CONCRETE SHRINKAGE EFFECT ON THE COMPOSITE
STEEL–CONCRETE STATE OF STRESSES

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Abstract. The size of concrete shrinkage deformation depends, similar to
creep, upon many factors: concrete composition, concrete quality, water/cement
ratio, aggregate nature and grain size, compaction manner, humidity of ambient
environment. In the steel–concrete composite structures, the phenomena of creep
and shrinkage affect concrete behaviour and lead to shortening in the concrete
slab. As concrete is rigidly tied to steel, the shortening is partially stopped and
brings about a redistribution of unit stresses inside the section. The redistribution
of the stresses leads to concrete unloading and increase in steel stress in the
compressed area. The paper presents the mechanism through which shrinkage
stresses develop in the case of a simply supported steel–concrete composite
girder. The calculation parameters, the specific deformations and modulus of
elasticity for concrete at various times since pouring are assessed according to
Eurocodes EC2 and EC 4.

Key words: composite girders; concrete shrinkage; stresses due to
shrinkage.

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1. Introduction

Using two materials with differing mechanical characteristics – steel and concrete – joined continuously together, leads to the occurrence of a state of self-balanced stresses in every section of the composite structure, irrespective of the external loads.

The composite structure slab concrete cannot deform from free shrinkage because the slab is joined together to the metal structure not connecting members.

Consequently, the tendency of shortening the concrete slab is stopped and in the composite structure internal stresses are induced overlapping those produced by shrinkage in the slab area.

The paper presents the mechanism of developing shrinkage stresses in the case of a simply supported steel–concrete composite girder.

The calculation parameters, the specific deformations and modulus of elasticity for concrete at various times since pouring are assessed according to Eurocodes EC2 and EC 4.

2. Stresses Produced by Concrete Shrinkage

The size of concrete shrinkage deformation depends, similar to creep, upon many factors: concrete composition, concrete quality, water/cement ratio, aggregate nature and grain size, compaction manner, humidity of ambient environment. The shrinkage deformations begin to be manifest immediately after concrete putting into place, independent of the size of unit stresses in the concrete.

The mechanism of developing shrinkage stresses is exemplified on the simply supported steel-concrete composite girder.

First, the slab is taken as separate from the metal section, so slab shortening can freely occur with not stresses taking place in the slab (Fig. 1 a).

Slab shortening will be given by relationship:

\[ \Delta L_c = \varepsilon_c L. \]  \hspace{1cm} (1)

To cancel this deformation, it is accepted to introduce a fictitious tensile force \( N_c \) (Fig. 1 b) acting upon all ends of the concrete slab of section area \( A_c \) (in the weight centre), to compensate for the shrinkage deformation

\[ N_c = -\varepsilon_c E_c A_c. \]  \hspace{1cm} (2)

After the first two stages – free shrinkage and fictitious tensile force – the concrete slab does not exhibit deformations and implicitly, no stresses in the joining members occur.
To cancel the fictitious force, \( N_c \), in the weight centre of the slab, a force equal and of opposite sign (compression force) is applied, \( N_m \); this force acts upon the composite section with an eccentricity noted \( z_{cm} \).

![Diagram showing the development of stresses due to concrete shrinkage.](image)

**Fig. 1 – Development of stresses due to concrete shrinkage.**

The eccentric compression force, \( N_m \), is equivalent to a centric compression force (acting upon the weight centre of the composite section) and a bending moment, \( M_m \), which acts upon the composite section.

It means that the following stresses act upon the composite cross section:

\[
N_m = -N_c = \varepsilon_c E_c A_c \quad \text{– compression force;}
\]

\[
M_m = N_m z_{cm} \quad \text{– positive bending moment.}
\]

In short, concrete compression is replaced by a tensile force in the concrete slab and an axial compression force, and a bending moment which acts upon the composite cross section. The overlapping of effects leads to finding the unit stresses in the concrete slab and steel girder.

