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EXTREME ANNUAL DISCHARGE EVALUATION WITH DIFFERENT EXCENDANCE AND RETURN PERIOD

BY

RALUCA-CATRINEL GIURMA-HANDLEY* and IOAN CRĂCIUN

“Gheorghe Asachi” Technical University of Iaşi
Faculty of Hydrotechnical Engineering, Geodesy and Environmental Engineering

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Abstract. The hydrological practice involve to obtain the primary informations abouth the hydrological parameters which can be use in water infrastructure design. By a statistical processing of the data from hydrological found we can obtain the parameters for designing and exploitation of hydraulic structures and not the last for delivery the prognosys.

The designing of the civil engineering structures demand the knowledge of the basically designing parameters (discharge, level, rainfall, volume, etc.) with exceeding calculus and checkup probabilities. The aim of this paper is to show that some basic concepts and methods used in designing flood-related hydraulic structures assuming a normal distribution and, in particular, the concepts of return period and risk are formulated by extending the geometric distribution to allow for changing exceeding probabilities over time with examples and applications. The applications demonstrate that the return period and risk estimates for nonstationary situations can be quite different than those corresponding to stationary conditions. Annual extreme discharge occur from one year to another, without condition the mutual, but to reach the homogeneity necessary like string value is required to be made up of values generated by the same hydrometeorological conditions (rainfall or snowmelt or both). Special

*Corresponding author: *e-mail*: rgiurma@tuiasi.ro

attention should be given to the highest values due to difficulties in measurement and their calcululm.

To establish the annual extreme flows can be used as Gumbel, Gauss-Laplace or Gama probability distributions. The normal and log-normal distributions give a solution regarding the appropriate assessment of the risk of a hydraulic structure during the designing project.

Key words: hydrological extreme parameters; statistical distribution; risk; design parameter.

1. Introduction

The hydrological practice involve to obtain the primary informations abouth the hydrological parameters which can be use in engineering design.

By a statistical processing of the data from hydrological found we can obtain the parameters for designing and exploitation of hydraulic structures and not the last for delivery the prognosys. The designing of the civil engineering structures demand the knowledge of the basically designing parameters (discharge, level, rainfall, volume etc.) with exceeding calculus and checkup probabilities (Giurma *et al*., 2001; Giurma, 2004).

Annual extreme discharge occurs from one year to another, without condition the mutual, but to reach the homogeneity necessary like string value is required to be made up of values generated by the same hydro-meteorological conditions (rainfall or snowmelt or both). Particular attention should be given to the highest values, however due to difficulties in measurement and their calculation.

Current practice using probabilistic methods, applied for designing hydraulic structures generally assume that extreme events are stationary. However, many studies in the past decades have shown that hydrological records exhibit some type of such as extreme values and return period (Salas & Obeysekera, 2014; Crăciun *et* al., 2011).

To establish the distribution of annual extreme discharge can use one of the following types of probabilities: Gumbel, Gauss-Laplace or Gama (Fürst, 1996). The normal and the exponential distributions appear as limiting cases of high coefficient of variation of the statistical population. Testing of these theoretical results with numerous hydrological data sets on several scales validates the applicability of the maximum entropy principle, thus emphasizing the dominance of uncertainty in hydrological processes. Both theoretical and empirical results show that the state scaling is only an approximation for the high return periods, which is valid when processes have high variation on small time scales. In other cases the normal distributional behaviour, which does not have state scaling properties, is a more appropriate approximation (Lu *et al*., 2013; Krishnamoorthy, 2006). Precipitation frequency distributions are generally skewed rather than normally distributed (Bao *et al*., 2011; You *et al*., 2007).

2. Gumbel Probability Distribution

This method is used to determine values with different probabilities of extreme hydrologic elements as: maximum rainfall in 24 hours, the maximum annual flow, annual minimum flow levels, annual maximum, annual minimum levels.

