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THEORETICAL AND NUMERICAL INVESTIGATION REGARDING THE BENDING BEHAVIOUR OF SANDWICH PLATES

BY

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Abstract. The paper presents a review of the main theories utilized in the bending behavior analysis of inhomogeneous sandwich plates, as well as their limits of application. Seeing how in the current practice, sandwich panels are made from various materials and with great dimensional variety, it is necessary to investigate, both numerically and experimental, in order to adequately establish the bending response parameters of the above mentioned plates. The numerical analysis carried out refer to the maximum transverse deflection of a sandwich panel, with simply supported boundary conditions, and uniformly loaded. The total thickness of the panel remains constant, but the thickness of the extruded polystyrene core and that of the aluminum facings vary. The results obtained through the use of numerical methods (FDM, FEM) are compared with the analytical solutions available in literature. Finally, comments are made related to the design of these types of structures.

Key words: plate theories; sandwich composite; governing equations; flexural deflection; numerical methods.

1. Introduction

Sandwich panels are used extensively in numerous technical and industrial areas, whenever the structures are required to be light, but also

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strength efficient: aerospace industry, shipbuilding industry, offshore platforms, transportation industry, sporting and recreational equipment, etc. (Plantema, 1966; Vinson, 1999, 2005; Newaz *et al.*, 2013). In the construction industry, sandwich panels are used as phonic and thermo-isolators and as claddings and roofs for steel buildings (production and storage facilities, sheds, showrooms, business centers, hypermarkets, malls, buildings with social, cultural and sporting destinations etc.), as well as interior partitioning for production spaces in various industries, the creation of cold rooms etc. (www.panoterm.ro; www.topanel.ro; panouri.kingspan.ro; Avó de Almeida, 2009).

American Society for Testing and Materials (ASTM) defines a sandwich structure as follows: "A structural sandwich is a special form of a laminated composite comprising of a combination of different materials that are bonded to each other so as to utilize the properties of each separate component to the structural advantage of the whole assembly" (Birman, 2010; Newaz *et. al.*, 2013; www.angelfire.com). ASTM standard C274/C274M covers the terminology necessary for a basic uniform understanding and usage of the language peculiar to structural sandwich constructions.

Typically a sandwich composite consist of three main parts: two thin, stiff and strong faces (skins), separated by a thick, light and weaker core. The faces are adhesively bonded to the core to obtain a load transfer between the components (www.angelfire.com).

Commonly used materials for the skins are metallic (steel, stainless steel and aluminum alloys) and nonmetallic (plywood, reinforced plastic, fiber composites, cement, veneer etc.).

The cores used in load carrying sandwich constructions can be divided into four main groups: corrugated, honeycomb, balsa wood and foams (polyurethane, polystyrene produced by expansion or by extrusion etc.). A variety of adhesives can be used for special purpose bonding: epoxy resins, phenolics, polyurethanes, urethane acrylates, polyester and vinylester resins.

The skin material usually has a high stiffness, whereas the core typically has high compressive and shear strength. When these are bonded together, this combination gives the sandwich structure a high flexural modulus (www.twi_global...).

2. Plate Theories Used for the Analysis of Sandwich Plates

An overview of main plate theories and their limits of applications can be found in (Wang *et al.*, 2000; Qatu, 2004; Reddy, 2004).

The classical laminate theory (CLT) and the first-order shear deformation theory (FSDT) are the most commonly used theories for analyzing laminated or sandwich beams, plates and shells in engineering applications (Altenbach *et al.*, 2004; Pandya & Kant, 1988; Bari & Bajaj, 2014).

Classical laminate theory is a widely accepted macromechanical approach for the determination of the mechanical behavior of composite laminate (Sezgin, 2008).

A typical laminate consist of two or more laminae bonded together to behave as an integral structural element (Fig. 1).

Laminated composite materials are generally orthotropic and have very good properties in the direction of the reinforcing fibers, but weak properties transverse to the fibers (Sezgin, 2008). As a consequence, the laminas are oriented in different directions, so the properties of the whole laminate match the design requirements.

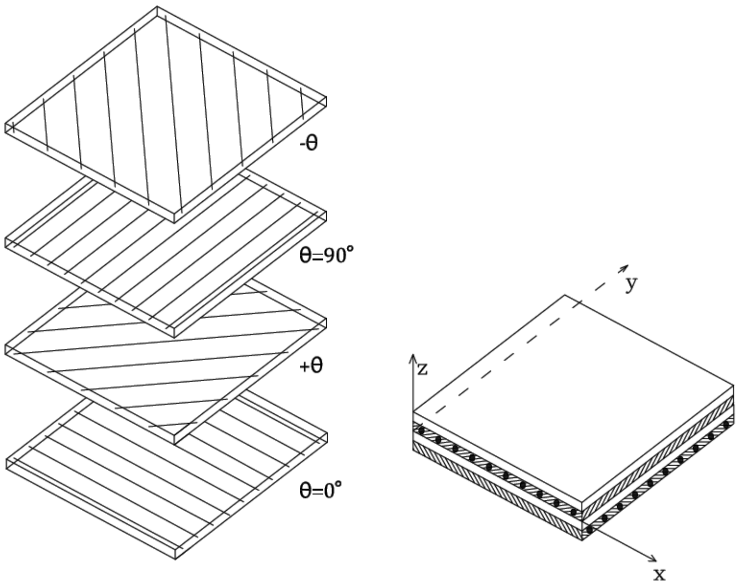


Fig. 1 – A typical laminate with different fiber orientations.

