BULETINUL INSTITUTULUI POLITEHNIC DIN IAȘI
Publicat de
Universitatea Tehnică „Gheorghe Asachi” din Iași
Tomul LIX (LXIII), Fasc. 6, 2013
Secția
CONSTRUCȚII. ARHITECTURĂ

MODELLING METHODS FOR UNREINFORCED MASONRY STRUCTURES

BY

LUCIAN SOVEJA*, MIHAI BUDESCU and IONEL GOSAV

“Gheorghe Asachi” Technical University of Iași
Faculty of Civil Engineering and Building Services

Received: November 5, 2013
Accepted for publication: November 26, 2013

Abstract. Studies concerned with evaluation and rehabilitation of historical masonry structures consider structural analysis in order to better understand the seismic behaviour of these buildings, to define the causes for present damage and to asses the safety level for a large variety of actions. Structural analysis such as linear, plastic or nonlinear analysis contributes to all the stages and activites related to the evaluation and rehabilitation process. This paper follows a description of the modelling methods of masonry structures taking into consideration different modelling strategies in correlation with the complexity of the analysed elements and the expected type of results. Advantages and disavantages of different modelling techniques and the applicability of these methods in the field of old masonry structures assessment are presented.

Key words: masonry structures; modelling methods; strategy.

1. Introduction

Depending on the complexity of the problem and the results expected from the analysis it is necessary to establish a type of idealization for the

*Corresponding author: *e-mail*: LucianSoveja@yahoo.com

behaviour of masonry based on the force-displacement curves. Thus, in this field of study, elastic, plastic and nonlinear analysis have been developed. The decision considering one of these methods of analysis is based on the performance level chosen in the assessment process. If the analyst evaluates the structure for the performance level of damage limitation associated to serviceability limit state, the material and geometric nonlinearities can be neglected and the analysis regards to the elastic behaviour. However, if the structure is analysed under severe loading conditions that can lead to reaching the ultimate limit state or the precolaps limit state, nonlinear analysis or plastic analysis are required.

Linear analyses are not appropriate due to the complexity of historical masonry structures, which, besides the anisotropic character behaves different in tension and compression. Moreover, due to almost zero strength of masonry in tension, linear analysis can be considered inaccurate even at low levels of loads. Masonry structures, such as arches and vaults, by the failure or dislocation process, creates various subsystems that are no longer subject to initial conditions therefore can not be simulated. However, the linear model is effective in identifying the global tendency of building behaviour, modal characteristics and areas in which the structure is subjected to stress concentrations able to interrupt the continuity of the masonry. The input data required relates to masonry weight and modulus of elasticity. Despite all the restrictions, in recent decades, linear analyses were used to simulate the structural behavior of a large number of masonry buildings with high cultural value (Pappas, 2012).

Plastic analysis or limit analysis aims to assess the maximum load that a structure can sustaine (load limit) and consider three hypotheses: tensile strength of masonry is null, the compressive strength of masonry is infinite and slipping between masonry blocks is impossible. It is based on two theories on which two analysis methods have been developed: graphical equilibrium methods (considering static theory) and collapse mechanisms method (considering kinematic theory).

Recommended as the most appropriate method of historical masonry structures analysis, nonlinear analysis can capture the full response of the structure throught the elastic stage, cracking and dislocation, to complete collapse. This type of analysis requires the definition of the elastic and inelastic mechanical properties of masonry. Different types of nonlinearities can be considered in nonlinear computation: mechanical (related to material nonlinearity), geometrical (due to the change of the load application point) and contact (characterizing the interaction between bodies). Nonlinear analysis of masonry structures requires proper handling of the major difficulties regarding the constitutive laws for masonry nonlinear behaviour to enable an efficient and stable implementation in the computer program.

2. Masonry Modelling Strategy

Masonry is a composite material with different directional properties directly dependent on the geometry and mechanical properties of its constituents (brick or stone and mortar). Due to the low resistance of the mortar both in tension and compression, joints of masonry are forming horizontal and vertical failure surfaces.

In the literature, research into modelling of masonry elements was based on consideration of masonry structure as heterogeneous with separate descriptions of each constituent material. Later, in structural analysis, modeling has had a phenomenologically character, representing throught constitutive eqs. at macroscopic level, collective behavior of masonry components.

