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DIRECT STRESSES ON LAYERS OF A SANDWICH BEAM WITH FOAM CORE

BY

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Abstract. The study investigates the variation of direct stresses on layers of a sandwich beam, considering the effect of increasing the thickness and the foam core density. The paper presents the numerical modeling of multi layered sandwich beams, with thin identical steel faces and rigid polyurethane foam core with density of 40, 60, 80 and 100 kg/m^3, loaded in bending, in static conditions. Assumptions like linear elastic behavior of the faces and core and also the isotropy of the layers' materials are accepted in the numerical simulations. The obtained results are centralized in table and the variation of direct stresses are illustrated with suggestive diagrams.

Key words: direct stresses; sandwich elements; foam core; numerical modeling.

1. Introduction

Sandwich elements with external steel facings and polyurethane foam core offer many advantages compared to the elements made from traditional materials. Therefore, they are widely used as cladding and structural elements in civil and industrial engineering (Țăranu, 1978).

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The purpose of this analysis is to study the influence of the core and the variation of direct stresses on layers of a sandwich beam with foam core, by alternating the thickness and the density of the intermediate layer.

Behavior of sandwich elements with foam core subjected to different loading actions is very complex and it is substantially different than the classical solutions with homogeneous materials. For this reason, it has been the focus of recent studies and numerical analysis.

For example, flexural tests of sandwich elements on transverse direction are carried out on specimens with standardized dimensions, according to ASTM standards, having as main purpose the determination of the overall bending stiffness of the multi layered panels (Marta, 2007).

The flexural behavior of sandwich panels made of polyurethane core and glass fiber reinforced polymer skins and ribs was a theme of research (Tarek, 2010).

The load-deflection behavior of a sandwich plate under static loading conditions has been analysed by (Feldhusen *et al.*, 2008), using the four-point bending test. The linear elastic behavior and the different failure of sandwich plate conditions have also been investigated.

2. Distribution of Direct Stresses on the Layers of the Sandwich Beam

The distribution of the direct stresses along the cross section of the sandwich beam is based on the general theory of flexure, adapted to the multi-layered structure of the sandwich element (Allen, 1969).

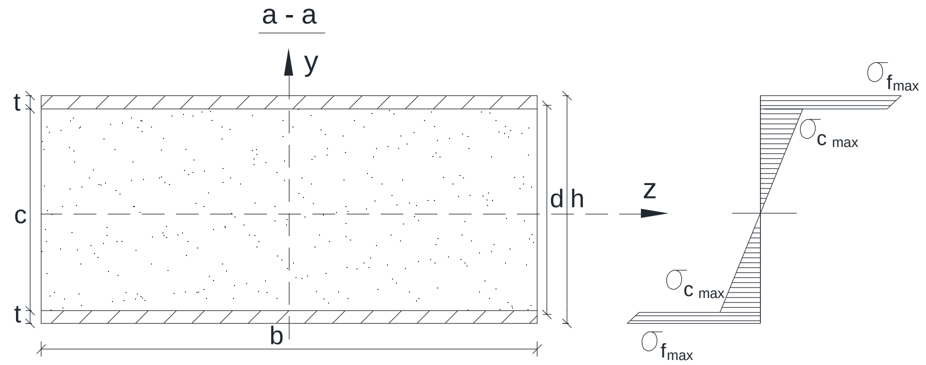

Fig. 1 – The distribution of direct stresses on the layers of the sandwich beam.

The direct stresses on the core thickness can be calculated with the following relation:

$$\sigma_c = \frac{ME_c}{D} \cdot z \text{, for } \left[-\frac{c}{2} \le z \le \frac{c}{2} \right], \tag{1}$$

where: σ_c is the direct stress in the core, [N/mm^2]; E_c – the longitudinal Young's modulus of the core material, [N/mm^2]; M – the bending moment, [N.mm]; D – the stiffness of the sandwich beam, [N·mm^2]; c – the thickness of the core, [mm].

The direct stresses in the facings can be calculated with the following relation:

$$\sigma_f = \frac{ME_f}{D} \cdot z \text{, for } \left[-\frac{c}{2} \le z \le -\frac{h}{2} \text{ and } \frac{c}{2} \le z \le \frac{h}{2} \right], \tag{2}$$

where: σ_f is the normal stress in the facings, [N/mm^2]; E_f – the longitudinal Young's modulus in the facings, [N/mm^2]; h – the height of the sandwich beam, [mm].

