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TOPOLOGICAL OPTIMIZATION OF BEAMS IN A STEEL FLOOR

BY

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Abstract. Optimization of floor girders is a complex nonlinear problem for which a simple computational procedure has been suggested. The Generalized Reduced Gradient (GRG). Nonlinear algorithm available in Excel Solver were utilized to optimize the design of plate girder for minimum weight and maximum allowable distance between secondary beams, given the span and grade of the material of the girder. Optimizing a girder for bending moment is achieved by moving material away from the neutral axis of the beam, in other words, by making the web more slender. When lateral supports are used to prevent from lateral torsional buckling, all the forms of the compressed flange instability will become the critical failure mechanism. Due to high slenderness values, the deflection of the beam is not a gouverning condition. In this paper, the maximum spacing between lateral supports against lateral-torsional buckling were maximized meanwhile respecting the provisions of the norm SR EN 1993-1-1. Results of the numerical computations considering various lengths of beams and girders are presented herein. Girders are considered to be efficiently designed with respect of the presented technique of optimization.

Key words: design optimization; plate girder design; EN 1993-1-1.

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1. Introduction

Plate girders are used either for long spans where the steel plates need splices for building up the sections, thus resulting an inefficiency in design, or in the case of carrying heavy loads. It is important to mention that, although plate girders may be lighter than other forms of compound beams, fabrication costs are likely to be much higher.

The design of a built-up steel girder is a tedious and time-consuming matter for the designing engineer. In common practice, steel plate girders are designed through a trail- and error approach due to the complex design rules.

Optimization is the process of reaching to "the best" solution to a problem based on the evaluation of a given criterion, or in other words, structural optimization is the process of obtaining the "optimum" solution to a structural problem. This optimization is attained through specific analysis and design theories suited in parallel with the research of the optimum method to solve the problem.

Azad, (1980), Vachajitpan & Rocky, (1978), propose design procedures based on elastic analysis to find an economical sizing of welded built-up I girders with symmetrical cross sections, based on the determination of the minimum weight, cross-sectional dimensions or cost optimization of I beams or girders for given values of elastic bending moments and shear forces.

In this paper, a comprehensive algorithm for optimum design of built-up wide-flange sections is presented. The design is performed in accordance with the European Standards Eurocodes specifications (EN 1990; EN 1993-1-1; EN 1993-1-5) in order to satisfy the requirements of both ultimate and serviceability limit states. The optimum solution for the different parts of the girders is closed-form so that the section sizing is direct and simple. The solution for a specific part will be rejected only when the section proportioning from the general part violates the imposed constraints on section dimensions.

In either case the solution is governed by singularly activating the bending moment, or the moment and shear together. An algorithm for the optimum design process is presented and the process is illustrated with the help of an example problem which is implemented on a spreadsheet program like Excel using Solving Toolbox.

2. Existing Design Methods for Base Plates

2.1. Design Objective

The dependent variables for the problem (Fig. 1) that can be evaluated once the design of the girder is specified are identified as it follows: the main objective is to maximize the distance between lateral supports of the girders (distance L_s) given by floor beams, while the weight of the girder is imposed by the design constraints.

To minimize the weight of the girder, the cross section was minimized *via* analytical methods of optimization, subject to the provisions in EN 1993-1-1 and EN 1993-1-5.

Then, the mathematical programming can be expressed as follows:

$$\text{Max } L_s$$
$$\text{subjected to: } \begin{cases} h(\mathbf{x}) = 0\,, \\ g(\mathbf{x}) \le 0 \end{cases} \quad (\text{NLP}), \tag{1}$$

where: **x** is a vector of continuous variables, defined within the compact set X and h(**x**) and g(**x**) – nonlinear constraints (NLP) derived from EC3.