The stress assessment method for the stresses occurring during concrete shrinkage can be adapted to Eurocodes as follows:

a) Concrete shrinkage, \( \varepsilon_c \), is assessed according to EC 2, respectively it is replaced by \( \varepsilon_{cs} \):

\[
\varepsilon_{cs} = \varepsilon_{cd} + \varepsilon_{ca},
\]

where: \( \varepsilon_{cd} \) is the final deformation due to shrinkage; \( \varepsilon_{cd} \) – the deformation due to shrinkage in time; \( \varepsilon_{ca} \) – the deformation due to initial elastic shrinkage.
b) The concrete modulus of elasticity is replaced by:

\[ E_c = \frac{n_u E_{cm}}{n_S}. \]  

(5)

The equivalence coefficient, \( n_S \), for the effect of shrinkage is assessed with the relationship:

\[ n_S = n_{E,0.55} = n_0 (1 + 0.55 \varphi(t,t_0)) \]

\[ \varphi(t,t_0) = \varphi_0 \beta_{(t,t_0)}; \]

\[ n_0 = \frac{E_e}{E_{cm}}. \]

c) The equivalence coefficient, \( n_{\varphi} \), is replaced by \( n_S \).

It yields:

\[ N_c = -N_m = -\varepsilon_{cs} \frac{n_m E_{cm} A_c}{n_S}. \]  

(6)

The unit stresses in the concrete slab and metal girder:

1. *In the concrete:*

\[ \sigma_c = -\frac{N_c}{A_c} + \frac{1}{n_S} \left( \frac{N_m}{A_m} + \frac{M_m}{I_m} z \right); \]  

(7)

2. *In the steel:*

\[ \sigma_a = \frac{N_m}{A_m} + \frac{M_m}{I_m} z. \]  

(8)

\( A_m \) and \( I_m \) represent the area and the inertia moment of the composite concrete–steel section where the equivalence coefficient taken is \( n_S \).

3. **Numerical Example**

The state of stresses is assessed in the cross section of a steel-concrete composite girder following the shrinkage in the slab.

The following data are known:

a) composite girder cross section (Fig. 2);

b) concrete grade: C40/50;

c) S420 steel.

The web of the metal girder is provided with longitudinal stiffening bars so that the grade for the section is 3.

**SOLUTION**

Taking into account the building stages (use of intermediate supports to pour the concrete slab), the shrinkage deformation is calculated before
removing supports (piles), respectively at 28 days and the shrinkage deformation in infinite time (practically at 100 years).

Fig. 2 – Cross section of composite girder.

i) Calculus of shrinkage deformation

The initial elastic deformation (due to endogenous shrinkage):

\[
\varepsilon_{ed}(t) = \beta_{as}(t)\varepsilon_{ed}(\infty) = 0.65 \times 75 \times 10^{-6} = 4.875 \times 10^{-5};
\]

\[
\varepsilon_{ed}(\infty) = 2.5(f_{ck} - 10)10^{-6} = 2.5(40 - 10)10^{-6} = 75 \times 10^{-6};
\]

\[
\beta_{as}(t) = 1 - e^{-\alpha t^\beta} = 1 - e^{-0.2\sqrt{t}} = 0.65.
\]

The deformation due to shrinkage during drying in time is

\[
\varepsilon_{ed}(t) = \beta_{as}(t,t_s)\varepsilon_{ed,\infty} = \beta_{as}(t,t_s)k_{cd} = 0.092 \times 0.72 \times 2.4 \times 10^{-6} = 1.6 \times 10^{-5};
\]

\[
\beta_{as}(t,t_s) = \frac{(t-t_s)}{(t-t_s) + 0.04\sqrt{h_0}} = \frac{(28-1)}{(28-1) + 0.04 \times \sqrt{400^3}} = 0.092
\]

(it is supposed that the concrete age \(t_s\) at which shrinkage begins is of one day)

\(\varepsilon_{ed,0} = 0.24 \times 10^{-3} = 2.4 \times 10^{-4}\) (Table 3, Annex 2, for RH = 80%)

\(h_0 = 2A_c / u = 2 \times 600 \times 20 / 600 = 40\) cm = 400 mm \(\Rightarrow k_b = 0.72\) (EC2).