The maximum direct stresses in the core can be computed with the relation:

$$\sigma_{c_{\max}} = \pm \frac{ME_c}{D} \cdot \frac{c}{2}. \tag{3}$$

The maximum direct stresses in the facings can be determined with the following relation:

$$\sigma_{f_{\max}} = \pm \frac{ME_f}{D} \cdot \frac{h}{2}. \tag{4}$$

3. Numerical modeling

The numerical modeling enables the analysis of the variation of direct stresses on layers of the sandwich beam, depending on the thickness and the density of the core. This analysis is realized by changing the thickness of the intermediate layer and also by modifying the density of the polyurethane foam core. The numerical simulations assumptions are based on static loading conditions, on linear elastic behavior of the faces and core and on the isotropy of the materials.

3.1 Geometry description

A simply supported sandwich beam of span L, acted by a concentrated load P at the mid span is considered (Fig. 2). The sandwich element is composed by two identical thin external layers made of steel, of thickness t, and an internal thick layer of polyurethane foam, of thickness c. The cross section **a-a** of the sandwich beam is shown in Fig. 1. The width of the cross section is b, while the height of the section is h. The distance between the neutral axes of the faces is d.

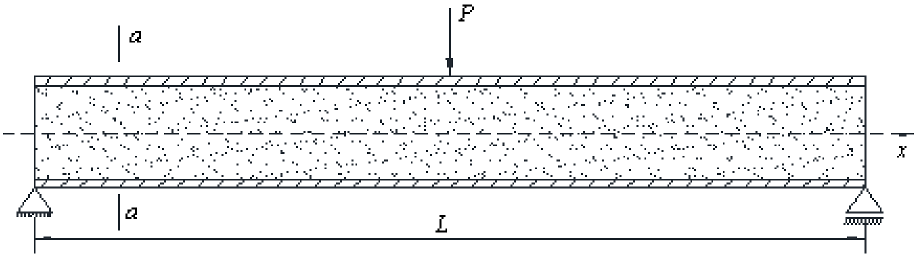

Fig. 2 – The static scheme and the loading conditions of the sandwich beam.

a) *Geometrical characteristics*

The geometrical characteristics of the sandwich beam used in the numerical modeling have constant values, except for the thickness of rigid polyurethane foam core and implicitly the distance between the neutral axes of the faces. The geometrical characteristics are presented below:

L = 1440 mm; b = 200 mm; t = 1 mm;
c = 40 mm; 60 mm; 80 mm; 100 mm;
d = 41 mm; 61 mm; 81 mm; 101 mm.

b) *Flexural rigidity*

According to the presented geometrical characteristics, the sandwich beam with $d << b$ is considered a wide beam, therefore the elastic modulus of the materials will be computed with the following relation:

$$E' = \frac{E}{1-\nu^2}, \tag{5}$$

where: E is the longitudinal Young's modulus of the material, [N/mm^2]; and ν – the Poisson's ratio.

Therefore, the flexural rigidity of the sandwich beam is defined according to the relation:

$$D = 2E_f^{'}\left(\frac{bt^3}{12}+\frac{btd^2}{4}\right)+E_c^{'}\frac{bc^3}{12}, \tag{6}$$

where: D is the flexural rigidity of the sandwich beam, [N·mm^2]; $E_f^{'}$ – the longitudinal Young's modulus in the facings, [N/mm^2]; $E_c^{'}$ – the longitudinal Young's modulus of the core material, [N/mm^2].

c) *Materials*

The sandwich element is composed of two thin steel facings and a continuous polyurethane foam core. The numerical modeling of the layered sandwich beam is realized by selecting the polyurethane foam densities, with values from 40 to 100 kg/m^3, that also modifies the elastic moduli of the layers materials and implicitly the flexural rigidity of the sandwich beam. A major influence on the stiffness of the multi-layered element has the thickness of the core, because while increasing the distance between the facings, the flexural rigidity of the beam is also increased.

Table 1
The Characteristics of the Materials Used in the Numerical Modeling

Materials for sandwich layers	ν	E N/mm^2	E' N/mm^2	ρ kg/m^3
Steel	0.3	2.10×10^5	2.30×10^5	7850
Polyurethane 40	0.33	1.63	1.83	40
Polyurethane 60	0.33	3.79	4.25	60
Polyurethane 80	0.33	10.65	11.95	80
Polyurethane 100	0.33	19.12	21.45	100

d) *Loads*

The sandwich beam is subjected to bending, acted at the mid span, by a concentrated load equal with 3,000 N, in the vertical direction, perpendicular to the longitudinal axis of the beam (Fig. 2).

e) *The geometry and the mesh of the model*

The geometry and the mesh of the model presented in the Figs. 3 and 4 are realized using the specialised software ANSYS Design Modeler and ANSYS Meshing. Fig. 3 presents the geometry of the sandwich beam, with respect to the geometrical characteristics and the materials described before**.**

The mesh of the model is realized with a fine rectangular grid: 123,019 nodes and 23,040 cells (Fig. 4).

