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CONSTRUCŢII. ARHITECTURĂ



# THIN AND CURVED SELF-BEARING SURFACES

BY

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Edification practice proves that an aesthetic fulfillment of a building may ignore structural laws – Greek architecture – same as correct buildings are fulfilling beautiful – iron architecture. The degree of participation of the structure to artistic impression is different and the buildings situated at those two extremes have a structural component that is destined to influence the aesthetic reaction. Through structure a building can emit a semiotic message, a noun-verbal communication as a product of the universal recognition or the compliance of a function. Pure structural message are coming from our intuitive understanding of a structural behavior, that is generated both by physical experience with structural action and by perception of the constructive forms of the nature.

## 1. Introduction

Several materials, and a unique expression – that is the arch. An arch is just a flipped cable, frozen in his deflected shape. The tension in the cable becomes compression in the arch, and the tension outside the cable becomes the thrust inside the arch which force it not to open. In order to freeze the shape of the arch, that is the so-called "anti-funicular curve", of the loading, it is necessary to stiffen the element by increasing the depth and width, otherwise it will buckle, just like any other thin elements subjected to compression.

Because the arch must be stiffened to avoid collapse under large stresses, it doesn't needs stiffener beams. It keeps its shape under variable forces, therefore it is stable comparing to a cable without stiffeners. Since an arch is compressed within his whole all loads including the self weight as well, acts downwards – it can be build with materials with high strength in compression resistance, like stone, brick or concrete. A masonry arch is built from separated blocks which need previously support, usually made of timber. The construction of the arch starts from both ends of the supporting element. When the vault head is locked within the last two elements, the supporting structure can be removed because any half of the arch supports on the other half.

Nature knows better the resistance laws and uses it whenever it is necessary. The arches stand at the basis of curved shapes and generate them by rotation or moving along an axis (usually a vertical one). Gauss classified the curved shapes in three categories: the cupola, cylinder, and the settle.

At the cupola, the cross-section shape has the curve downwards and if we cut it in two on any direction we recall the same cross-section shape. A cupola shape surface has the curves downwards in all radius directions. It is a unfolding surface and it has been used for a very long time to cover large spans (Fig. 1).

At cylinders, the vertical cross-sections have the curve downwards except one: the cross-section along its longitudinal axis is a straight line. The cylinder has the curvature null along its axis.

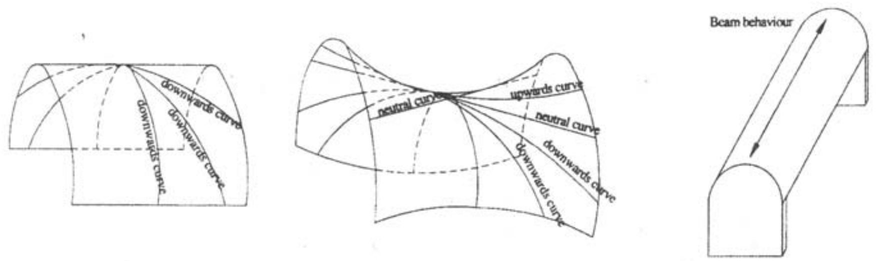


Fig. 1

At settles we have two null curve directions, and when those two tend to get closer, the settles become cylinders and if it is given a downwards curve, the cylinder becomes cupola. We can therefore consider the cylinder as a breaking line between the cupola and the settle.

One of the most common combinations of cylindrical surfaces is the intersected vault from the ecclesial architecture. That consists in an intersection between two cylindrical vaults, under a 90° angle, leaned on four marginal arches and intersects along some diagonal curved dollies which end at the four corner pillars, on which the vault leans. So the vault is called the vault with outer edges and the dollies were often visual outstanding and possible structural, with the help of the nervures, but even if those nervures can be important in an aesthetic perspective, they aren't necessary to support the cupola because of its self-bearing curve. When we eliminate the marginal arches we obtain a cupola with outer edges, that is a kind of cupola with a square basis and the vertical cross-section semi-circle also called monastery-vault.

## 2. Cupola as a Structure

Any signification we can give to the cupola, its structural behavior must be understood before it can be appreciated its usefulness in architecture (Fig. 2). For that we must ignore the minor shape differences given by the cupola in its historical development and to consider as a perfect hemisphere with reduced thickness reported to the span. Whether it stands on the central part of the church, or on the ground, the cupola must support its self-weight and also the live loads.