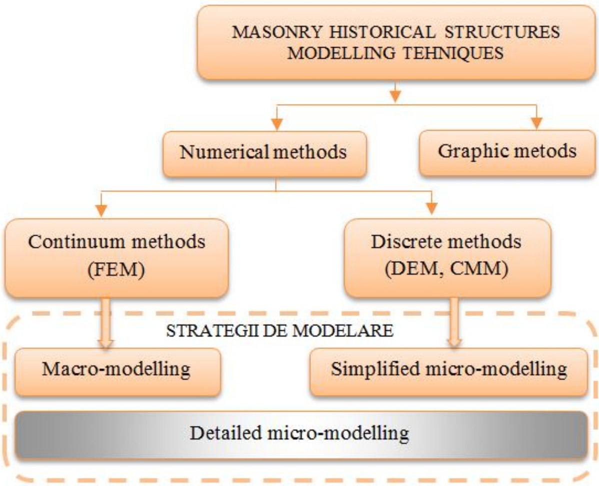


Fig. 1 – The strategy in modelling masonry.

Depending on the complexity level corresponding to the structural analysis, three main strategies have been developed for masonry modelling:

a) *detailed micro-modelling*, in which, starting from the description of each individual material, units and mortar are modeled as continuous materials bonded with contact discontinuous interfaces representing possible plans of failure;

b) *simplified micro-modelling* by geometric expansion of the units separated by discontinuous elements that simulate the behavior of the mortar joints and unit-mortar interface;

c) *macro-modelling* or homogeneous material model removes the difficulty of describing the units and the mortar joints separately, considering the masonry as homogeneous and anisotropic continuum based on the homogeneization concept.

The decision of choosing a suitable modelling strategy depends on the expected calculation accuracy and on the size of the model (Fig. 1). Micro-modelling provides a more realistic representation of the structural behaviour of masonry, but it prezents a prohibitive character due to the large number of degrees of freedom used, the increased volume of input data and the complexity in defining the failure criterion for masonry. Thus, this method has proved suitable for studying the local behaviour of masonry structures, with a low level of complexity, especially for modelling masonry elements tested experimentally, being able to capture all possible failure modes of masonry.

Between micro- and macro-modelling, the homogenization concept plays an important role in the analysis of masonry, referring to a unique continuous medium and aiming to determine the mechanical parameters of a fictitious homogeneous material which is able to simulate the real heterogeneous material. The applicability of the concept lies both in elastic and plastic limits of the material behavior. This procedure was implemented by deriving the global behavior from the behaviour of individual masonry units and mortar in multiple steps (successive vertical and horizontal joints) or in a single step with the real geometry of masonry (Costa, 2014).

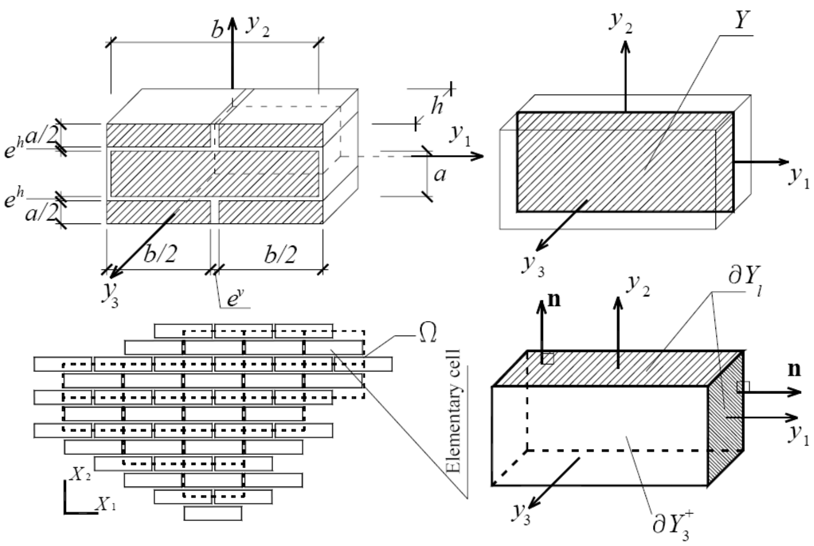


Fig. 2 – The repeating volume element in the homogeneization process: macroscale „X” and microscale „y”.

