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PASSIVE TUNED MASS DAMPER FOR SEISMIC PROTECTION

BY

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Abstract. One of the approaches to reduce excessive oscillation on buildings due to dynamic forces is represented by installing a passive mechanical device called tuned mass damper (TMD). This paper presents a study on the effectiveness of TMD in reducing the response of structures that are subjected to seismic excitation. The earthquake accelerograms of El Centro'40 and Kobe'95 are considered, and a two-dimensional linear-elastic model with TMD on the top is used in performing dynamic analysis.

Key words: passive tuned mass damper (TMD); passive control; vibration control; earthquake.

1. Introduction

When a structure is subjected to an earthquake, or to the action of strong winds, a certain amount of energy is introduced into the structure. For moderate to strong earthquake or wind excitations, a substantial amount of the total energy introduced in the structure is dissipated as hysteretic energy. Physically, it represents energy dissipated through nonlinear behaviour of the structural members after yielding, and the more hysteretic energy dissipated, the higher

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the structural damage. The displacements and the accelerations can become very large especially if the dynamic load frequency matches the structure's natural frequency, resulting in resonance.

Passive energy dissipative devices (PED) try to concentrate the dissipation of hysteretic energy in a few chosen and well-designed elements, and the remaining portion can be absorbed by the structure in elastic or near elastic behaviour. The tuned mass damper (TMD) is such an energy absorber consisting of a mass, a spring, and a viscous damper. Although active vibration control nowadays has received significant attention from many researchers, a passive control technique is still considered. One of the reasons for the acceptance of such devices is that they are very reliable since external power sources are not required for their operation. The motion of its mass is activated when the natural frequency of the TMD is tuned to be in or near resonance with the predominant frequency of the main structure. This tuning results in excellent reductions in displacement for loads applied at the resonant frequency but is less effective for loads at varying frequencies such as seismic loads.

This paper presents a study regarding the effectiveness of TMD in reducing the response of structures that are subjected to seismic excitation. Frequency domain responses and time domain responses are computed using a Matlab program that relies on the state space formalism which is widely used in control system theory. The effectiveness of the TMD is evaluated by comparing the response: displacement and acceleration, with and without TMD.

2. Analytical Formulation of a Tuned Mass Damper

The concept of the Tuned Mass Damper dates back to the 1940's (Hartog, 1947). It consists of a secondary mass with properly tuned spring and damping elements (s. Fig. 1), providing a frequency-dependent device that decreases response in the primary structure. In this study we will consider the motion of the structure in only one dimension. We will, for example, not consider the motion of the building in the vertical direction, and furthermore will assume the torsional effects on the building to be negligible.

According to d'Alembert principle, the differential eqs. of motion for each mass from Fig. 1 take the form:

$$\begin{cases} m\ddot{x}(t) + c\dot{x}(t) + kx(t) - c_d\dot{x}_d(t) - k_d x_d(t) = -m\ddot{x}_g(t), \\ m_d\ddot{x}_d(t) + c_d\dot{x}_d(t) + k_d x_d(t) + m_d\ddot{x}(t) = -m_d\ddot{x}_g(t). \end{cases} \tag{1}$$

It's convenient to represent the vibrating structural system response by a state space model in order to apply passive, semi-active or active techniques. In our case, the eqs. (1) of motion in which it's substituted $z_1 = x$,

$z_2 = \dot{x}$, $z_3 = x_d$, $z_4 = \dot{x}_d$, can be written in the following state space description as:

$$\begin{Bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \\ \dot{z}_4 \end{Bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -\dfrac{k}{m} & -\dfrac{c}{m} & \dfrac{k_d}{m} & \dfrac{c_d}{m} \\ 0 & 0 & 0 & 1 \\ \dfrac{k}{m} & \dfrac{c}{m} & -\dfrac{k_d}{m_d}-\dfrac{k_d}{m} & -\dfrac{c_d}{m_d}-\dfrac{c_d}{m} \end{bmatrix} \begin{Bmatrix} z_1 \\ z_2 \\ z_3 \\ z_4 \end{Bmatrix} + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \ddot{x}_g . \quad (2)$$

