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CONCRETE SHRINKAGE EFFECT ON THE COMPOSITE STEEL–CONCRETE STATE OF STRESSES

BY

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Abstract. The size of concrete shrinkage deformation depends, similar to creep, upon many factors: concrete composition, concrete quality, water/cement ratio, aggregate nature and grain size, compaction manner, humidity of ambient environment. In the steel–concrete composite structures, the phenomena of creep and shrinkage affect concrete behaviour and lead to shortening in the concrete slab. As concrete is rigidly tied to steel, the shortening is partially stopped and brings about a redistribution of unit stresses inside the section. The redistribution of the stresses leads to concrete unloading and increase in steel stress in the compressed area. The paper presents the mechanism through which shrinkage stresses develop in the case of a simply supported steel–concrete composite girder. The calculation parameters, the specific deformations and modulus of elasticity for concrete at various times since pouring are assessed according to Eurocodes EC2 and EC 4.

Key words: composite girders; concrete shrinkage; stresses due to shrinkage.

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1. Introduction

Using two materials with differing mechanical characteristics – steel and concrete – joined continuously together, leads to the occurrence of a state of self-balanced stresses in every section of the composite structure, irrespective of the external loads.

The composite structure slab concrete cannot deform from free shrinkage because the slab is joined together to the metal structure not connecting members.

Consequently, the tendency of shortening the concrete slab is stopped and in the composite structure internal stresses are induced overlapping those produced by shrinkage in the slab area.

The paper presents the mechanism of developing shrinkage stresses in the case of a simply supported steel–concrete composite girder.

The calculation parameters, the specific deformations and modulus of elasticity for concrete at various times since pouring are assessed according to Eurocodes EC2 and EC 4.

2. Stresses Produced by Concrete Shrinkage

The size of concrete shrinkage deformation depends, similar to creep, upon many factors: concrete composition, concrete quality, water/cement ratio, aggregate nature and grain size, compaction manner, humidity of ambient environment. The shrinkage deformations begin to be manifest immediately after concrete putting into place, independent of the size of unit stresses in the concrete.

The mechanism of developing shrinkage stresses is exemplified on the simply supported steel-concrete composite girder.

First, the slab is taken as separate from the metal section, so slab shortening can freely occur with not stresses taking place in the slab (Fig. 1 *a*).

Slab shortening will be given by relationship:

$$\Delta L_c = \varepsilon_c L \, . \tag{1}$$

To cancel this deformation, it is accepted to introduce a fictitious tensile force N_c (Fig. 1 *b*) acting upon all ends of the concrete slab of section area A_c (in the weight centre), to compensate for the shrinkage deformation

$$N_c = -\varepsilon_c E_c A_c \, . \tag{2}$$

After the first two stages – free shrinkage and fictitious tensile force – the concrete slab does not exhibit deformations and implicitly, no stresses in the joining members occur.

To cancel the fictitious force, N_c , in the weight centre of the slab, a force equal and of opposite sign (compression force) is applied, N_m ; this force acts upon the composite section with an eccentricity noted z_{cm}.

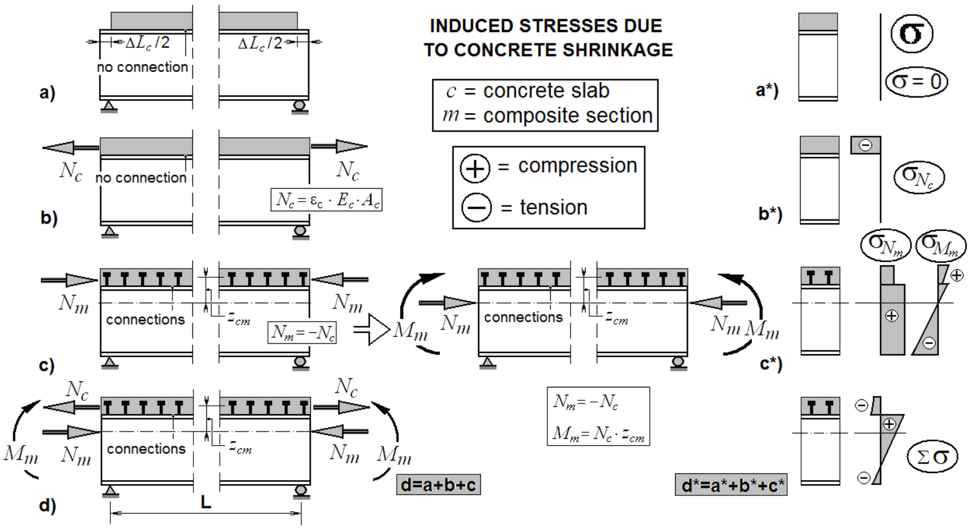


Fig. 1 – Development of stresses due to concrete shrinkage.