The elements of hydrological extreme parameters of the random variables, as is characterized by the property that their values, are chronologically successive independent of each other. The extreme values of random variables distributed according to a specific law (eq. type Gumbel) distribution law different from other values. To use the eq. of type Gumbel random variables must be selected from a large number of other independent variables (*e.g*. maximum flow each year is selected from the daily flow 365).

For preparation of Gumbel probability curve is considered a statistical series data X_i, $i = 1,...,n$ in descending order ($X_i > X_{i+1}$). For the statistical series data is calculated arithmetic mean and standard deviation.

In eq. (1) for the statistical series in descending ordered the empirical probabilitie with formula of Weibull

$$p_i = \frac{i}{n+1}, \tag{1}$$

where i is the serial number of terms in the series; n – the total number of terms in the series.

Empirical probabilities corresponding to each reduced variable Y_i is calculated using eq.

$$Y_i = -\ln[-\ln(1-p_i)], \tag{2}$$

Values with different probabilities Q_{pi} is determined by the eq.

$$Q_{p_i} = Q_d + \frac{Y_i}{\alpha}, \tag{3}$$

where Q_d is the mode, α – the standard deviation of the extreme values and Y_i – the reduced variable.

In the Table 1 is presented a case study for a statistical series data of discharge (column 2). The graphically value pairs *(p_i, Q_{p_i}*) give the Gumbel probability curve.

Table 1
Empirical Probability Values and Annual Maximum Flows with Different Probabilities Corresponding for Gumbel Method

Year	Q_i^{max} in descending order, [m^3/s]	$p_i = \frac{i}{n+1}$	Y_i=–ln[–ln(1–p_i)]	$Q_{p_i} = Q_d + Y_i / \alpha$ m^3/s
1987	30.21	0.050	3.113	29.54813
1988	29.22	0.100	2.397	28.75152
1989	28.87	0.150	1.968	28.27389
1990	28.74	0.200	1.655	27.92615
1991	28.66	0.250	1.406	27.64896
1992	28.23	0.300	1.196	27.41581
1993	28.09	0.350	1.014	27.21249
1994	27.78	0.400	0.850	27.03042
1995	27.67	0.450	0.700	26.86398
1996	27.61	0.500	0.561	26.70921
1997	26.56	0.550	0.430	26.56318
1998	26.45	0.600	0.304	26.42355
1999	26.34	0.650	0.183	26.28837
2000	26.31	0.700	0.064	26.15588
2001	26.23	0.750	–0.055	26.02433
2002	26.21	0.800	–0.174	25.89189
2003	26.13	0.850	–0.295	25.75635
2004	26.07	0.900	–0.423	25.61483
2005	25.89	0.950	–0.559	25.46302

3. Gauss-Laplace Probability Distribution

Relationship which gives us maximum annual flows with a certain probability after Gauss-Laplace

$$Q_{pi} = Q_{\text{med}} + \sigma Y_i \, , \tag{4}$$

where Q_{med} is the average value, σ – the standard deviation and Y_i – the reduced variable.

For the series of annual maximum values shown in Table 2 (column 2) will be the values of the columns 3 and 5, resulting in Gaussian-Laplace probability curve.

Table 2
Empirical Probability Values and Annual Maximum Flows with Different Probabilities Corresponding Gauss-Laplace Method

Year	$Q_i^{\max}$ in descending order, [m^3/s]	$p_i = \frac{i}{n+1}$	Y_i=–ln[–ln(1–p_i)]	$Q_{p_i} = Q_d + Y_i / \alpha$ m^3/s
1987	30.21	0.050	3.113	31.16975
1988	29.22	0.100	2.397	30.14781
1989	28.87	0.150	1.968	29.53507
1990	28.74	0.200	1.655	29.08896
1991	28.66	0.250	1.406	28.73336
1992	28.23	0.300	1.196	28.43426
1993	28.09	0.350	1.014	28.17343
1994	27.78	0.400	0.850	27.93985
1995	27.67	0.450	0.700	27.72633
1996	27.61	0.500	0.561	27.52778
1997	26.56	0.550	0.430	27.34044
1998	26.45	0.600	0.304	27.16131
1999	26.34	0.650	0.183	26.98790
2000	26.31	0.700	0.064	26.81793
2001	26.23	0.750	–0.055	26.64917
2002	26.21	0.800	–0.174	26.47927
2003	26.13	0.850	–0.295	26.30539
2004	26.07	0.900	–0.423	26.12383
2005	25.89	0.950	–0.559	25.92908