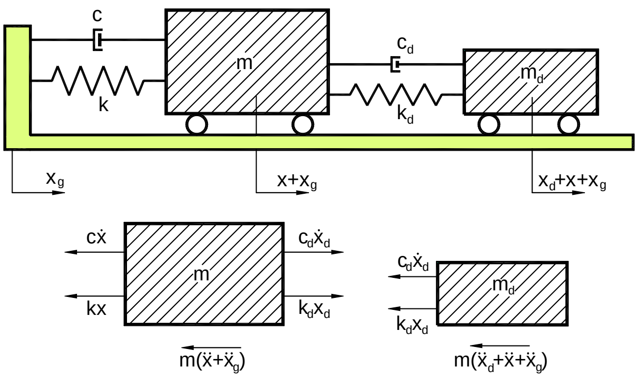

Fig. 1 – Mathematical model of TMD.

Hence, the description can be modified to

$$\begin{cases} \dot{z}(t) = Az(t) + B\ddot{x}_g(t) \\ y(t) = Cz(t) + D\ddot{x}_g(t) \end{cases} \quad (3)$$

where: $z(t)$ is the vector of the state variables, $y(t)$ – the vector of the measurable variables, $\ddot{x}_g(t)$ – the ground acceleration, A – the system matrix, B – the input matrix, C – the output matrix, D –the connection matrix between control input, $\ddot{x}_g(t)$, and output, $y(t)$.

The first eq. of the system (3) is in fact a system of four ordinary differential eqs. with following solution:

$$z(t) = \Phi(t,0)z(0) + \int_0^t \Phi(t,\tau)B\ddot{x}_g(t)\mathrm{d}\tau , \quad (4)$$

where $\Phi(t,\tau) = e^{A(t-\tau)}$ and initial condition $z(0) = z_0$.

Besides a time domain approach, which is a good way for evaluating the response, a transfer function analysis can provide further insight into the response of structure as the parameters of TMD are varied. The transfer function $H(s)$ is given by

$$H(s) = C(sI - A)^{-1}B + D, \tag{5}$$

where: I is identity matrix.

3. Response of the SDOF System Using Tuned Mass Damper (TMD)

We will consider three SDOF models which have the critical damping ratio ξ = 2% and the following natural periods: T = 1.2 s, T = 0.8 s, T = 0.5 s. In the first case, the ratio, μ, between the mass of the TMD and the mass of the system it is chosen to be 1/100, and the damping ratio of TMD, ξ_d, is varied. The structure stiffness, k, structure damping, c, TMD stiffness, k_d, and TMD damping, c_d, can be calculated as following:

$$k = (2\pi f)^2 m;$$
$$c = 2\xi(2\pi f)m = 4\pi\xi fm;$$
$$k_d = (2\pi f_d)^2 m_d = (2\pi\rho f)\mu m;$$
$$c_d = 2\xi_d(2\pi f_d)m_d = 4\pi\xi_d \rho f \mu m,$$

where: f is the structure nature frequency, and ρ – the ratio between frequency of the structure and the TMD, which is chosen typically 1 (Lee *et al.*, 2006; Sadek *et al.*, 1997).

The transfer functions of the ground acceleration to the displacement of the system as the damping ratio of TMD is varied are presented in Fig. 2. If the damping is increased beyond 8% of critical the response of the three systems will increase, and also if the damping decreases less than 4% the frequency interval of the effectiveness of TMD will be reduced. In particular, for large damping ratio the effect of TMD is minimal, the behaviour of the model is just the same as for the model without TMD.