The homogeneization process may consist in choosing a representative and repeating volume element (RVE) of the masonry structure at the microscopic level considering the effects on the macroscopic behaviour (Fig. 2).

The mechanical properties of the composite are defined in terms of stress tensor (Σ) and strain tensor (E):

$$E = < \varepsilon > = \frac{1}{V} \int \varepsilon(u) \mathrm{d}Y, \quad (1)$$

$$E = < \sigma > = \frac{1}{V} \int_Y \sigma \mathrm{d}Y, \quad (2)$$

where: V is the volume of a representative cell (Fig. 4); σ – local stress tensor; ε – local strain tensor; <…> – averaging operator.

The periodicity conditions are imposed to the stress and strain field (σ and u) with the relations:

$$u = E_y + u^{\mathrm{per}}, \; u^{\mathrm{per}} \text{ on } \partial Y, \quad (3)$$

$$\sigma_n \text{ antiperiodic on } \partial Y, \quad (4)$$

where: u is the total displacement field, y – the relative position between two points on the representative volume element borders, u^{per} – the field of periodic strain, σ_n – the microstress vector.

In this framework, bricks and mortar are assumed rigid-perfectly plastic materials with associated flow rule. As the lower bound theorem of limit analysis states and under the hypotheses of homogeneization, S^{hom} can be derived by means of the following (nonlinear) optimization problem:

$$S^{\mathrm{hom}} = \left\{ \sum \left\{ \begin{array}{l} E = < \sigma > = \frac{1}{A} \int_Y \sigma \mathrm{d}Y \quad \text{(a)} \\ \mathrm{div}\sigma = 0 \quad \text{(b)} \\ [[\sigma]] n^{\mathrm{int}} = 0 \quad \text{(c)} \\ \sigma_n \text{ anti-periodic on } \partial Y \quad \text{(d)} \\ \sigma_{(y)} \in S^m, \forall y \in Y^m, \sigma_{(y)} \in S^b, \forall y \in Y^b \quad \text{(e)} \end{array} \right\} \right\}. \quad (5)$$

Here, $[[\sigma]]$ is the jump of micro-stresses across any discontinuity surface of normal nint. Conditions (a), (d) are derived from periodicity, condition (b) imposes the micro-equilibrium and condition (e) represents the yield criteria for the components (brick and mortar).

Constitutive structural macro-models (Fig. 3) are relatively simple to use, require less input data and the failure criterion for masonry is defined, in general, by a simplified law. Constitutive eqs. of the material, in this case, are suitable for studying the behavior of the entire masonry structures because it reduces computation time and performance. The difficulties in macro-modelling relates to the formulation of quasi-brittle materials behaviour laws considering, in general, different failure criteria in tension and compression (Roca, 2013).

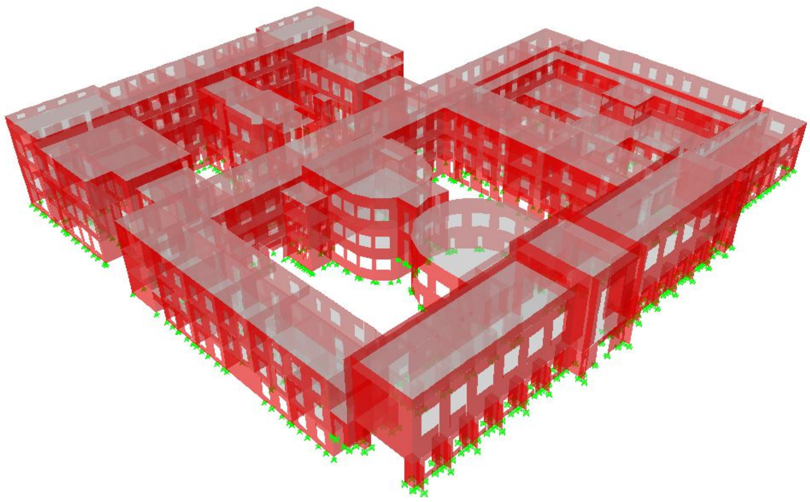


Fig. 3 – Structural analysis of "Al.I. Cuza" University, based on macro-modelling strategy (Gosav & Soveja, 2013).