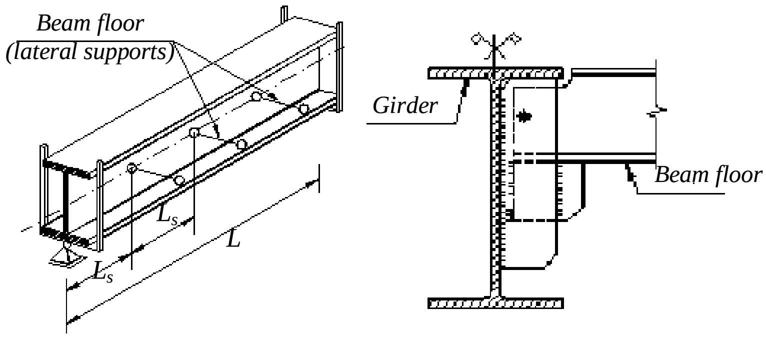

Fig. 1 – Girder configuration.

2.2. Constraints

In the context of structural optimization, these constraints fall into two categories:

1º Behavior constraints that limit the plastic/elastic moment, shear force and stability conditions:

a) sectional classification/ proportional limitation;

b) moment resistance (flanges only method);

c) shear capacity (strength);

d) lateral torsional stability between lateral supports;

e) serviceability.

2º Design constraints imposed on design variables to put practical limits on some dimensions of elements.

Further, to simplify the formulation, it is necessary to introduce dependent design variables. These variables give rise to additional constraints, called *equality constraints.*

2.3. Design Parameters

The fixed values arise from designed usage of the structure and code specifications, therefore could not be varied. In this case the fixed values are the girder span and the steel yield strength.

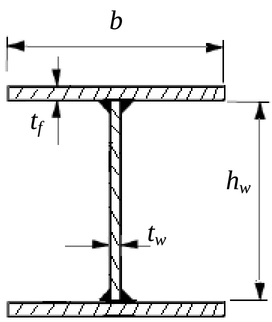

Fig. 2 – Variables parameters of the problems.

The design variables in this structural optimization problem were the dimensions which control the cross-sectional area. These are the parameters that determine the geometry of the optimized girder and the spacing between lateral supports:

1. Flange width and thickness (b and t_f).
2. Web height and thickness (h_w and t_w).
3. Spacing between lateral supports (L_s).

3. Design Constraints Derived from EC3

Girders of length L, depth h, simply supported and carrying a point load P_k at the supports are considered in his paper.

3.1. Section Class – Local Buckling Constraints

The bending moment, shear force, and point load bearing resistances the girders made up of thin plate elements, slender enough as to be significantly influenced by local buckling failure modes. Classification into 1, 2, 3, or 4 class of resistance reflects the ability of the elements to resist to local buckling and is based on the comparison of the compressed walls as parts of the cross section having a limit slenderness $\lambda_0 = (c/t)\varepsilon$, with the appropriate limits expressed in Table 5.2 of EN 1993-1-5, c/t being the width-to-thickness ratio, $\varepsilon = \sqrt{235/f_y}$ (f_y in N/mm^2) and f_y – the nominal yield strength for the section.

In this example, only cross-section of Class 2 is considered. Therefore, the local web buckling constraint is expressed by

$$\lambda_w = \frac{h_w}{t_w} \leq \lambda_{0w}, \tag{2}$$

where limited web slenderness has values $\lambda_{0w} \leq 83\varepsilon$ and for the compressed flange the limited plate slenderness is $\lambda_{0f} \leq 10\varepsilon$.

An adequate bracing against lateral buckling is adopted if the following condition is accomplished:

$$M_{Ed} \leq M_{c,Rd}, \tag{3}$$

where relation

$$M_{c,Rd} = W_{pl}\frac{f_y}{\gamma_{M_0}} \quad \text{for Class 1 and 2} \tag{4}$$

is satisfied; in the relationship M_{Ed} is the maximum design moment, $M_{c,Rd}$ – the design section moment resistance, W_{pl} – the plastic section modulus, f_y – the nominal yield strength for the section, and $\gamma_{M0} = 1$ – the partial safety factor.

3.2. Shear Capacity Constraints (without Tension Field Action)

For the shear resistance verification, the inequality:

$$V_{Ed} \leq V_{c,Rd} \tag{5}$$

must be satisfied, in which V_{Ed} is the maximum design shear force, and $V_{c,Rd}$ the design shear resistance. For a stocky web with $h_w/t_w \leq 72\sqrt{235/f_y}$, to which the elastic shear stress distribution is approximately uniform (as in the case of an equal flanges I-section), the uniform shear resistance $V_{c,Rd}$ is usually given by

$$V_{c,Rd} = h_w t_w \frac{f_y/\sqrt{3}}{\gamma_{M_0}}, \tag{6}$$

in which $f_y/\sqrt{3}$ is the shear yield stress and A_v – the shear area of the web defined in Clause 6.2.6(3) of EN 1993-1-1.