The deformation from shrinkage in the moment of traffic opening \((t = 28\) days) is:

\[
\varepsilon_{cs}(t) = \varepsilon_{ed}(t) + \varepsilon_{ed}(t) = 1.6 \times 10^{-5} + 4.875 \times 10^{-5} = 6.475 \times 10^{-5} \approx 0.65 \times 10^{-4}.
\]
The deformation arising from final shrinkage \((t = \infty)\) will be

\[ \varepsilon_{cs}(\infty) = \varepsilon_{cd}(\infty) + \varepsilon_{ca}(\infty) = 1.728 \times 10^{-4} + 0.75 \times 10^{-4} \approx 2.5 \times 10^{-4}. \]

It is noticed that 74% of the deformation occurs by drying and 26% by concrete setting.

The slab deformation calculated infinitely is used in combinations of actions to check the composite girder.

ii) Calculus of equivalence coefficient

a) According to SR EN 1994-2:

The equivalence coefficient for short term loads:

\[
E_{cm} = \frac{210,000}{22,000 \left( \frac{f_{cm}}{10} \right)^{0.3}} = \frac{210,000}{22,000 \left( \frac{48}{10} \right)^{0.3}} = \frac{210,000}{35,220} = 5.96.
\]

The equivalence coefficient for other loads:

\[ n_0 = n_0 \left[1 + \Psi_L \varphi(t,t_0)\right] \]

where: \(\Psi_L\) is the multiplier for the creep:

\[
\Psi_L = \begin{cases} 
1.1 & \text{permanent loads;} \\
0.55 & \text{shrinkage effects;} \\
1.5 & \text{imposed deformations,}
\end{cases} \]

\(\varphi(t,t_0)\) – slow flow coefficient.

The creep coefficient, \(\varphi(t,t_0)\): creep occurs in the interval between time \(t_0\) to time \(t\).

The relative humidity, \(RH = 80\%\) is taken into consideration

\[
\varphi_{RH} = \left[1 + \frac{1 - RH/100}{0.1\sqrt{h_0}}\right] \alpha_1 = \left[1 + \frac{1 - 80/100}{0.1\sqrt{400}}\right] 0.94 = 1.14;
\]

\[
\alpha_1 = \frac{35}{f_{cm}} = 0.80; \quad \alpha_2 = \frac{35}{f_{cm}} = 0.94; \quad \beta(f_{cm}) = \frac{16.8}{\sqrt{f_{cm}}} = 2.42;
\]
\[
\beta(t_0) = \frac{1}{0.1 + t_0^{0.2}} = \frac{1}{0.1 + 1} = 0.909; \quad t_0 = 1 \text{ day};
\]

\[
\beta_{c(t_0)} = \left( \frac{t - t_0}{\beta_H + t - t_0} \right)^{0.3} = 1 \quad \text{for} \quad t = \infty;
\]

\[
\varphi_0 = \varphi_{RH} \beta(f_{cm}) \beta(t_0) = 1.14 \times 2.42 \times 0.909 = 2.50;
\]

\[
\varphi(t, t_0) = \varphi_0 \beta_{c(t_0)} = 2.50 \times 1 = 2.50.
\]

b) The equivalence coefficient for shrinkage:

\[
n_S = n_{L(\varphi=0.55)} = n_0 \left[1 + 0.55 \varphi(t, t_0) \right] = n_0 (1 + 0.55 \times 2.50) = 2.375 n_0 = 14.15.
\]

iii) Evaluation of stresses produced after shrinkage

The results reached by applying SR EN 1994-2 will be used:

\[
n_0 = 5.96; \quad n_S = 2.375 n_0 = 14.15; \quad \epsilon_{cs} = 2.5 \times 10^{-4}; \quad E_{cm} = 35,220 \text{ N/mm}.
\]

a) Steel equivalent section

Steel equivalent width of the concrete slab:

\[
b_{ei}^* = \frac{b_{ei}}{n_S} = \frac{6,000}{14.15} = 424 \text{ mm}.
\]

The steel equivalent composite section is represented in Fig. 3.

