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THE OPERATIONAL FIABILITY IN THERMAL SYSTEMS THE WEIBULL DISTRIBUTION MODEL

BY

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Abstract. This paper investigates the reliability, durability, and maintainability for heating systems. More precisely, we are talking about sure functioning and security for heating systems. The heating systems are very different with specific problems and with many equipments. The warranty of reliability raises the durability of equipments. The information about reliability require observation and notification to the process. The results about reliability research are *post-factum*, but can have an important part in determining the real reliability for a product or a process.

Key words: reliability; durability; security; heating systems.

1. Introduction

The systems for the production and distribution of thermal energy that are greatly frequent and also very different, equally present specific particularities which set them apart from the majority of the production systems and services, by using many pre-fabricated components for which third party

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suppliers guarantee a certain fiability (Cirstoloveanu, 2009). By the introduction of the fiability's warranty, the probability of the malfunction due to the failure of the prefabricated elements has significantly diminished. Thus, in the case of the heating installations (Cirstoloveanu, 2009) based on independent systems which are small or middle-sized, and for which the heated water generation centrals, produced by highly specialized companies, present fiability warranty, the number of malfunctions has considerably diminished, thus increasing the average of the good functioning operational time without failing. Although the method of the in-use observation of the products involves difficulties in the precise gathering of the data, it still is a useful way, actually representing real-life trials of the products meant to identify aspects of their fiability with an increased efficacy. For the success of such trials (Mocanu & Ungureanu, 1982; Ungureanu, 2000), special programs are needed which must be set up and achieved in such a way that the selected data during the functioning of the products should be pertinent, should present a high degree of confidence in order for the results to be generalized. In the situation in which the gathering of the data proves to be too costly, another possibility is that of the trials on statistic samples of products, on special workbenches and/or by ensuring special functioning conditions in relationship to the nature of the process.

Thus, one could achieve (Ungureanu, 2000): increases of the functioning speed; these are also called *accelerated trials*. Some processes are accelerated such as the ones which involve wear and tear, tiredness, corrosion, etc. If we refer to the systems of production and distribution of the thermal energy, in experiments, the acceleration can be achieved by the functioning of prototypes throughout the year, thus in every season.

Experiments on components, sub-assemblies of the system or of the installation which they prove to be more economic.

2. Weibull Distribution Model

It is a fiability model of products having components which in time suffer physical wear and tear. If running-in period is admitted in the normal exploitation period due to the wear and tear, the surviving curve needs to indicate a deviation from the stationary mode described by the model of the exponenial distribution; thus, a deformation occurs in the sense of a slight decrease, still remaining close to the horizontal. The Wiebull model generalizes the model of the exponential distribution moving from one to two parameters (Ungureanu, 2000; Cirstoloveanu, 2009). The probability density of the Wiebull model with two parameters is

$$f(t,\beta,\lambda)=\beta\lambda t^{\beta-1}\mathrm{e}^{-\lambda t^{\beta}}, \tag{1}$$

$$\beta > 0,\ \lambda > 0,\ t \geq 0,\ (\beta \text{ is called } \textit{mode parameter}). \tag{2}$$

The $F(t)$ repartition function results:

$$F(t,\beta,\lambda) = \int_{-\infty}^{t} f(t,\beta,\lambda)\mathrm{d}t \tag{3}$$

where $F(t,\beta,\lambda) = 0,$ for $t < 0$ and $1 - \mathrm{e}^{-\lambda t^{\beta}}$, for $t \geq 0.$

The intensity (breaking down rate) is obtained with the same definition:

$$Z(t) = \frac{F'(t,\beta,\lambda)}{F(t,\beta,\lambda)} = \beta\lambda t^{\beta-1}. \tag{4}$$

The function of fiability results according to the relation

$$R(t) = 1 - F(t,\beta,\lambda) = \mathrm{e}^{-\lambda t^{\beta}}. \tag{5}$$

Given the probability of functioning without breaking down, the average of the time of good functioning or the life cycle of the product $T = T_0 =$ $=$ MTBF is obtained as follows:

$$\mathrm{MTBF} = T = \int_{0}^{\infty} \beta\lambda t^{\beta-1}\mathrm{e}^{-\lambda t^{\beta-1}}\mathrm{d}t = \frac{\Gamma\left(\dfrac{1}{\beta}+1\right)}{\lambda^{1/\beta}} \tag{6}$$

Γ is the Euler function known especially as the gamma function. The dispersion of the time of good functioning, is

$$D(t) = \sigma^2 = \frac{\Gamma\left(\dfrac{2}{\beta}+1\right) - \Gamma^2\left(\dfrac{1}{\beta}+1\right)}{\lambda^{2/\beta}}. \tag{7}$$

In Figs. 1 and 2 the evolution of the Wiebull density probability is presented depending on the β parameter, the variation of the breaking down rate with the same parameter, respectively (Ungureanu, 2000; Cirstoloveanu, 2009).

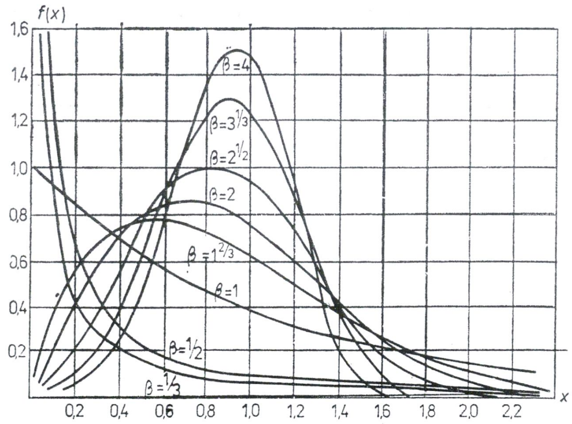

Fig. 1 – The evolution of the Wiebull density.

