E_{rs}} = e_s, \quad (7)$$

Eqs. (5) and (6) becomes

$$I_s = I_m \frac{h_s}{h_e} e_s, \quad (8)$$

$$I_{rs} = I_r \frac{L_s}{L} \left(\frac{L}{l} \right)^3 \mu \rho_f e_s. \quad (9)$$

In the case of monolithic reinforced concrete walls, the piers and the connecting beams are made from the same material, hence the moduli of elasticity, E_m and E_r , are equal.

For the masonry walls, the piers are realized of unreinforced masonry (ZNA) or reinforced masonry with tie-columns of reinforced concrete (ZC), while connecting beam can be made of only one member (lintelbeam) or mixed, made of the beam, lintel and the masonry in between.

In case of masonry walls, the modulus of elasticity of the piers is the modulus of elasticity of the unreinforced masonry (in case of ZNA masonry), respectively the equivalent modulus of elasticity for confined masonry, while the modulus of elasticity of the connecting beam is the modulus of elasticity of the concrete (if connecting beam is made of only one for the mixed lintel area).

The moduli of elasticity of the equivalent columns (E_s) and of the connecting beams (E_{rs}) can be equal to the modulus of elasticity of the piers and lintel beam of the walls or can be different, if relationship (7) is observed. Subunit values are recommended for the coefficient e_s to minimize the cross sections of the equivalent members, respectively smaller heights for the equivalent frame of the building.

Considering the equivalent columns of square cross section of side, a , from (5) it yields:

$$a = \sqrt[4]{12 I_m \frac{h_s}{h_e} e_s}. \quad (10)$$

The width of connecting beams are established as being equal with the pier thickness, and from eq. (6) will result:

$$h_{rs} = \frac{L}{l} \sqrt[3]{12 \frac{I_r}{b_{rs}} \cdot \frac{L_s}{L} \mu \rho_f e_s}. \quad (11)$$

Knowing the geometrical characteristics and related loads, the replacing frame can be calculated either by simplified methods or by means of software specific for plane frames.

In case of 3-D programs, in order to simplify analysis, all the equivalent frames will have equal spans and columns placed at the intersection of the axes (which can be equal to or different from the modular axes of the building) (Fig. 2).

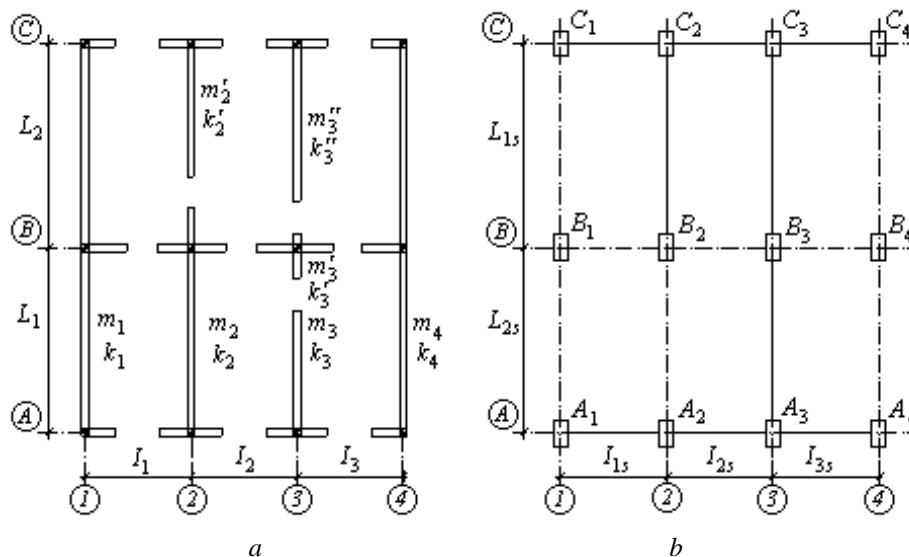


Fig. 2 – Masonry structure (a), equivalent frame (b);
 ——— real connecting beam, - - - - - connecting beam.

In the calculus along the transversal direction, the walls from axes 1 and 4 must be replaced by three columns (A_1 , B_1 and C_1 , respectively A_4 , B_4 , C_4), and piers m_2 by two columns (A_2 and B_2). These columns that replace an uncoupled wall or piers will be joined by fictitious connecting beams, which represent lintelbeams with very small sizes and stiffness that do not influence efforts coming from adjacent columns.

In order to define the dimensions of the replacing columns, the stiffness conditions are required.