3.3. Simplified Assessment of Buckling Resistance

A simplified design method assessing the lateral stability of beams in buildings with discrete restraints to the compression flange given in EN 1993-1-1, Clause 6.3.2.4, is adopted. In this method, the lateral buckling response of the compression flange of the beam to which one third of the compressed portion of the web is added as collaborating part, analysed as a strut, is assumed to represent the behaviour of the beam to lateral torsional buckling. Members with discrete lateral restraints to the compression flange are not susceptible to lateral-torsional buckling if the length, L_s, between restraints or the resulting relative slenderness, $\bar{\lambda}_f$, of the equivalent compression flange, satisfies the condition

$$\bar{\lambda}_f = \frac{k_c L_s}{i_{f,z}\lambda_1} \le \bar{\lambda}_{c0} \frac{M_{c,\,Rd}}{M_{y,\,Ed}}, \tag{7}$$

where: k_c is a parameter depending on the restraining conditions and the allure of the bending moment diagram, values of which are given in Clause 6.3.2 of EN 1993-1-1, $M_{y,\,Ed}$ – the maximum design value of the bending moment within the restraint spacing, $\bar{\lambda}_{c0}$ = 0.5 (maximum value), $i_{f,z}$ – the radius of gyration about the minor axis of the section (*z-z*) of the equivalent compressed flange to which a 1/3 of the compressed part of the web area is added and $\lambda_1 = 93.9\varepsilon$ – a slenderness limit.

The basic definition of non-dimensional (relative) slenderness of the beam $\bar{\lambda}_f$ (eq. (7)) requires the explicit calculation of design section moment resistance $M_{c,\,Rd}$.

4. Numerical Parametric Study

The girder configuration is adopted as in Figs. 2 and 3. The optimization takes into account different conditions, namely the structural steel grade S235, various defined spans *L*, subject to an imposed point load, *Q*, [kN], (from weight floor beam and live loads) and an uniformly distributed dead load, the self-weight, q_k, [kN/m].

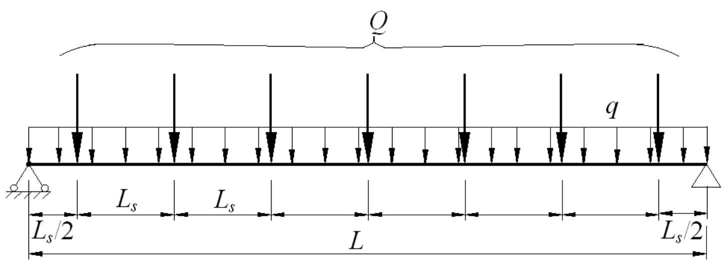

Fig. 3 – Imposed load and beam floor position on the girder.

The primary objective of this paper is to maximize the distance between lateral supports of the girder while an optimum weight of the girder is kept constant.

Concerning the goal of mass optimization as well, the maximization process of the distance L_s is combined with an optimal analytical solution for the previously mentioned design variables (height, width and thicknesses of the web and of the flange – see Fig. 2). Therefore, the optimum height of the beam is determined with the following relationship (Martin & Purkiss, 2007):

$$h_w = \sqrt[3]{2\lambda_w \frac{M_{y,Ed}}{f_{yd}}}\,. \tag{8}$$

The remaining parameters are determined by constraints related to allowable flexural stress, transverse and lateral flexural deformation.

Computational optimal solution has been accomplished through Microsoft Excel spreadsheet Solver (GRG Nonlinear): the solver was set to 150 iterations and since the problem was nonlinear, quadratic estimate was also used. The tolerance was 0.5 while the convergence was set to 0.0001. The problem was scaled automatically by the solver and the search pattern was Newton Search using forward derivative.