In a second case, the same three SDOF are considered but this time the damping ratio of TMD ξ_d is maintained constant at 4% and the mass ratio, μ, is varied. Fig. 3 shows the frequency domain responses between the excitation and displacement of the system when the mass ratio μ is varied.

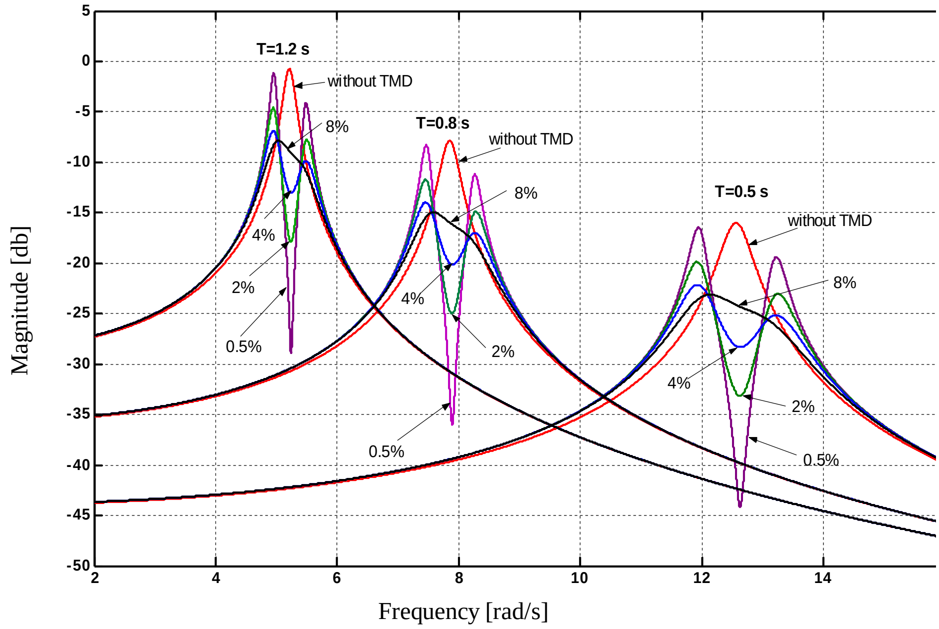

Fig. 2 – Frequency domain responses of structural model without and with TMD, μ=0.01 and the damping ratio of TMD, ξ_d, is varied.

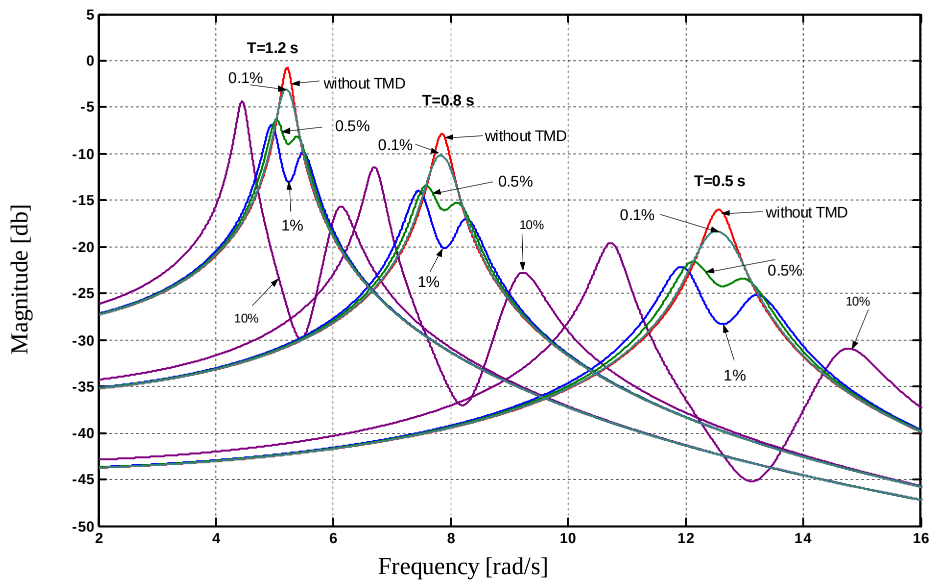

Fig. 3 – Frequency domain responses of structural model without and with TMD, ξ_d=*4%* and mass ratio, μ, is varied.