![Cross section characteristics](image)

The axial stress developed from shrinkage will be:

\[
N_{c} = -N_m = -\epsilon_{cs} \frac{n_0}{n_S} E_{cm} A_c = -2.5 \times 10^{-4} \times \frac{1}{2.375} \times 0.3522 \times 10^6 \times 12,000;
\]

\[
N_{c} = -N_m = -449 \times 10^3 \text{ daN}.
\]
The bending moment:

\[ M_m = N_m z_{cm} = 449 \times 10^3 \times 79.7 = 357.85 \times 10^5 \text{ daN.cm} \]

The unit stresses found from the shrinkage phenomenon are:

a) in the concrete:
   a1) in the upper fibre:
   \[
   \sigma_{cs up} = -\frac{N_c}{A_c} + \frac{1}{n_f} \left( N_m \frac{M_m}{I_m} z_{cs} \right) = \]
   \[
   = \frac{449 \times 10^3}{12,000} + \frac{1}{14.15} \left( \frac{449 \times 10^3}{2,768} + 357.85 \times 10^5 \right) = -14.6 \text{ daN/cm}^2;
   \]

   a2) in the lower fibre:
   \[
   \sigma_{cs inf} = -\frac{N_c}{A_c} + \frac{1}{n_f} \left( N_m \frac{M_m}{I_m} z_{ci} \right) = \]
   \[
   = \frac{449 \times 10^3}{12,000} + \frac{1}{14.15} \left( \frac{449 \times 10^3}{2,768} + 357.85 \times 10^5 \right) = -17.1 \text{ daN/cm}^2;
   \]

b) in the steel:
   b1) in the upper fibre:
   \[
   \sigma_{as up} = \frac{N_m}{A_m} + \frac{M_m}{I_m} z_{as} = \frac{449 \times 10^3}{2,768} + 357.85 \times 10^5 \]
   \[
   = 286.9 \text{ daN/cm}^2;
   \]

   b2) in the lower fibre:
The unit stress diagram arising from shrinkage is given in Fig. 4.

4. Final Remarks and Conclusions

The magnitude of concrete deformation due to shrinkage depends, similarly to creep, upon numerous factors, such as: concrete composition, concrete quality, water/cement ratio, aggregates nature and grain size, compaction, humidity of surrounding environment.

The phenomena of creep and shrinkage influence the concrete behaviour and produce a shortening of the concrete slab. As concrete is stiffly tied to steel, the shortening is partially prevented and leads to the redistribution of stresses inside the cross section.

The redistribution of stresses leads to concrete discharge and increase of steel tensioning in the compressed area.

Calculations show that about 75% of the deformation takes place by drying and about 25% , during concrete setting.

The slab deformation calculated for the infinite time is used in combinations of actions during composite girder check.

The shrinkage influence cannot be neglected when calculating the connectors in sections where the sliding forces at the contact between the concrete slab and the steel girder are high.

According to EC4, the effects of shrinkage and creep of the concrete slab can be neglected in the calculations for the ultimate limit states other than fatigue for members whose all cross sections belong to Class 1 or Class 2 and where no approximations are necessary for the lateral buckling, in the limit states of normal service.

REFERENCES

EFECTUL CONTRACŢIEI BETONULUI ASUPRA STĂRII DE EFORȚURI DIN STRUCTURILE COMPUSE OȚEL–BETON

(Rezumat)

Mărimea deformației betonului din contracție depinde, ca și pentru curgerea lentă, de numeroși factori: compoziția betonului, calitatea cimentului, raportul apă/ciment, natura și granulozitatea agregatelor, modul de compactare, umiditatea mediului ambient.

În structurile compuse oțel–beton, fenomenele de curgere lentă și contracție influențează comportarea betonului și produc o scurtare a dalei de beton. Deoarece betonul este legat rigid de oțel, această scurtare este parțial împiedicată și conduce la o redistribuire a eforturilor unitare în interiorul secțiunii. Distribuirea eforturilor conduce la o descărcare a betonului și o creștere a solicitării oțelului în zona comprimată.

Se prezintă mecanismul de dezvoltare a eforturilor din contracție pe structura unei grinzi compuse oțel–beton simplu rezemate, de tip cheson închis.

Evaluarea parametrilor de calcul, respectiv deformațiile specifice și modulele de elasticitate ale betonului, la diferite perioade de la punerea în operă, se face în conformitate cu euronormele EC2 și EC 4.