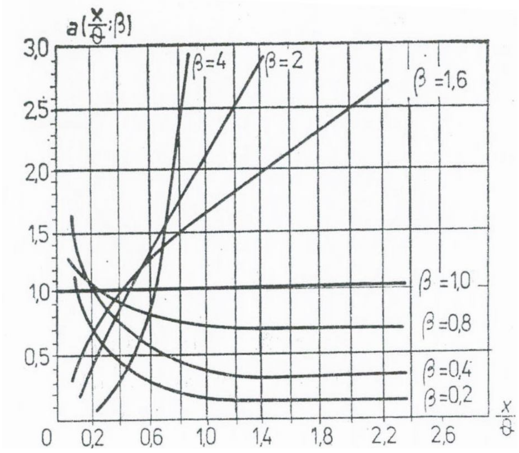

Fig. 2 – The evolution of the Wiebull density.

3.The Regularized Form of the Weibull Model

Due to some advantages in results application and interpretation, the regularized form is more advantageous. It is noted with

$$\lambda = \frac{1}{\theta^{\beta}}, \tag{8}$$

where θ can be expressed as

$$\theta = \frac{1}{\lambda^{1/\beta}}, \tag{9}$$

representing the real scale parameter.

The function of the distribution density becomes

$$f\left(\frac{t}{\theta},\theta\right)=\frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}\mathrm{e}^{-\left(\frac{t}{\theta}\right)^{\beta}}; \tag{10}$$

using the distribution density the repartition function becomes

$$\begin{gathered}F\left(\frac{t}{\theta},\beta\right)=\int_{-\infty}^{T} f\left(\frac{t}{\theta},\beta\right)\mathrm{d}t\\ F\left(\frac{t}{\theta},\beta\right)=0,\ \text{for } t\le 0,\\ F\left(\frac{t}{\theta},\beta\right)=1-\mathrm{e}^{-\left(\frac{t}{\theta}\right)^{\beta}},\ \text{for } t>0.\end{gathered} \tag{11}$$

To continue, the fiability function is

$$R\left(\frac{t}{\theta},\beta\right)=\mathrm{e}^{-\left(\frac{t}{\theta}\right)^{\beta}}, \tag{12}$$

and the breaking down rate is

$$\lambda(t)=\frac{\beta}{\theta}\left(\frac{t}{\theta}\right). \tag{13}$$

The average functioning time (MTBF) and the dispersion are determined as follows:

$$\mathrm{MTBF}=T_m=\frac{\Gamma\left(\frac{1}{\beta}+1\right)}{\lambda^{1/\beta}}=\theta\Gamma\left(\frac{1}{\beta}+1\right), \tag{14}$$

$$D(t)=\sigma^2=\theta^2\left[\Gamma\left(1+\frac{2}{\beta}\right)-\Gamma^2\left(1+\frac{1}{\beta}\right)\right]. \tag{15}$$

For $\beta > 1$ the breaking down rate is monotonous increasing from zero, and for $\beta < 1$ the breaking down rate decreases. The law of the exponential distribution is a special case of the Weibull law (Fig. 3).

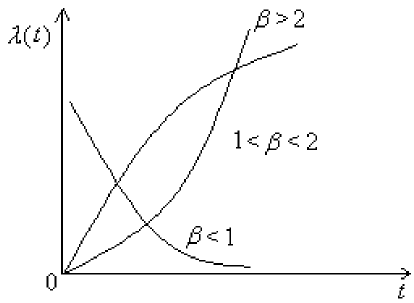

Fig. 3 – The synthesis of the variation of the breaking down rate for β representative fields.

4. Conclusions

Research has demonstrated that the mechanic, hydraulic, electric, electronic elements, constitutive of the thermal energy production and distribution systems in the process of normal functioning, accidental malfunctions are consequences of the exponential distribution law. Without generalizing this distribution law, it is more frequently met and very often leads to results which are close to reality. It can be noticed that for the same phenomena different laws of distribution can be used, without many results being essentially different. The Weibull distribution law having an *extra* parameter than the exponential one approximates better the experimental data. It should be mentioned that a three-parameter Weibull distribution was developed which presents a reduced interest in the case of the studied systems.

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FIABILITATEA OPERAȚIONALĂ ÎN INSTALAȚII TERMICE
APLICAREA MODELULUI DE DISTRIBUȚIE WEIBULL

(Rezumat)

Prin determinarea fiabilității înțelegem determinarea timpilor de bună funcționare, a durabilității, definite în acelaş context, a timpilor de reparații (cădere) şi mai cuprinzător a mentenabilității. Apare astfel necesitatea determnării fiabilității operaționale pe care o considerăm oportună şi de eficacitate şi a determinării fiabilității experimentale în mod selective. Modelul de distribuție Weibull este un model de fiabilitate a produselor având component care suferă în timp uzură fizică. Dacă se admite rolajul în perioada de exploatare normal datorită uzurii, curba de supraviețuire trebuie să indice o abatere de la regimul staționar descries de modelul distribuției exponențiale (model de distribuție operațională); se produce o deformare în sensul unei descreşteri uşoare, rămânând totuşi aproape de orizontală. Modelul Wiebull generalizează modelul distribuției exponențiale trecând de la un parametru la doi parametric.