3.2. The Analysis of the Numerical Results

The numerical results present the values for the direct stresses, for the 16 numerical models. The sandwich beam with the thickness of 60 mm and the density of the polyurethane core of 80 kg/m^3, shown in Fig. 5, is representative to ilustrate the deformed shape.

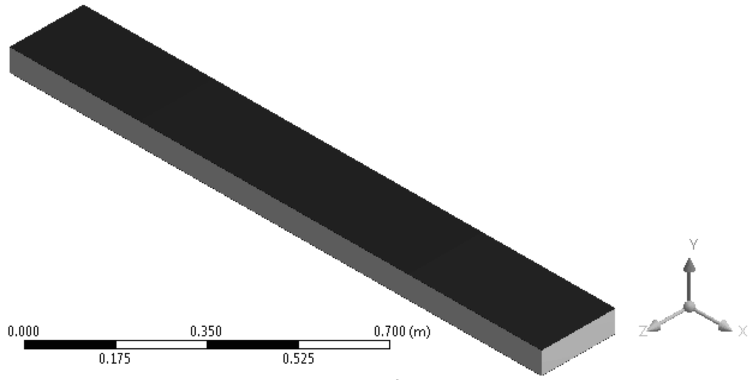

Fig. 3 – The geometry of the sandwich beam.

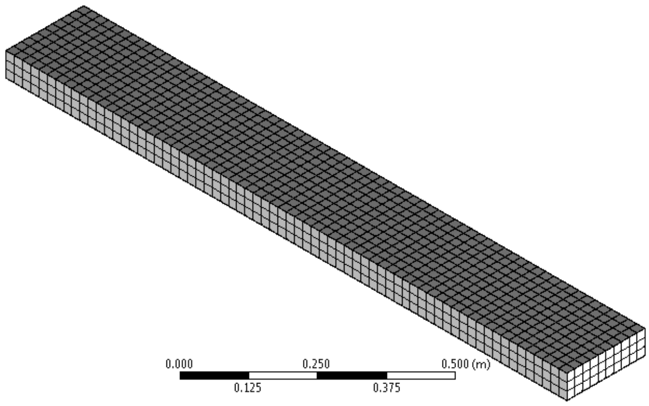

Fig. 4 – The mesh of the computing model.

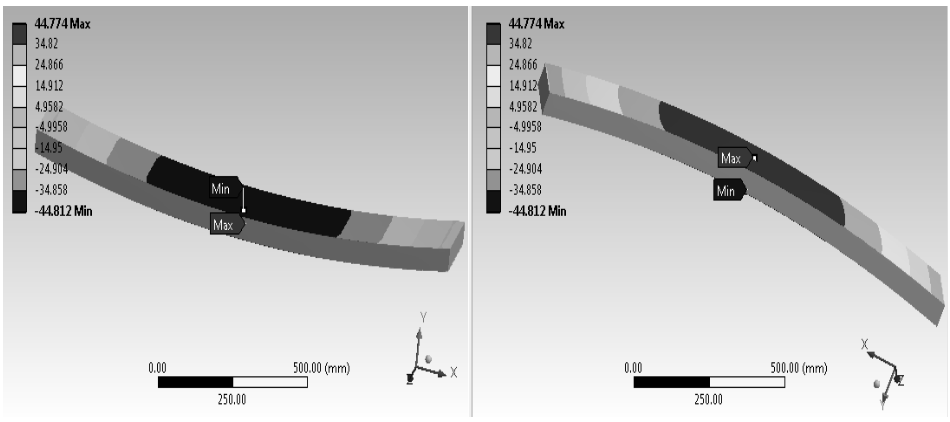

Fig. 5 – Numerical results for the direct stresses of the beam.

According to the loading conditions and supports (Fig. 2), the distribution of direct stresses on layers of a sandwich beam is realistic, with maximum values in faces and reduced in the intermediate layer, with a zero value at the neutral axis level. The superior face of the sandwich element generates direct compressive stresses, while the inferior face presents direct tensile stresses. Moreover, the maximum direct stresses occur at the midspan of the element, where the bending moment is maximum, suggesting that the numerical model follows the studied theories of strength of materials.

The values for the maximum direct stresses of the sandwich beams resulted from the numerical modeling are centralised in Table 2.