The study of the system response only under harmonic excitation can't provide enough information about its real behaviour. For this reason, it will be analysed the time history response of SDOF model with TMD under earthquake acceleration. Two earthquake signals, depicted in Fig. 4, are considered:

a) El Centro earthquake signal: North-South component recorded at Imperial Valley Irrigation District substation in El Centro, California, during the Imperial Valley, California earthquake of May 18, 1940. The magnitude is 7.1 and the maximum ground acceleration is 0.3495 m/s^2.

b) Hyogo-ken Nanbu (Kobe) earthquake signal: North-South component recorded at Kobe Japanese Meteorological Agency (JMA) station during the Hyogo-ken Nanbu (Kobe) earthquake of Jan. 17, 1995. The magnitude is 7.2 and the maximum ground acceleration is 0.8337 m/s^2.

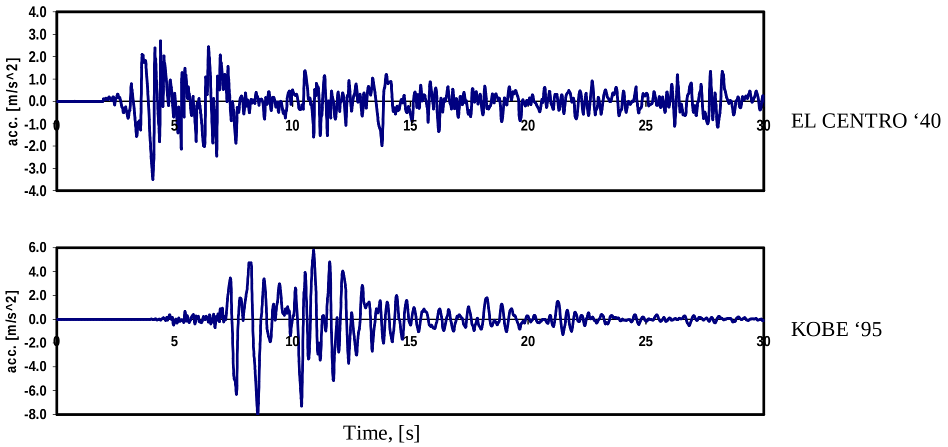

Fig. 4 – Earthquake signals.

The response of the structure is highly dependent on the frequency content of earthquake accelerations. Fig. 5 shows the Fourier spectrum for the two signals, scaled at the same peak of maximum ground acceleration, 3 m/s^2. The spectrum was elaborated using a Matlab function (FFT*)* that returns the discrete Fourier transform, computed with a fast Fourier transform (FFT) algorithm (Cooley & Tukey, 1965).

Fig. 6 depicts the comparison of the time domain responses with uncontrolled structure and structure controlled with TMD, and Fig. 7 presents the maximum response of the structural models with periods from 0.1 s up to 1.8 s, for different mass ratios.

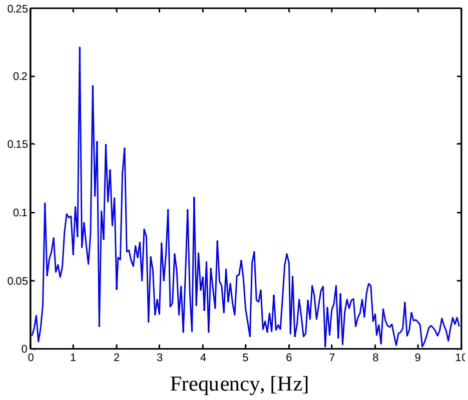

Fig. 5 *a* – Fourier spectrum
of the El Centro'40 ground acceleration.