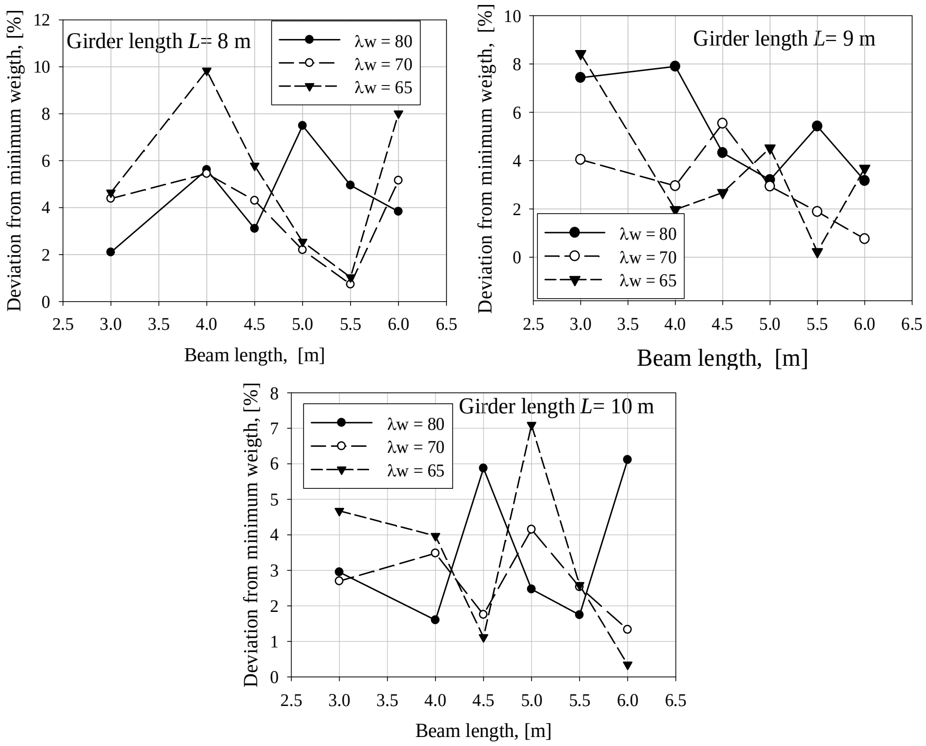

Fig. 4 – Design sensitivity of web slenderness, λ_w, with respect to the optimum girder weight.

The height of the girder (eq. (8)) is sensitive to the initial choice of the web slenderness, λ_w, value that can reduce sufficient amount of weight.

The optimizer depending upon the web slenderness, λ_w , the design sensitivity tries in the first place to modify gradually the initial values of this parameter. The sensitivity to design, meaning the deviation from the weight minimization criterion is studied by comparing the optimal solution using spacing criterion with the minimizing of the mass criterion, once the maximum allowable spacing is known.

The alteration of the optimum area and spacing with various spans may be seen in Figs. 4 and 5. From Fig. 5, it results that for a lower web slenderness, λ_w , greater distances are allowed. This is logical since lateral-torsional moment capacity is dependent on the flange area and depth of girder. Smaller height of the beam leads to wider flange and higher values of the radius of gyration. However, large distances result in a smaller number of floor beams, meaning labor manufacturing economy.

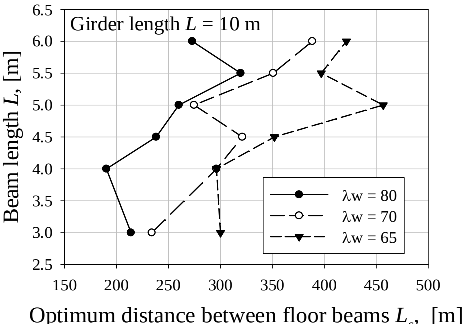

Fig. 5 – Design sensitivity of web slenderness, λ_w, with respect to the maximum distance between beam floors.

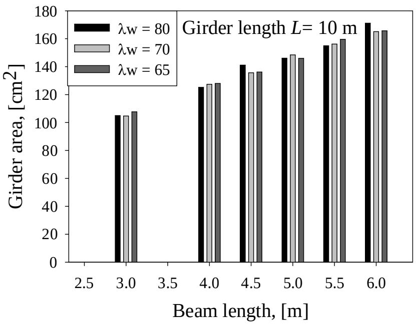

Fig. 6 – Design sensitivity of web slenderness, λ_w, with respect to the optimum area.