3. Modelling Methods for Historical Masonry Structures

A wide variety of modelling methods have been introduced in the analysis of historical masonry structures with the ability to simulate the behaviour of masonry in the early stages of loading, but limiting in current research area is the analysis in the proximity of general collapse. Numerical methods can be separated into two categories, developed on the homogeneous material eqs. (continuum methods) and considering the discontinuities given by the mortar joints as areas of crack propagation (discrete methods).

Continuum methods, which include the finite element method and boundary element method, are based on a continuous medium approximation of masonry in order to reduce the resource requirements and computation time. If the masonry nature is a periodic type with similar units and mortar joints uniform distributed, the homogeneous continuum model leads to efficient and realistic solutions. In this method there were also used, with reliable results, macro-models based on the theory of plasticity and the theory of fracturing (Edmans *et al*., 2013).

The study of masonry structures through discrete methods consider the heterogeneous character of masonry and possible failure plans at the interface of the units using discotinuous elements. Applications of these methods are preferred especially in analysis of collapse mechanisms.

3.1. Finite Element Method

In general, unreinforced masonry structures are characterized by complexe structural system because of the architectural forms which correspond to the structure and the massive walls bonded with arches and vaults. The structural overstrength evaluation, especial to seismic action, is made by the identification of the areas with stress concentration. Thus a realistic modelling of the structural system is required.

The finite element method (FEM) is a technique based on numerical analysis in order to obtain approximate solutions further used to determine the variation of parameters characterizing continuous media (field displacements, strains, stresses).

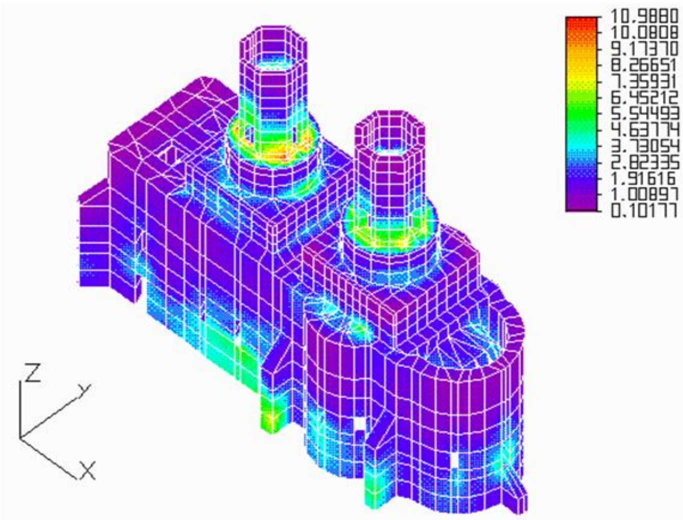
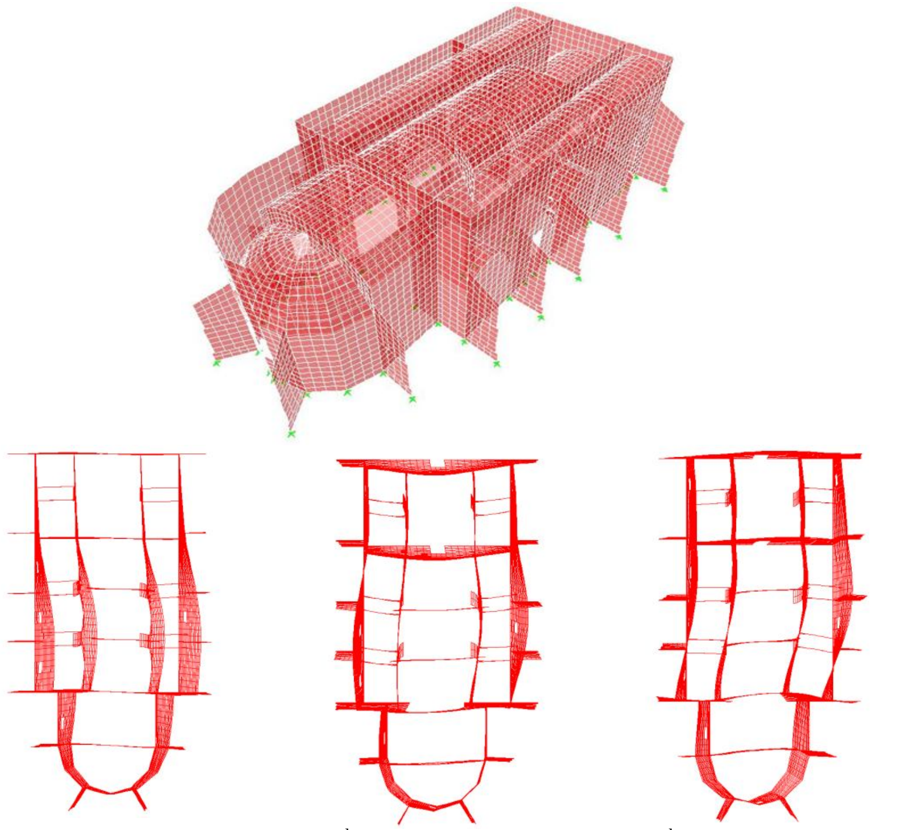