4. Gama Probability Distribution

In this case to determine the annual maximum flows with different probabilities can use the relationship:

$$Q_{p_i} = Q_{\text{med}}\left[1 + C_v\sqrt{2}\left(\frac{1}{2}Y_i - 1\right)\right], \tag{5}$$

where Q_{med} is the averagge, C_v – the coefficient of variation and Y_i – the reduced variable.

For the statistical series data the annual maximum flow given in Table 3 can calculate the coefficient of variation and the module coefficient (K_i). C_v has the value 0.06366 for the the statistical series data from Table 3 (column 2).

Fig. 1 shows a comparison of Gumbel, Gauss-Laplace and Gama probability curves distribution.

From the Fig.1 we can find the values of the hydrological parameter with different probabilities of execeedance based on the three distribution curves (Table 4).

Table 3

Empirical Probability Values and Annual Maximum Flows with Different Probabilities Corresponding Gama Method

Year	Q_i^{max} in descending order, [m^3/s]	$p_i = \frac{i}{n+1}$	$K_i = \frac{Q_i^{max}}{Q_{med}}$	Y_i	Q_{p_i} m^3/s
1987	30.21	0.050	1.122	3.113	28.0662
1988	29.22	0.100	1.085	2.397	27.2048
1989	28.87	0.150	1.047	1.968	26.6882
1990	28.74	0.200	1.047	1.655	26.3122
1991	28.66	0.250	1.047	1.406	26.0124
1992	28.23	0.300	1.047	1.196	25.7603
1993	28.09	0.350	1.047	1.014	25.5404
1994	27.78	0.400	1.010	0.850	25.3435
1995	27.67	0.450	1.010	0.700	25.1635
1996	27.61	0.500	1.010	0.561	24.9962
1997	26.56	0.550	1.010	0.430	24.8382
1998	26.45	0.600	0.972	0.304	24.6872
1999	26.34	0.650	0.972	0.183	24.5410
2000	26.31	0.700	0.972	0.064	24.3978
2001	26.23	0.750	0.972	–0.055	24.2555
2002	26.21	0.800	0.972	–0.174	24.1123
2003	26.13	0.850	0.972	–0.295	23.9657
2004	26.07	0.900	0.972	–0.423	23.8127
2005	25.89	0.950	0.935	–0.559	23.6485

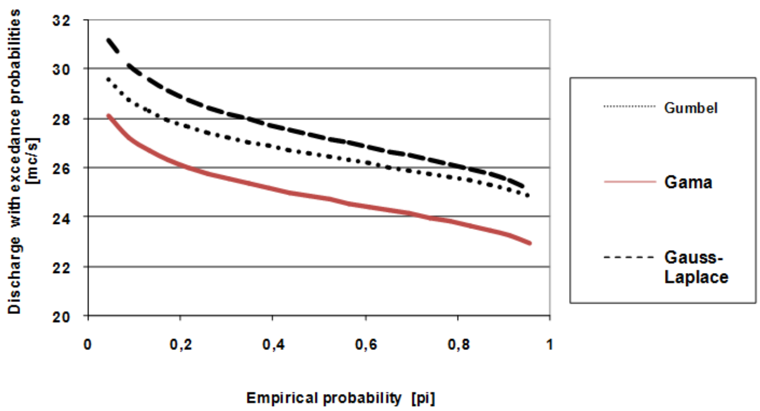

Fig. 1 – The Gumbel, Gauss-Laplace and Gama probability curves.