The eccentric compression force, N_m , is equivalent to a centric compression force (acting upon the weight centre of the composite section) and a bending moment, M_m, which acts upon the composite section.

It means that the following stresses act upon the composite cross section:

$$N_m = -N_c = \varepsilon_c E_c A_c \text{ – compression force;} \tag{3 a}$$

$$M_m = N_m z_{cm} \text{ – positive bending moment.} \tag{3 b}$$

In short, concrete compression is replaced by a tensile force in the concrete slab and an axial compression force, and a bending moment which acts upon the composite cross section. The overlapping of effects leads to finding the unit stresses in the concrete slab and steel girder.

The stress assessment method for the stresses occurring during concrete shrinkage can be adapted to Eurocodes as follows:

a) Concrete shrinkage, ε_c, is assessed according to EC 2, respectively it is replaced by ε_{cs}:

$$\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca}, \tag{4}$$

where: ε_{cs} is the final deformation due to shrinkage; ε_{cd} – the deformation due to shrinkage in time; ε_{ca} – the deformation due to initial elastic shrinkage.

b) The concrete modulus of elasticity is replaced by:

$$E_c = \frac{n_o}{n_S} E_{cm} . \tag{5}$$

The equivalence coefficient, n_S, for the effect of shrinkage is assessed with the relationship:

$$n_S = n_{L(\Psi=0.55)} = n_0 (1 + 0.55 \varphi(t, t_0))$$

$$\varphi(t, t_0) = \varphi_0 \beta_{c(t,t_0)} ; \; n_0 = \frac{E_a}{E_{cm}} .$$

c) The equivalence coefficient, $\overline{n}_{\varphi}$, is replaced by n_S.

It yields:

$$N_c = - N_m = - \varepsilon_{cs} \frac{n_o}{n_S} E_{cm} A_c . \tag{6}$$

The unit stresses in the concrete slab and metal girder:

1. *In the concrete*:

$$\sigma_c = - \frac{N_c}{A_c} + \frac{1}{n_S} \left(\frac{N_m}{A_m} + \frac{M_m}{I_m} z \right) ; \tag{7}$$

2. *In the steel*:

$$\sigma_a = \frac{N_m}{A_m} + \frac{M_m}{I_m} z . \tag{8}$$

A_m and I_m represent the area and the inertia moment of the composite concrete–steel section where the equivalence coefficient taken is n_S.

3. Numerical Example

The state of stresses is assessed in the cross section of a steel-concrete composite girder following the shrinkage in the slab.

The following data are known:

a) composite girder cross section (Fig. 2);

b) concrete grade: C40/50;

c) S420 steel.

The web of the metal girder is provided with longitudinal stiffening bars so that the grade for the section is 3.

SOLUTION

Taking into account the building stages (use of intermediate supports to pour the concrete slab), the shrinkage deformation is calculated before removing supports (piles), respectively at 28 days and the shrinkage deformation in infinite time (practically at 100 years).

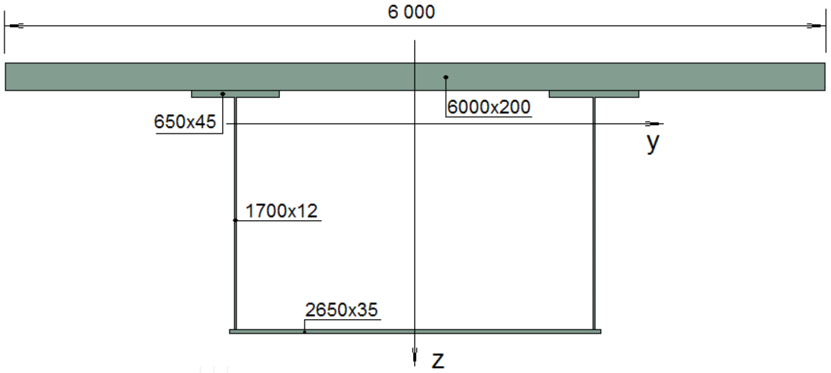


Fig. 2 – Cross section of composite girder.