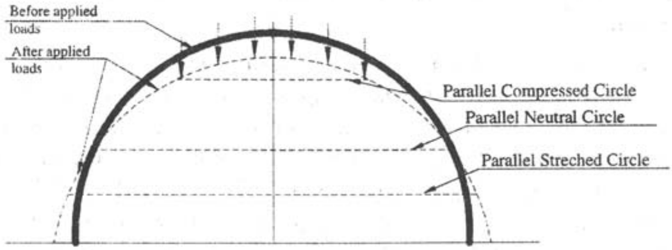


Fig. 2

Those loads must be channeled towards the ground and likewise the arch the transfer is made along its curves in a vertical plane, the meridians. The cupola is a sum of identical arches disposed along a circular basis and unifying at the height point, where a common "key" ends it. The loads sum each other from the vault head towards the basis along the meridians within the vertical planes, which are more and more compressed while closeting to the cupola's support. The difference between the arch and cupola consists in the thickness much more diminished of the cupola reported to the same span arch as much as the lack of buttresses or the tie rods to hinder outer forces. What concerns the cupola behave differently is that the hypothetical arches from which it is composed are reunited along the cupola's vertical cross-section, making out of it a monolithic structure. Reducing the thickness can go from 1/20...1/30 at the arches radius to 1/200...1/300 at cupola. Continuity of the cupola's surface allows this kind of thickness reduction introducing an action along its horizontal cross-sections or parallel, which prevents the meridian arches to open. Under stresses, the upper points of the cupola move towards the inside and the lower one towards the exterior. Those deflections, which develop freely at an arch, are avoided at cupola by horizontal rings.

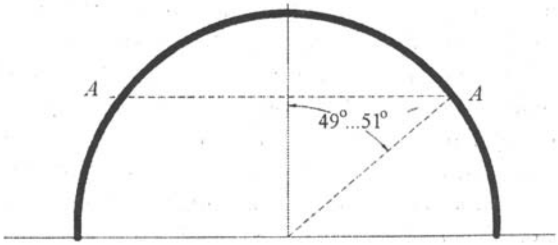


Fig. 3

At the top part, the parallel circles are compressed opposing the deflections towards the inner, this leading to a possible radius reduction, while at the bottom part they are subjected to tension opposing the deflections towards outside, that leading to bending. In the case of the cupola those deflections are extremely small (Fig. 3).

Preventing the meridian deflections of the cupola by compression or tension of its parallel circles has two consequences.

Firstlly it is necessary that the cupola be much more resistant and prevents the buckling of the compressed meridians, which leads to a considerable reduction of its cross-section.

Secondly it prevents the opening the cupola at the basis, eliminating the necessity of buttresses or tie-rods. The base stripes which work at tension, work just as any other tie-rods that hinder the cupola to open. If the upper circles shrink and the bottom ones lengthen, there is an equilibrium curve which is a neutral circle which manifests in the two situations. Under its self-weight, this circle makes a 52° angle with the cupola's vertical axis. This thing has not been correctly understood in the past, such as the majority of the old cupola presents vertical cracks at their basis due to low masonry resistance to tension (Fig. 4).

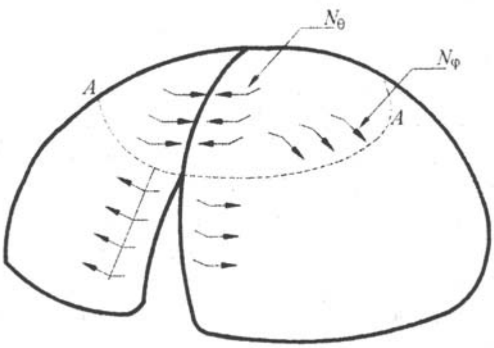


Fig. 4

Another leading character of the cupola compared with the arch refers to its behavior while the forces applied change signs. There is a single shape of arch subjected to compression for a given group of loads through a combination of compression and bending along its meridians, acting partially as an arch, partially as a beam. From this point of view the cupola presents the advantage consisting in a curve element made out of a monolithic material that prevents sliding of a meridian arch to the next one. This mechanism permits the cupola to take not only one kind of loads, but any kind, without changing its shape and without developing bending tensions, with the condition that the loads must be continue and axially symmetric. From this point of view, the cupola is a structure extremely efficient regarding the use of materials. The most of the cupolas are stiffened at the bottom part by a strong ring which prevents deflections for all practical situations. This ring prevents the opening of the cupola under stresses, bends in a necessary way the cupola's surface in its very closeness and introduces in it a small quantity of bending stresses, usually only on 5% of cupola's surfaces and develops bending tensions in the very closeness of the support, while in the rest develops just compression and tension.

## 3. Cupolas

Covering the spaces of rectangular shapes can be achieved by structures that are capable to take loads with respect to the size of the span, cylindrical surfaces offering a good solution in this case; sometimes the layer is sustained by firm frames or spatial beam networks. For covering circular spaces it appears the necessity of thin layers that works on two directions and supports along all of the contour, like cupolas, the classic type symmetric rotational form. Analogical to the thin plane plates reinforced on two directions, the cupola works likewise on the two directions and supports on its contour, permitting a reasonable distribution of efforts. If the symmetrical load (the kind of self-loads uniformly distributed) and conditions of "membrane" state are respected, in the layers appear axial efforts, $N_m$, directed along the meridian and, $N_p$, directed along the parallel circles (tangent at the respective curves).