Table 4

Discharge Values with Probability of 0.05% of Exceedance with using Gumbel, Gauss-Laplace and Gama Distributions

Gumbel distribution	Gauss-Laplace distribution	Gama distribution
$Q_{max}^{0.05\%}$ =29.5 m^3/s	$Q_{max}^{0.05\%}$ =31.2 m^3/s	$Q_{max}^{0.05\%}$ =28.1 m^3/s

5. Log-Normal Probability Distribution

In the hydrological practice the frequency analysis purpose the estimating of the relationship between a hydrological event and the return period, T, of this event.

The return period, T, can be asimilate with the excendance probability of the hydrological parameter, h, P_H, where $H \geq h$, is given by the eq.

$$T = \frac{1}{P_H}, \text{ where } H \geq h. \tag{6}$$

$$T = \frac{1}{P_H}, \text{ where } H \geq h$$

In another way, T is related to the cumulative distribution function of h, P_H ($H < h$), by the relation

$$T = \frac{1}{1 - P_H}, \text{ where } H < h. \tag{7}$$

$$T = \frac{1}{1 - P_H}, \text{ where } H < h$$

In the hydrological practice many cumulative distribution functions cannot be expressed analytically and are tabulated as function of normalized variables, H', where

$$H' = \frac{H - H_{\text{med}}}{\sigma_h}, \tag{8}$$

$$H' = \frac{H - H}{\sigma_h}$$

H_{med} is the mean and σ_h – the standard deviation of the hydrological parameter series.

The cumulative probability distribution of H_{med}, $P_{H'}$ ($H' < h'$), for many distributions depends only by the h' parameter and the skweness coefficient, C_s, of the statistical population.

The value of H' with the return period T by h'_T $h'_T$for many probability distributions is

$$h'_T = K_T(T, C_s), \tag{9}$$

$$h'_T = K_T(T, C_s)$$

where $K_T(T,C)$ is derived from the cumulative distribution function of H' and can be considered a frequency factor.

From the eqs. (8) and (9) we can find the probability of the H with return period T, namely

$$h_T = H_{\text{med}} + K_T \sigma_T \,. \qquad (10) \qquad h_T = \bar{H} + K_T.\sigma_T$$

The frequency factor, can be used in many engineering applications bun not for all probability distributions (Moisello, 2007; Renard *et al*., 2006).

6.1. The Normal Distribution

In the case of using the normal distribution the variable H, is equal to the standard normal deviation, and has a $N(0,1)$ distribution (Douglas & Vogel, 2006; Limpert *et al*., 2001). The frequency factor can be approximated by the empirical eq. (Abramowitz & Stegun, 1965)

$$K_T = w - \frac{2.5255 + 0.8028w + 0.01032w^2}{1 + 1.43278w + 0.189269w^2 + 0.001308w^3} \,,$$

$$(11) \; K_T = w - \frac{2.5255 + 0.8028w + 0.01032w^2}{1 + 1.43278w + 0.189269w^2 + 0.001308w^3}$$

where

$$w = \left[ln\left(\frac{1}{p^2}\right)\right]^{0.5} \qquad w = \left[ln\left(\frac{1}{p^2}\right)\right]^{0.5}$$

with the parameter, p, has values situated between 0 and 0.5; the parameter p is the exceedance probability

$$p = \frac{1}{T} \,. \qquad (12) \; p = \frac{1}{T}$$

The particulary situation when p is greather than 0.5, $1 - p$ is substuited for p in the eq. (12) and the value of the frequency factor can be computed by the eq. (11) is given a negative sign. The error when using the eq. (11) to estimate the frequency factor is less than 0.045%, acceptable in the hydrologically practice to establish the designing parameters (West, 2009; Aitchison & Brown, 1957).