i) *Calculus of shrinkage deformation*

The initial elastic deformation (due to endogenous shrinkage):

$$\varepsilon_{ca}(t) = \beta_{as}(t)\varepsilon_{ca}(\infty) = 0.65\times 75\times 10^{-6} = 4.875\times 10^{-5};$$
$$\varepsilon_{ca}(\infty) = 2.5\left(f_{ck}-10\right)10^{-6} = 2.5\left(40-10\right)10^{-6} = 75\times 10^{-6};$$
$$\beta_{as}(t) = 1-\mathrm{e}^{(-0.2t^{0.5})} = 1-\mathrm{e}^{(-0.2\sqrt{28})} = 0.65.$$

The deformation due to shrinkage during drying in time is

$$\varepsilon_{cd}(t) = \beta_{ds}(t,t_s)\varepsilon_{cd,\infty} = \beta_{ds}(t,t_s)k_h\varepsilon_{cd,0} = 0.092\times 0.72\times 2.4\times 10^{-4} = 1.6\times 10^{-5};$$

$$\beta_{ds}(t,t_s) = \frac{(t-t_s)}{(t-t_s)+0.04\sqrt{h_0^3}} = \frac{(28-1)}{(28-1)+0.04\times\sqrt{400^3}} = 0.092$$

(it is supposed that the concrete age t_s at which shrinkage begins is of one day)

$\varepsilon_{cd,0} = 0.24\times 10^{-3} = 2.4\times 10^{-4}$ (Table 3, Annex 2, for RH = 80%)

$$h_0 = 2A_c/u = 2\times 600\times 20/600 = 40\ \text{cm} = 400\ \text{mm} \Rightarrow k_h = 0.72\ \text{(EC2)}.$$

The deformation from shrinkage in the moment of traffic opening (t = = 28 days) is:

$$\varepsilon_{cs}(t) = \varepsilon_{cd}(t)+\varepsilon_{ca}(t) = 1.6\times 10^{-5}+4.875\times 10^{-5} = 6.475\times 10^{-5} \approx 0.65\times 10^{-4}.$$

The deformation arising from final shrinkage ($t = \infty$) will be

$$\varepsilon_{cs}(\infty) = \varepsilon_{cd}(\infty) + \varepsilon_{ca}(\infty) = 1.728\times10^{-4} + 0.75\times10^{-4} \approx 2.5\times10^{-4}.$$

It is noticed that 74% of the deformation occurs by drying and 26% by concrete setting.

The slab deformation calculated infinitely is used in combinations of actions to check the composite girder.

ii) *Calculus of equivalence coefficient*

a) *According to SR EN 1994-2:*

The equivalence coefficient for short term loads:

$$n_0 = \frac{E_a}{E_{cm}} = \frac{210{,}000}{22{,}000\left(\dfrac{f_{cm}}{10}\right)^{0.3}} = \frac{210{,}000}{22{,}000\left(\dfrac{48}{10}\right)^{0.3}} = \frac{210{,}000}{35{,}220} = 5.96.$$

The equivalence coefficient for other loads:

$$n_L = n_0\left[1 + \Psi_L \varphi(t,t_0)\right]$$

where: Ψ_L is the multiplier for the creep:

$$\Psi_L = \begin{cases} 1.1 \text{ – permanent loads;} \\ 0.55 \text{ – shrinkage effects;} \\ 1.5 \text{ – imposed deformations,} \end{cases}$$

$\varphi(t,t_0)$ – slow flow coefficient.

The creep coefficient, $\varphi(t,t_0)$: creep occurs in the interval between time t_0 to time t.