Table 2

Maximum Direct Stresses of the Sandwich Beams, $|\sigma|_{\max}$ MPa

Thickness of the core *c*, [mm]	Density ρ, [kg/m^3]			
	40	60	80	100
40	118.08	84.688	67.975	60.643
60	74.761	56.539	44.812	39.585
80	56.339	42.59	33.961	30.086
100	45.165	34.189	27.077	23.851

The values presented in Table 2 show a decrease of the maximum direct stresses, while increasing the thickness and the density of the core. This aspect is ilustrated also in the following diagrams. Fig. 6 presents the variation of the maximum direct stresses of the sandwich element, depending on the thickness of the core, for different densities of the polyurethane foam, while Fig. 7 illustrate the variation of the maximum normal stresses for different thicknesses of the intermediate layer.

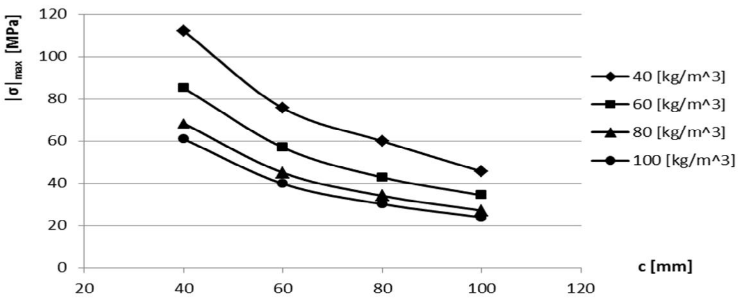

Fig. 6 – The variation of the maximum direct stresses, function of the thickness of the core, for different densities of the polyurethane foams.

For the thicknesses and the densities adopted, it is noticed that the thickness of the core influence more than density the decreasing of the direct stresses.

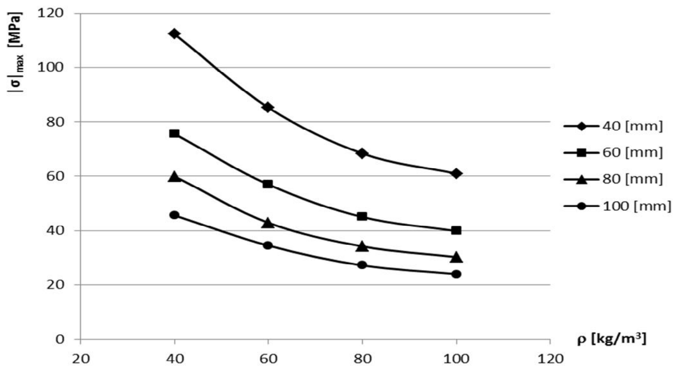

Fig. 7 – The variation of the maximum direct stresses, function of the density of the polyurethane foam, for different thicknesses of the core.

4. Conclusions

The results reveal a significant decrease of the direct stresses, while increasing the thickness and the density of the polyurethane foam. This study indicates that the thickness of the core influence more than density the decreasing of the direct stresses on the layers of the sandwich beam.

The variation of density of the polyurethane foam core modifies the elastic moduli of the layers' materials and implicitly the flexural rigidity of the sandwich beam. Moreover, the thickness of the core has a major influence on the flexural rigidity of the multi-layered element, following the principles of the I-beam analogy.

As a consequence of increasing the foam core density, a brittle behaviour regarding failure and fracture of the sandwich beams is sustained by the specialized literature (Nasirzadeh, 2014).

Recent appropriate studies of the authors (Dupir, 2014) mention the comparative analysis of sandwich beams with different characteristics of the intermediate layer, focused on the influence of the core upon the rigidity and the weight of the sandwich elements.

Further researches can take into consideration the non linear characteristics and the anisotropy of the materials, to obtain more accurate results.

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TENSIUNI NORMALE IN STRATURILE UNEI GRINZI SANDVIŞ CU MIEZ CONTINUU

(Rezumat)

Scopul studiului este de a investiga variația tensiunilor normale în straturile unei grinzi sandviş, sub efectul creşterii grosimii şi densității miezului. Se prezintă formularea teoretică şi modelarea numerică pentru evaluarea tensiunilor normale ale unei grinzi stratificate de tip sandviş, cu fețe exterioare subțiri din oțel şi miez din spumă rigidă poliuretanică, cu densitatea de 40, 60, 80 şi 100 kg/m^3, solicitată la încovoiere, în condiții de încărcare statică. Rezultatele numerice obținute sunt centralizate în tabel, iar variația tensiunilor normale este ilustrată prin diagrame.