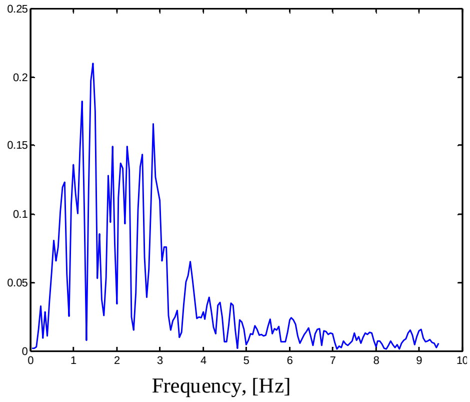

Fig. 5 *b* – Fourier spectrum
of the Kobe'95 ground acceleration.

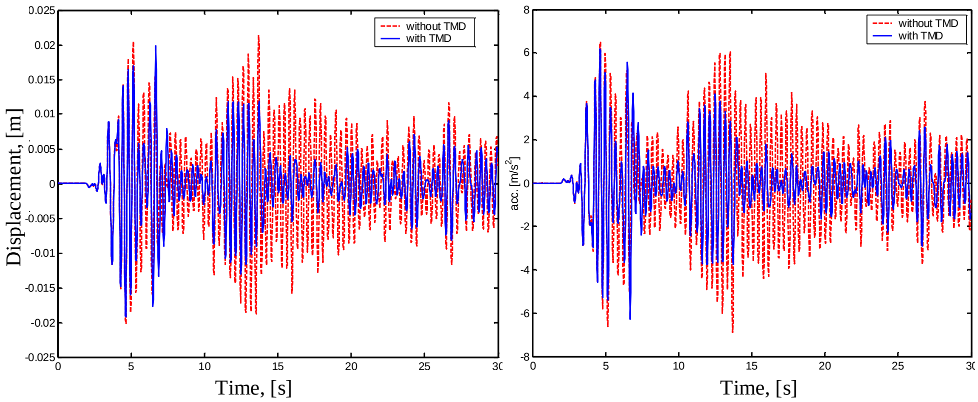

Fig. 6 *a* – Numerical simulations of the structural model
under El Centro earthquake acceleration, (T = 0.35 s, μ = 1%).

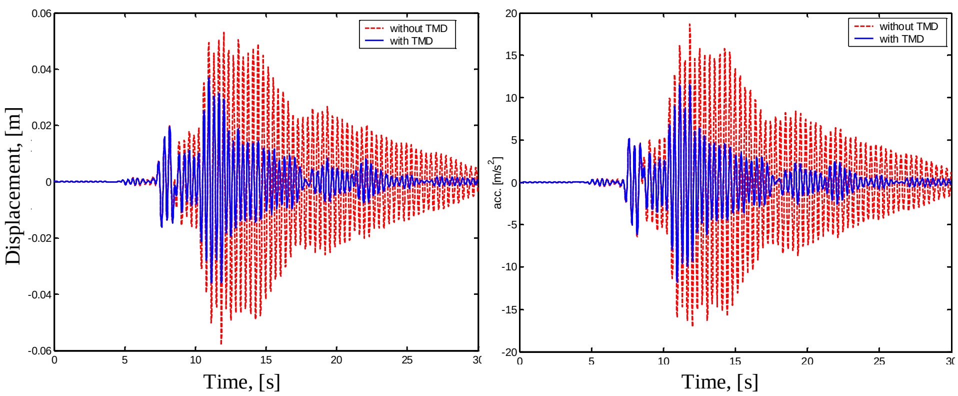

Fig. 6 *b* – Numerical simulations of the structural model
under Kobe earthquake acceleration, (T = 0.35 s, μ = 1%).