Fig. 4 – Structural analysis of "Trei Ierarhi" Church (Iaşi) using solid elements-Tresca stress map (Budescu *et al*., 1994).

The basic idea of the FEM is based on the possibility of describing the real strain field through their values in a finite number of points. Usually finite elements are defined in the process of meshing, which occur as the result of decomposition of a domain into several compatible subdomains with disjunct interior. The method was widely used in the evaluation of historic masonry buildings in linear and nonlinear analysis, offering good results in describing the structural response of the analysed buildings (Varum *et al*., 2014).

Considering mechanical analysis, masonry historical buildings have structures characterized by: high degree of static indetermination, complex geometry given by the overlapping elements, high variations of transversal cross section, rigidity or masses.

Geometric representation of the structure can be made using two-dimensional elements (shell elements) (Fig. 5) or three-dimensional elements (solid elements) (Fig. 4). No element is superior to the other, the decision of choosing one depending entirely to the complexity of the problem.



1st Mode , T = 0.134 s 2nd Mode, T = 0.096 s 3rd Mode, T = 0.093 s

Fig.5 – Structural analysis of Bogdana Church (Rădăuți) using shell elements (Soveja & Gosav, 2013).

Because the two-dimensional elements are defined by a small number of nodal connections, their use in modeling masonry structures leads to efficient and practical analysis. In contrast, solid-type elements allow the control of the stress evolution within the structural elements, a necessary information in analysing masonry structures with thick walls.

In order to assess historical masonry buildings, linear and nonlinear dynamic analysis, based on FEM, offers important information and contributes to their classification in seismic risk classes and in the development of the intervention solution. In these analysis relevant influences are given by the construction age, the presence of closed old dislocations, the joint mortar filling degree, brick or stone material properties and dimensions, connections between the masonry blocks and properties of block–mortar interface. Because of the large number of variables considered in the dynamic analysis, finite element models need to be calibrated with the dynamic characteristics determined experimentally (Debranjan & Pradip, 2012).

3.2. Graphic Equilibrium Methods

Formulated by Karl Culmann in 1866, this method considers the static theorem of limit analysis and have been widely used in stability assessment of masonry structures subjected mainly to compression efforts. Static theorem of limit analysis assumes that the structure is safe, meaning that a state of collapse will not be reached, if a permitted static equilibrium condition can be found. An allowable static state of equilibrium is achieved when a thrustline can be determined, in equilibrium with external forces, within the borders of the structure. These are theoretical lines drawn within the element, in the direction of the compressive forces resultants. The equilibrium state is achieved when the resulting polygonal shape is within the transversal section of the element. Equilibrum graphic methods allows a graphical description and visualization of the structural equilibrium in a masonry element (Block *et al*., 2006).

3.3. Colapse Mechanism Method

This method, which lies in the field of plastic analysis or limit analysis, is based on a kinematic theorem: if a kinematically admissible mechanism can be found (by arbitrarily introducing a sufficient number of hinges) for which the work developed by external forces is positive or null, then the structure will reach collapse.