The relative humidity, RH = 80% is taken into consideration

$$\varphi_{RH} = \left[1 + \frac{1-\mathrm{RH}/100}{0.1\sqrt[3]{h_0}}\alpha_1\right]\alpha_2 = \left[1 + \frac{1-80/100}{0.1\sqrt[3]{400}}0.80\right]0.94 = 1.14;$$

$$\alpha_1 = \left[\frac{35}{f_{cm}}\right]^{0.7} = \left[\frac{35}{48}\right]^{0.7} = 0.80; \quad \alpha_2 = \left[\frac{35}{f_{cm}}\right]^{0.2} = \left[\frac{35}{48}\right]^{0.2} = 0.94;$$

$$\beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} = \frac{16.8}{\sqrt{48}} = 2.42;$$

$$\beta(t_0)=\frac{1}{0.1+t_0^{0.2}}=\frac{1}{0.1+1}=0.909\,;\ t_0=1\text{ day};$$

$$\beta_{c(t,t_0)}=\left(\frac{t-t_0}{\beta_H+t-t_0}\right)^{0.3}=1\qquad\text{for } t=\infty;$$

$$\varphi_0=\varphi_{RH}\beta(f_{cm})\beta(t_0)=1.14\times2.42\times0.909=2.50\,;$$

$$\varphi(t,t_0)=\varphi_0\beta_{c(t,t_0)}=2.50\times1=2.50\,.$$

b) *The equivalence coefficient for shrinkage*:

$$n_S=n_{L(\Psi=0.55)}=n_0\left[1+0.55\,\varphi(t,t_0)\right]=n_0(1+0.55\times2.50)=2.375n_0=14.15\,.$$

iii) *Evaluation of stresses produced after shrinkage*

The results reached by applying SR EN 1994-2 will be used:

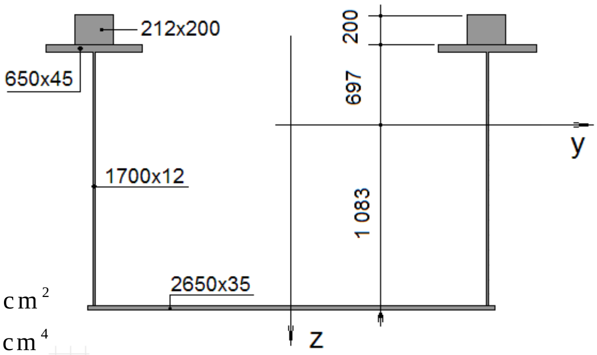
n_0 = 5.96; n_S = 2.375n_0 = 14.15; ε_{cs} = 2.5 × 10^{-4}; E_{cm} = 35,220 N/mm.

a) *Steel equivalent section*

Steel equivalent width of the concrete slab:

$$b_{\text{eff}}^{*}=\frac{b_{\text{eff}}}{n_S}=\frac{6{,}000}{14.15}=424\ \text{mm}\,.$$

The steel equivalent composite section is represented in Fig. 3.



$A_m=2{,}768\ \text{cm}^2$

$I_m=2\times10^7\ \text{cm}^4$

Fig. 3 – Cross section characteristics.

The axial stress developed from shrinkage will be:

$$N_c=-N_m=-\varepsilon_{cs}\frac{n_0}{n_S}E_{cm}A_c=-2.5\times10^{-4}\frac{1}{2.375}0.3522\times10^6\times12{,}000\,;$$

$$N_c=-N_m=-449\times10^3\,\text{daN}\,.$$

The bending moment:

$$M_m = N_m z_{cm} = 449\times10^3\times79.7 = 357.85\times10^5\ \text{daN.cm}$$

The unit stresses found from the shrinkage phenomenon are:
a) *in the concrete:*
a$_1$) in the upper fibre:

$$\sigma_{c.\sup} = -\frac{N_c}{A_c} + \frac{1}{n_S}\left(\frac{N_m}{A_m} + \frac{M_m}{I_m} z_{cs}\right) =$$

$$= -\frac{449\times10^3}{12{,}000} + \frac{1}{14.15}\left(\frac{449\times10^3}{2{,}768} + \frac{357.85\times10^5}{2\times10^7}89.7\right) = -14.6\ \text{daN/cm}^2;$$

a$_2$) in the lower fibre:

$$\sigma_{c.\inf} = -\frac{N_c}{A_c} + \frac{1}{n_S}\left(\frac{N_m}{A_m} + \frac{M_m}{I_m} z_{ci}\right) =$$

$$= -\frac{449\times10^3}{12{,}000} + \frac{1}{14.15}\left(\frac{449\times10^3}{2{,}768} + \frac{357.85\times10^5}{2\times10^7}69.7\right) = -17.1\ \text{daN/cm}^2;$$