In the transversal direction:

a) The stiffness sum of the equivalent column A_1 , B_1 and C_1 (from axis 1) should be equal with the stiffness of the wall:

$$k_{A_1} + k_{B_1} + k_{C_1} = k_1. \quad (12)$$

b) The stiffness sum of the equivalent column A_2 and B_2 (from axis 2) should be equal with stiffness of the pier m_2 , while stiffness of the column C_2 is equal with stiffness of the pier m_2 .

$$k_{A_2} + k_{B_2} = k_2, \quad (13)$$

$$k_{C_2} = k_{m_2}. \quad (14)$$

Stiffness k_{A_1} , k_{B_1} and k_{C_1} , respectively k_{A_2} and k_{B_2} are arbitrary choosed, respecting eqs. (12) and (13) and considering that sizes of the columns should be determined considering the stiffness equality on longitudinal direction.

c) In axis 3 exists a biunivoque correspondence between the number of equivalent columns and the number of piers, resulting

$$k_3 = k_{m_3}, \quad (15)$$

$$k_{B_3} = k'_{m_3}, \quad (16)$$

$$k_{C_3} = k''_{m_3}. \quad (17)$$

Axis 4 is identically with axis 1.

By imposing the equality conditions of the stiffness of the replacing columns with the stiffness of the piers along the two directions, for every column, it yields a system of two eqs. whose unknowns are the dimensions of the columns.

Noting with a_x and a_y , k_{xx} and k_{yy} the dimensions and the stiffness values of a replacing column along the two directions, respectively with k_{mx} and k_{my} the stiffness values of a piers, by imposing the stiffness conditions along the two directions it results

$$k_{xx} = k_{mx}, \quad (18)$$

$$k_{yy} = k_{my}, \quad (19)$$

From eqs. (16) and (17) it results the size of equivalent column:

$$a_x = \sqrt[8]{\frac{144I_{my}^3 \left(\frac{h_s}{h_e}\right)^2}{I_{mx}}} e_s^2, \quad (20)$$

$$a_y = \sqrt[8]{\frac{144I_{mx}^3 \left(\frac{h_s}{h_e}\right)^2}{I_{my}}} e_s^2. \quad (21)$$

For connecting beams dimensions are established from eq. (6).

By establishing the dimensions of the equivalent columns and connecting beams, it results a frame structure, having the stiffness along the two main directions equal to those in the real building.

The final efforts in the cross sectional direction in the wall in axis 1 and pier m_2 result by summing up the values in the same direction in the replacing pier A_1 and B_1 and C_1 , respectively A_2 and B_2 .

For example in wall 1 are valuable the following relations:

a) bending moment: $M = M_{A_1} + M_{B_1} + M_{C_1}$;

b) shear forces: $T = T_{A_1} + T_{B_1} + T_{C_1}$;

c) axial load is zero.

In the same way are determined the efforts in pier 2.

In the other piers and in the calculus for the longitudinal direction, efforts are equal to those in the replacing piers.

3. Conclusions

The calculus of the structural walls with average-sized openings through the equivalent frame method presents the benefit of being processed either with software for frames or with software for every independent frame (plane frame software) or with 3-D software. Analysis can be simplified if specific software is designed to define the geometrical characteristics of the equivalent frames.

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ASPECTE PRIVIND CALCULUL PEREȚILOR STRUCTURALI DIN ZIDĂRIE LA ÎNCĂRCĂRI ORIZONTALE FOLOSIND METODA CADRELOR SUBSTITUENTE

(Rezumat)

Calculul pereților structurali cu goluri mijlocii se poate face prin metode simplificate (metoda cadrului înlocuitor, metoda structurii continue echivalente) sau prin metode mai exacte (metoda elementului finit).

În lucrarea de față se propune calculul pereților structurali cu goluri mijlocii la încărcări orizontale prin metoda cadrului substituent modificat. Cadrul substituent al unui perete cu goluri mijlocii are numărul de niveluri egal cu cel al clădirii iar numărul de stâlpi egal cu numărul montanților peretelui, în timp ce riglele se consideră că au secțiunea constantă.

Caracteristicile geometrice ale cadrului substituent modificat (deschideri, înălțimea riglelor, dimensiunile secțiunilor stâlpișorilor și riglelor) se stabilesc din condiția egalității între rigiditatea peretelui și a cadrului la încărcări orizontale. Eforturile secționale din elementele cadrului substituent modificat sunt egale cu cele din elementele peretelui.

Avantajul metodei propuse constă în faptul că pentru calculul static al cadrului substituent modificat se utilizează programe de calcul existente pentru calculul cadrelor ortogonale.

În cazul programelor de cadru plan (2-D), unde fiecare perete se consideră separat, din considerente de simplificare a calculelor stâlpilor pot fi aleși cu secțiunea pătrată, iar în cazul programelor spațiale (3-D) secțiunile stâlpilor vor fi dreptunghiulare, stabilite din condiția rigidităților egale pe ambele direcții ale clădirii.