The assumptions underlying the calculation are the followings:

a) masonry has no tensile strength (although this assumption can be considered conservative, mortar joints are capabile of very small tensile forces);

b) masonry has infinite compression strength (in most cases collapse of masonry structures is due to yielding in tension, not in compression);

c) shear behavior of masonry joints is assumed perfectly plastic;

d) load limit is reached at small displacements.

Following these considerations, the only collapse mechanism is the rotation of adjacent blocks around a common point, masonry behaving like a set of perfectly rigid blocks, in equilibrium due to the compressive forces normal to the contact surface.

The collapse coefficient (λ) is defined as a gravity loads multiplier equalizing the maximum lateral force. Collapse mechanism methods are used in the structural analysis of masonry buildings at the ultimate limit state, considering the structure composed of rigid-solid bodies (macro-elements) with discontinuities only at their borders.

The division of the structure in macro-elements can be based on several considerations, including: geometrical characteristics, damage pattern observed *in situ* due to previous seismic activities or a finite element analysis which shows the cracks pattern on a three-dimensional model.

With the above mentioned principles, the load bearing capacity of masonry structures depends on macro-components which are portions of the structure connected by failure surfaces (existing or potential), behaving as a whole and following a kinematic mechanism. The collapse will occur not by exceeding the material strength but due to loss of structural stability (Borri *et al.*, 2010).

3.4. Discrete Element Method

Based on explicit numerical integration of differential eqs. of rigid blocks motion, the analysis of historic structures with discrete element method is characterized by modeling the material as a collection of individual elements interacting with each other through the contact surface with the possibility of progressive failure, crack propagation, large displacements and blocks rotations. This method can be applied to masonry structures composed of brick or stone with regular shapes and allows the simulation of the collapse mechanisms of masonry structures as a result of joint slipping or the rotation of discrete elements.

According to researchers, the method can be applied only if it allows finite displacements and rotations of the blocks including complete detachment of the elements and if it can estimate, during the calculation, the change of the contact between blocks. Automatic rounding of the contact points is possible to avoid the problem of interlocking blocks during their rotation, making the discrete element method a convenient method for the analysis of masonry structures.

In this method it is possible to simulate blocks as "deformable blocks" (using a network of finite elements inside the block) or "rigid blocks". The type of the contact surfaces can be "soft contact" (in which tensions are derived from the relative displacement between the blocks and small blocks overlays are allowed in compression, "rigid contact" or using springs supports on the surface of the blocks (Wolfram & Tammam, 2010).

3.5. Equivalent Frame Method

Among the possibilities of nonlinear analysis of masory structures, the equivalent frame method is found as a viable method in wich the in plane failure mechanism for masonry walls was defined idealizing two-dimensional elements of the walls (spandrels and piers) in one-dimensional elements interconnected by rigid nodes, simulating a response similar to that of an equivalent frame structure.

Masonry is considered a homogeneous and isotropic material and the local nonlinearity of each pier or spandrel is defined by elasto-plastic joints described experimental deduced force–displacement curves. The rigid nodes is a feature in the model derived from *in situ* damage observation of masonry structures subjected to past seismic actions.

In literature, different effective lengths for spandrel and piers are considered. Specified in seismic codes, the equivalent frame method require a limited number of degrees of freedom, therefore a reduced computational effort, leading to affordable incremental nonlinear analysis of complex masonry structures (Sabatino & Rizzano, 2011).

4. Conclusions

Elastic analyses are not appropriate due to the complexity of historical masonry structures, which besides the anisotropic character behaves different in tension and compression. However, the linear model is effective in identifying the global tendency of building behavior, modal characteristics and areas in which the structure is subjected to stress concentrations are able to interrupt the continuity of the masonry. Recommended as the most appropriate method of historical masonry structures analysis, nonlinear analysis can capture the full response of the structure throught the elastic stage, cracking and dislocation, to complete collapse. This type of analysis requires the definition of the elastic and inelastic mechanical properties of masonry.

Depending on the complexity level corresponding to the structural analysis, three main strategies have been developed for masonry modelling: detailed micro-modelling, simplified micro-modelling and macro-modelling. The decision of choosing a suitable modelling strategy depends on the expected calculation accuracy and on the size of the model. Between micro- and macro-modelling, the homogeneization concept plays an important role in the analysis of masonry, referring to a unique continuous medium and aiming to determine the mechanical parameters of a fictitious homogeneous material which is able to simulate the real heterogeneous material

A wide variety of modelling methods have been introduced in the analysis of historical masonry structures with the ability to simulate the behavior of masonry in the early stages of loading, but limiting in current research area is the analysis in the proximity of general collapse.