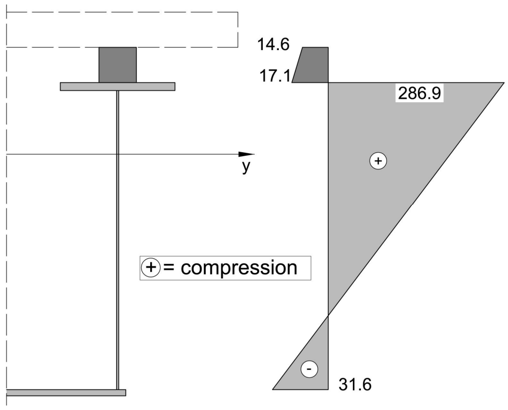


Fig. 4 – Stresses due to concrete shrinkage.

b) *in the steel:*
b$_1$) in the upper fibre:

$$\sigma_{a.\sup} = \frac{N_m}{A_m} + \frac{M_m}{I_m} z_{as} = \frac{449\times10^3}{2{,}768} + \frac{357.85\times10^5}{2\times10^7}69.7 = 286.9\ \text{daN/cm}^2;$$

b$_2$) in the lower fibre:

$$\sigma_{a.\inf} = \frac{N_m}{A_m} - \frac{M_m}{I_m} z_{ai} = \frac{449 \times 10^3}{2,768} - \frac{357.85 \times 10^5}{2 \times 10^7} 108.3 = -31.6 \text{ daN/cm}^2 .$$

The unit stress diagram arising from shrinkage is given in Fig. 4.

4. Final Remarks and Conclusions

The magnitude of concrete deformation due to shrinkage depends, similarly to creep, upon numerous factors, such as: concrete composition, concrete quality, water/cement ratio, aggregates nature and grain size, compaction, humidity of surrounding environment.

The phenomena of creep and shrinkage influence the concrete behaviour and produce a shortening of the concrete slab. As concrete is stiffly tied to steel, the shortening is partially prevented and leads to the redistribution of stresses inside the cross section.

The redistribution of stresses leads to concrete discharge and increase of steel tensioning in the compressed area.

Calculations show that about 75% of the deformation takes place by drying and about 25% , during concrete setting.

The slab deformation calculated for the infinite time is used in combinations of actions during composite girder check.

The shrinkage influence cannot be neglected when calculating the connectors in sections where the sliding forces at the contact between the concrete slab and the steel girder are high.

According to EC4, the effects of shrinkage and creep of the concrete slab can be neglected in the calculations for the ultimate limit states other than fatigue for members whose all cross sections belong to Class 1 or Class 2 and where no approximations are necessary for the lateral buckling, in the limit states of normal service.

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EFECTUL CONTRACŢIEI BETONULUI ASUPRA STĂRII DE EFORTURI DIN STRUCTURILE COMPUSE OŢEL–BETON

(Rezumat)

Mărimea deformaţiei betonului din contracţie depinde, ca şi pentru curgerea lentă, de numeroşi factori: compoziţia betonului, calitatea cimentului, raportul apă/ciment, natura şi granulozitatea agregatelor, modul de compactare, umiditatea mediului ambiant.

În structurile compuse oţel–beton, fenomenele de curgere lentă şi contracţie influenţează comportarea betonului şi produc o scurtare a dalei de beton. Deoarece betonul este legat rigid de oţel, această scurtare este parţial împiedicată şi conduce la o redistribuire a eforturilor unitare în interiorul secţiunii. Redistribuirea eforturilor conduce la o descărcare a betonului şi o creştere a solicitării oţelului în zona comprimată.

Se prezintă mecanismul de dezvoltare a eforturilor din contracţie pe structura unei grinzi compuse oţel–beton simplu rezemate, de tip cheson închis.

Evaluarea parametrilor de calcul, respectiv deformaţiile specifice şi modulele de elasticitate ale betonului, la diferite perioade de la punerea în operă, se face în conformitate cu euronormele EC2 şi EC 4.