In the case of a log-normal distribution, the random variable is first transformed using the relation

$$Y = ln H \qquad (13) \qquad Y = ln H$$

and the value of Y with return period T; y_T is given by

$$y_T = Y_{\text{med}} + K_T \sigma_y \,, \qquad (14) \; y_T = \bar{Y} + K_T \cdot \sigma_y$$

where Y_{med} and σ_y are the mean and respectivelly standard deviation of the statistical parameter Y and K_T – the frequency factor of the standard normal deviation with return period T.

The value of the original variable, H, with returd period T, h_T, is given by relation

$$h_T = ln^{-1} y_T = \mathrm{e}^{y\,T} . \tag{15}$$

An application of the log-normal distribution for a statistical data series. On for the mean annual rainfall at a rainfall station is normally distribuited with a mean of 760 mm and a standard deviation of 555 mm. A designing approach demands the knowledge of the 50-year annual rainfall.

In the case study the log-normal distribution provide a design parameter of 14.81%, higher than in the normal distribution which provides extrasafety.

6. Conclusions

Annual extreme discharge occurs from one year to another, without condition the mutual, but to reach the homogeneity necessary like string value is required to be made up of values generated by the same hydrometeorological conditions (rainfall or snowmelt or both). Special attention should be given to the highest values due to difficulties in measurement and their calculus.

To establish the annual extreme flows can be used as Gumbel, Gauss-Laplace or Gama probability distributions. The normal and log-normal distributions give a solution regarding the appropriate assessment of the risk of a hydraulic structure during the designing project.

In hydrological practice is important to consider the best hypothesis to determine the design parameters. The literature offers many possibilities to establish design parameters with different probabilities of exceeding Strabilia by Design specifications and it is imperative to include these parameters in assumptions and risks related to the period of exploitation of the work.

This paper presents the case considering two methods of establishing an important hydrological parameter in sizing civil engineering structures with different probabilities can be taken to overcome some methods amog others in terms of design specialists provide additional security.

The two probability distributions: normal distribution, respectively, log-normal distribution, analysed in this paper, provides professionals the opportunity to choose the safest option.

The case study analysed log-normal distribution provides a sizing parameter 14.81% higher than in the normal distribution which provides extra safety.

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DETERMINAREA DEBITELOR EXTREME (MAXIME, MINIME) ANUALE CU FRECVENŢE ŞI DIFERITE PROBABILITĂŢI DE DEPĂŞIRE

(Rezumat)

În practica hidrologică este important să se adopte cea mai potrivită ipoteză pentru a stabili parametrul de proiectare.

Literatura de specialitate oferă multe posibilităşi de a stabili parametrii de proiectare cu diferite probabilităşi de depăşire stabiliţi prin prescripţiile de proiectare şi este absolut necesar să se includă în aceşti parametri ipotezele şi riscurile aferente perioadei de exploatare a lucrării.

Lucrarea prezintă situaţia analizei unor metode de stabilire a unui parametru hidrologic important în dimensionarea structurilor de inginerie civilă cu diferite probabilităţi de depăşire putând fi asumate unele metode în dauna altora din prisma oferirii specialiştilor din proiectare a unei siguranţe suplimentare.

Debitele extreme anuale se produc de la un an la altul, fără a se condiţiona reciproc, dar pentru a se ajunge la omogenitatea necesară se impune ca şirul de valori să fie alcătuit din valori generate de aceleaşi condiţii hidrometeorologice (ploi sau topirea zăpezilor sau ambele situaţii). O atenţie deosebită trebuie acordată celor mai mari valori datorită dificultăţilor întâmpinate în măsurarea şi calculul lor.

Pentru stabilirea debitelor extreme anuale se pot utiliza distribuţii de probabilitate ca Gumbel, Gauss-Laplace sau Gama. Alte tipuri de distribuţii, respectiv distribuţia log-normală, oferă specialiştilor posibilitatea de a stabili frecvenţa de apariţie a unor valori extreme ceea ce oferă un surplus de siguranţă.