Fig. 6 presents very close values of the optimal area for the initial limit values. As recommendations, initial values λ_w = 70 are good choices for improvement of minimum weight girder and labor manufacturing process of the steel floor system.

4. Conclusions

Optimization of girders as part of floor systems is a complex nonlinear problem for which a simple computational procedure has been suggested. The worksheet with solver can take more than thousand variables and very small to large scale nonlinear optimization may be performed.

Additionally, the designed spreadsheet does not cross any limit and therefore robust design may be approached. Computational work sheet as well as numerical optimizer calculates optimum parameters in centimeters and most of the time design variables are not integer. Such problem can be solved by rounding up to a nearest natural number or introducing the integer constraint in the solver which could bring about fractional rise in the weight of the girder. In the floor design this methodology might help to save precious design time, not implying high computational cost. The numerical optimizer has been assessed for simple girders and variety of load cases and it has been established that computation time may increase to a moderate level. The optimization programs created for girder needs only parameter, loads and spans initialization.

REFERENCES

Azad A.k., *Continuous Steel I-Girders: Optimum Proportioning*. J. of the Struct. Division, ASCE, **106**, *ST7*, Proc. Paper 15575, 1543-1555 (1980).

Farkas J., Jármai K., *Analysis and Optimum Design of Metal Structures.* A.A. Balkema, Rotterdam, Netherlands, 1997.

Magnucki K., Rodak M., Lewinskia J., *Optimization of Mono- and Anti-Symmetrical I-Sections of Cold-Formed Thin-Walled Beams*. Thin-Walled Struct, Poland., *44*, 832-836 (2006).

Veljkovic M., Johannson B., *Design of Hybrid Steel Girders.* J. of Construct. Steel Res., **60**, 535-547 (2004).

Martin L., Purkiss J., *Structural Design of Steelwork to EN 1993 and EN 1994*. Third Ed., CRC Press, 2007.

Rao S.S., *Engineering Optimization: Theory and Practice.* Fourth Ed., John Wiley & Sons, Inc. New Jersey, 2009.

Vachajitpan P., Rocky K.C., *Design Method for Optimum Unstiffned Girders*. J. of the Struct. Div., ASCE, **104**, *ST1*, Proc. Paper 13459, 141-155 (1978).

* * * *Modeling with Excel + OML, a Practical Guide*. Amsterdam Optimization Modeling Group LLC, Erwin, 2009.

* * * *Eurocode 3: Design of Steel Structures, Part 1-1: General Rules and Rules for Buildings*. SR EN 1993-1-1.

OPTIMIZAREA TOPOLOGIEI GRINZILOR PLANŞEELOR METALICE

(Rezumat)

Optimizarea rețelelei de grinzi ale planşeelor cu structură metalică implică probleme neliniare complexe, prin urmare se propune o metodă simplă de implementare a tehnicii de optimizare în practica proiectării curente. Optimizarea poziției grinzilor secundare (maximizarea distanței) în lungul grinzii principale, cât şi minimizarea greutății grinzii principale pentru deschideri şi mărci de oțel prescrise s-a realizat cu ajutorul algoritmilor disponibili în toolbox-urile programului Excell, Metode neliniare de gradient (GRG Neliniar). Secțiunea optimă a grinzilor solicitate la încovoiere se obține distribuind materialul în planul de acțiune al momentului încovoietor, cât mai departe de axa neutră, rezultând secțiuni zvelte. Zvelteți mari ale elementului implică cedarea prematură la starea limită ultimă sau de serviciu, prin pierderea stabilității generale prin răsucire sau încovoiere-răsucire. In acest articol s-a studiat optimizarea distanței dintre grinzile secundare considerate ca fiind rezemări laterale intermediare ale grinzii principale, ținând cont de verificările impuse de standarul SR EN 1993-1-1. Sunt evaluate diferite situații impuse de diferitele deschideri ale rețelei de grinzi.