The theoretical model of a structure refers to the representation of its structural components taking into consideration the way they are assembled, their stiffness, inertia, damping and eventually the connection between the structure and the foundation soil. Using finite element method for meshing complex structures of historic masonry buildings, some characteristic features can easily result, features that can not be obtained with idealized models.

Using these methods individually or complementary in the analysis of historical masonry buildings, the complete structural behaviour response for seismic or gravitational actions can be obtained.

REFERENCES

Block P., DeJong M., Ochsendorf J., *As Hangs the Flexibile Line: Equilibrium of Masonry Arches*. Nexus Network J., **8**, *2* (2006).

Borri A., Cangi G., Caraboni M., Giancarlo A., Menghini F., Procaci L., *Sisma del 6 Aprilie 2009 in Abruzzo e BBCC: il casco della Chiesa di S. Silvestro a L'Aquilla*. Boll. Ing., *4* (2010).

Budescu M., Ciongradi I., Ciupală A.M., *Proposal of Intervention in Order to Rehabilitate the Resistance Structure of "Trei Ierarhi" Monastery"*. Bul. Inst. Politehnic, Iaşi, **XL (XLIV)**, *1-4*, s. Constr. Archit., 40-42 (1994).

Costa A.A., Quelhas B., Almeida J., *Numerical Modelling Approaches for Existing Masonry and RC Structures, Structural Rehabilitation of Old Buildings*. Build. Pathol. a. Rehab., **2**, 285-305 (2014).

Debranjan S., Pradip S., *Seismic Evaluation of Existing Unreinforced Masonry Buildings*. Proc. of the Internat. Symp. on Engng. under Uncertainty: Safety Asses. A. Manag., Kolkata, India, 2012, 1267-1276.

Edmans B.D, Alfano G., Nahai H., *Nonlinear Multi-Scale Homogenization with Different Structural Models at Different Scales.* Internat. J. for Numer. Meth. in Engng., **94**, 355-373 (2013).

Gosav I., Soveja L., *Expertiza tehnică la imobilul A al Universității Tehnice „Gh. Asachi" din Iaşi*, 2013.

Pappas A., *Calibration of the Numerical Material Behaviour of Multi-Leaf Stone Masonry Walls Based on Experimental Results.* M. Sc. Diss., Univ. of Padova, 2012.

Roca P., Cervera M., Pela L., Clemente R., Chiumentini M., *Continuum FE Models for the Analysis of Mallorca Cathedral*. Engng. Struct., **46**, 653-670 (2013).

Soveja L., Gosav I., *Lateral Stiffness Evaluation of Masonry Vaulted Churches.* C60 Internat. Conf., Cluj-Napoca, 2013.

Varum H., Tarque N., Silveira D., *Structural Behaviour and Retrofitting of Adobe Masonry Buildings*. Build. Pathol. a. Rehabil., **2**, 37-75 (2014).
Wolfram J., Tammam B., *Seismic Vulnerability of Historical Masonry Buildings for Different Earthquake Characteristics*. Case Study fo the Mosque of takiyya al-Sulaymaniyya, 2010.

METODE DE MODELARE A STRUCTURILOR DIN ZIDĂRIE NEARMATĂ

(Rezumat)

Studiile orientate către evaluarea şi consolidarea clădirilor istorice din zidărie au la bază analize structurale în scopul înțelegerii particularităților structurale de comportare, caracterizării cauzelor degradărilor existente şi determinării gradului de vulnerabilitate pentru o largă varietate de acțiuni. Analizele structurale contribuie la toate fazele şi activitățile orientate către conceperea lucrărilor de evaluare şi reabilitare eficiente a monumentelor istorice. Se urmăreşte o descriere a metodelor de modelare a structurilor din zidărie având în vedere diferite strategii de modelare în funcție de complexitatea elementelor analizate şi de rezultatele considerate. Sunt prezentate avanantajele şi dezavantajele diferitelor metode şi aplicabilitatea acestora în câmpul analizei structurilor istorice din zidărie.