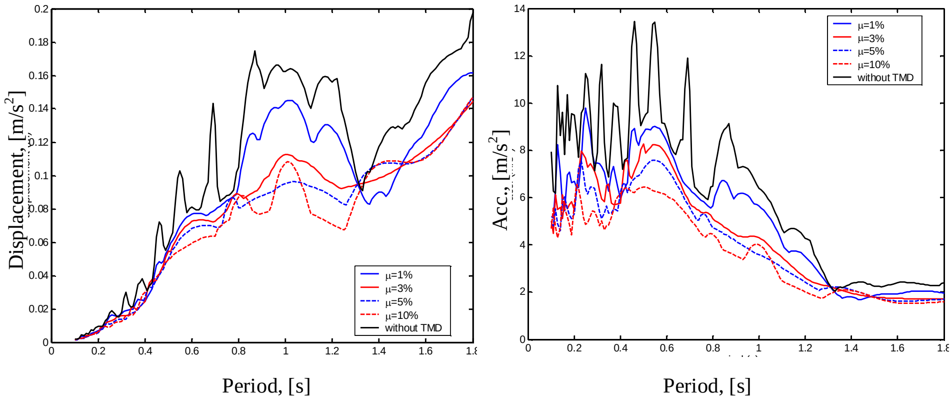

Fig. 7 *a* – The maximum structural response
under El Centro earthquake acceleration for different mass ratio, μ.

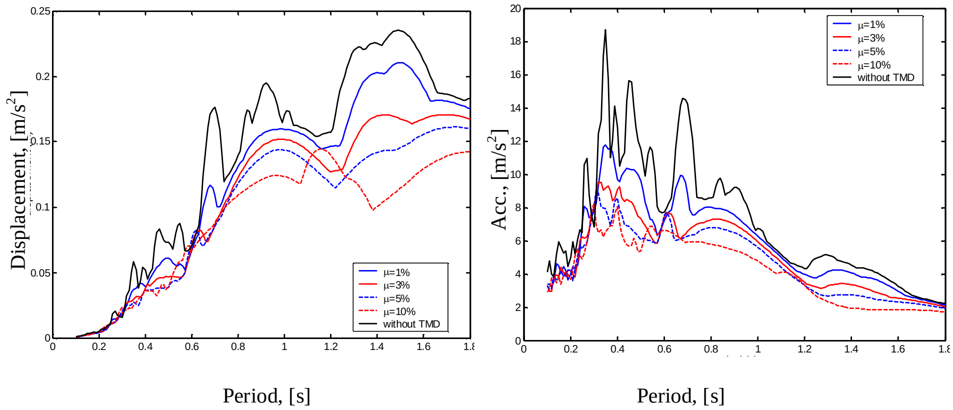

Fig. 7 *b* – The maximum structural response
under Kobe earthquake acceleration for different mass ratio, μ

4. Conclusions

Tuned mass damper (TMD) systems have been incorporated into many structures and dynamic systems throughout the world to effectively reduce undesirable oscillations. In this study, the effectiveness of the TMD using the proposed tuned parameters has been investigated through numerical example. A properly tuning leads to excellent reductions in displacement and acceleration for loads applied at the resonant frequency but is less effective for loads of different frequencies (see Fig. 6). The results of the study show that the responses are generally decreased. However, in some cases as we can see in

Fig. 7 the responses could be equal or even larger, thus using of TMD for reducing seismic response should be reconsidered.

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DISIPATOR CU MASA ACORDATĂ PENTRU PROTECȚIA SEISMICĂ

(Rezumat)

O metodă folosită pentru reducerea oscilațiilor excesive a clădirilor solicitate la acțiuni dinamice, o reprezintă instalarea unui dispozitiv mecanic pasiv, numit dispator cu masă acordată (TMD). Se prezintă eficacitatea unui TMD în reducerea răspunsului structurilor supuse la acțiunea seismică. Pentru analiza dinanică s-a considerat un model bidimensional liniar-elastic cu TMD la partea superioară, supus accelerogramelor cutremurelor El Centro'40 şi Kobe'95.