As for their way of behavior, along the parallels it appear, at the upper part, compression efforts, and at the bottom part, tension stresses, meanwhile along the meridians appear only compression efforts. The low tension resistance of the concrete cause meridian cracks at the bottom part, delimitated by a 90° angle, the cupola dividing the vaults.

The ring that delimitates the compressed part from the tensioned one, is found for loading with its self-weight to be 51°49′ from the vertical.

Insurance of taking those efforts of compression can be achieved with the help of pre-stressing, that of initial effort and juxtaposing with efforts in service.

Solving the bottom part depends on the way in which the loads are transmitted to the soil:

a) directly, the cupola supporting continuously on a circular foundation capable to take over the tension efforts;

b) indirectly, through counter-forces tangent to the cupola or through vertical pillars.

The most economical solutions are obtained obviously by supporting the cupola directly on the foundation, after that it can be adopted buttress solutions. From the point of view of the membrane state in the cupola, it is preferred that the distance between the buttresses or the pillars to be as small as it can be. With this goal, there is adopted a V or Y pillars solution, that creates many support points for the cupola, that leaves the lower level free for circulation.

The solving of the cupola itself can represent many variants: from a simple plate with constant thickness in the field and thickened on the edges, to the reticular cupolas, especially in the cases of large spans or solutions with prefabricated elements.

To avoid cofrate difficulties, due to its double curvature, it is adopted sometimes the type of cupola built from a succession of cylindrical surfaces. There is one more type of cupola made out of assembling thin surfaces, which allow as much the stiffness requirement, also solving the various functional problems and illuminating problems.

What is essential to remind for designing and achievement of thin layers is the desire to solve the transfer of the loads to the soil on the safest and the most direct method.

## 4. Arches Statically Undetermined – Composing Principles. Classification

As concerns the terminology, arches are bars with the axis plane curved, loaded by stresses in their plane. The characteristic of the arches from the point of view of the stresses is overall for the axial compression stresses, compared to the bending moments and the shearing forces. This, unlike the beams, where the effect of bending moments are preponderant.

The vault is that resistance element that works from the mechanical point of view as an arch structure and has mechanical stiffness due to clamping in the wall structure.

The arches are characterized also by the fact that they develop stresses in the supports, even if they are loaded with vertical stresses. Due to the stresses in the supports, it leads to an important reduction of the bending moments, compared to the simple supported beam with the same span.

Arches are in fact elements that characterize they self also by the fact that they develop thrusts even if the actions that are subjected are vertical and perfectly symmetrical to the vault or arch axis. Especially due to this thrusts, that are sectioned with different sign of the horizontal components of the reactions that appears in the supports, it leads to an important reduction if the bending moment in the arch, compared with the straight beams that allow as in the case of vaulting structures in the Medium Eve arches, utilization of the stone and brick masonry for a distinguished stylistic elegancy, capable to solve the functions and to withstand in time.

The arches are used as bearing elements at industrial buildings, expositional complexes, bridges, etc. The static determined arches often used in constructions are the triple articulated arch and the arch with tie-rod.

Thrust-stresses resulted by the triple articulated arches must be taken by the construction elements that support the arch. At the bridges, the arches are supported on large foundations that take the stresses in good conditions. At industrial buildings, for overtaking the thrust by the supporting elements (walls, pillars) becomes difficult, that why it is usually introduced a tie-rod, that takes a part of the stresses. To maintain the statically determination condition, the introduction of the additional connection is compensated by transforming the hinge from a support, in a simple support.

The axial stress in the tie-rod leads to a decreasing of the bending moments in the arch, compared to the simple supported beam, as in the case of the arch with triple articulation.

From the point of view of the support and the statically schema. the arches can be:

a) arches with two hinges (one statically undetermined);

b) arches with three hinges (statically determined);

c) fixed arches (triple static undetermined).

Besides the arch with a straight tie-rod at the level of the birth line, it is often used an arch with simple tie-rod or pre-stressed. The tie-rod over height at the arches that enter in the composition of some roofs permits to obtain a free height bigger in the industrial building.

If the arch has a hinge atone end and a simple support at the other one it works just as a system with thrusts only when the simple support is bent, so being a simple curved element loaded with stresses along its plane.

On the arch principle (as a system that develops thrusts) works and the structures composed out of straight or curved elements are commonly called triple hinged frames.

The terminology specific to the arches is the next one (Fig. 5):

a) Sections at the supports of the arch are called birth ends.

b) The line $A$-$B$ which joins the origins, is called the birth line. If the birth line is horizontal ($\alpha = 0$), the arch is called "leveled births".

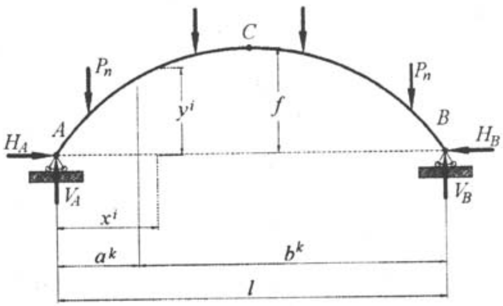


Fig. 5

c) The cross-section that is the most far on the vertical from the birth line is called key. At the key is usually found the intermediary hinge, $C$.

d) The distance measured on the vertical, or perpendiculary on the birth line, from the birth line to the key, is called arrow distance.

e) The distance between the births measured on the horizontal is called span of the arch.

f) The inner face of the arch is called intrados, and the outer face is called extrados. The same terminology is used at the vaults (birth, key, intrados and extrados).

g) The ratio between the arrow distance of the arch and its span ($f/l$) is called the flatness of the arch: flattened arches ($f/l = 5$); bolded arches ($f/l > 5$).

In constructions, the resistance structure on arches is placed usually in the last category. The ăxis of the arches is often parabolic, rarely circular. The coordinates of the arches axis are necessary in the static analysis; these ones can be considered in three ways (Fig. 6):

a) in the $xAy$ axis system, composed of the birth line and the vertical from $A$;

b) in the $xAy$ axis system, composed of the horizontal in the birth in $A$, and the vertical from $A$.

In the first case, the coordinates of a point, $I$, are:

a) $X_i$ measured perpendicularly to $Ay$ axis;

b) $Y_i$ measured parallel to $Ay$ axis to the birth line, or

c) $X_i$ measured perpendicularly to $Ay$ axis;

d) $Y_i$ measured perpendicularly to the birth line $Y_i = Y_i \cos \alpha$.

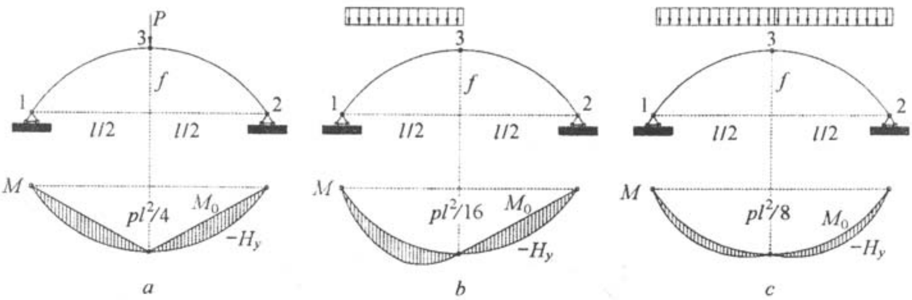


Fig. 6

## 5. Conclusions

As in the case of important constructions for Romanian national spirituality, whose integrity needs to be kept and controlled in the process of periodical rehabilitation, the structural behavior and strength is imperative and requires a careful analysis both in elastic and post-elastic domain, taking into consideration the process of ageing during time and other "traumas" that a building is subjected.

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REFERENCES

1. Levy M., Salvadori M., *De ce cad construcţiile* (transl. from English). Edit. Tehnică, Bucureşti, 1988.
2. Siegel C., *Forme structurale ale arhictecturii moderne* (transl. from German). Edit. Tehnică, Bucureşti, 1966.

3. S t r a t L., *Structuri*. Partea a II-a, Edit. Instit. Politehnic, Iaşi, 1980.
4. S t r a t L., *Mecanica şi proiectarea structurilor simple*. Edit. Instit. Politehnic, Iaşi, 1975.
5. C i o r n e i Al., *Cum concepem cosntrucţiile civile*. Edit. Instit. Politehnic. Iaşi. 2000.
6. M o i s e s c u A., *Betonul în arhitectură*. Edit. Tehnică, Bucureşti, 1964.

## SUPRAFEŢE PORTANTE SUBŢIRI ŞI CURBE

(Rezumat)

Practica demonstrează că o satisfacere estetică a clădirii poate ignora legile structurale – arhitectura greacă – aşa cum construcţiile corecte satisfac frumosul – arhitectura fierului. Gradul de implicare a structurii la impresia artistică diferă şi construcţiile situate la cele două extreme au o componentă structurală care are scopul de a influenţa răspunsul estetic. Prin structură o construcţie poate emite un mesaj semiotic, o comunicare non-verbală precum un produs al recunoaşterii universale sau satisfacerea unei funcţii. Mesaje pur structurale vin din înţelegerea noastră intuitivă a comportamentului structurilor, ce este generată atat de experienţa cu acţiunile structurale cât şi de percepţia formelor constructive naturale.