The stiffness of the composite laminate is derived from the properties of its constituent laminas. The procedure implies the analysis of the laminate, whose laminas have various orientations about the natural coordinate system or a chosen one. As a result, the general behavior of a multidirectional laminate is a function of the properties and the stacking sequence of its individual laminas.

Knowing the strain and stress profile on the laminate's thickness is crucial in order to define the flexural stiffness of the laminate. One of the hypotheses of CLT is that the laminas are perfectly glued together, but the adhesive layer is infinitesimally thin. As a result, the displacements are continuous between the lamina skins, so the lamina cannot slip and the laminate behaves as a whole.

The internal forces and moments of a laminate are obtained by integrating the stresses on each lamina across the laminate thickness.

The CLT is an extension of Kirchhoff's classical plate theory for homogeneous isotropic plates to laminated composite plates with a reasonably high width-to-thickness ratio (Altenbach *et al*., 2004).

Equations governing shear deformation theories are typically more complicated than those of the classical theory. Hence, it is desirable to establish exact relationships between solutions of the classical theory and shear deformation theories so that whenever classical theory solutions are available, the corresponding solutions of shear deformation theories can be readily obtained (Wang *et al.*, 2000). Thus, knowing the plate deflection in the classical bending theory (Kirchhoff), the deflection in the first order shear deformation theory (Mindlin) can be expressed, both for the homogeneous plate (single layer) and for the sandwich plate.

2.1. Bending Behavior of Isotropic Plates in Classical Plate Theory (CPT)

For homogeneous isotropic plates, Kirchhoff's theory is limited to thin plates with ratios of maximum plate deflection w to plate thickness $h < 0.2$ and plate thickness/minimum in-plane dimensions < 0.1 (Altenbach *et al*., 2004).

The response parameter of isotropic plates in bending is the flexural deflection, w, which in the classical theory is obtained from the biharmonic equation (Timoshenko & Woinowski-Krieger, 1968; Wang *et al.*, 2000):

$$\nabla^2\nabla^2 w = \frac{q}{D}, \tag{1}$$

where: ∇^2 is Laplace operator, which can be written in terms of the rectangular coordinates (x, y) for polygonal plates

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}, \tag{2}$$

q – the transverse load and D – the flexural rigidity of the plate:

$$D = \frac{Eh^3}{12(1-\upsilon^2)}, \tag{3}$$

where E is the Young modulus of the material, υ – Poisson's ratio, h – the thickness of the plate.

It should be noted that the deflection in any point of the plate is equal to the deflection of the middle plane ($z = 0$), as a consequence of the hypothesis of the normal, linear and inextensible segment (Kirchhoff), therefore:

$$w(x, y, z) = w_0(x, y)\,. \tag{4}$$

The well-known equation for isotropic Kirchhoff plate bending problem (1) can be written as a pair of Poisson equations (Wang *et al.*, 2000; Vrabie & Ungureanu, 2012):

$$\nabla^2 M = -q \quad \text{a)}; \qquad \nabla^2 w = -\frac{M}{D} \quad \text{b)}; \tag{5}$$

where: M is the moment sum (Marcus moment).

$$M = \frac{M_x + M_y}{1+\upsilon}\,. \tag{6}$$

In eq. (6) M_x, M_y are the bending moments in rectangular coordinates.

In the case of polygonal Kirchhoff plates with straight simply supported edges, the boundary conditions associated with Eqs. (5a) and (5b) are given by:

$$w = 0; \quad M = 0\,. \tag{7}$$

On a fixed (or clamped) edge the boundary conditions are given by:

$$w = 0; \quad \frac{\partial w}{\partial n} = 0\,, \tag{8}$$

and for an unloaded free edge:

$$V_n^* = V_n + \frac{\partial M_{ns}}{\partial s} = 0; \quad M_n = 0\,. \tag{9}$$

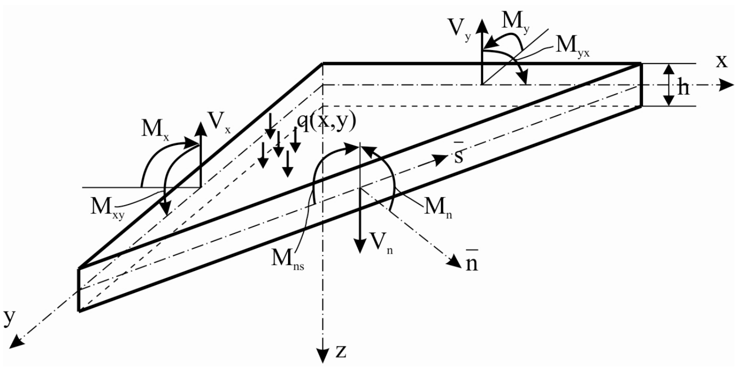


Fig. 2 – The internal moments and forces and the loading on a plate element.

In conditions (8) and (9), for generality, the direction of the normal to the edge was denoted with “n”, “s” is the parallel direction to the edge, V_n^* is the generalized shear force, V_n , M_n , M_{ns} are the shear force, bending moment and the torsion moment, respectively, on the edge with the "*n*" normal (Fig. 2).

If the flexural displacement w is determined by integrating eq. (1), the internal moments and shear forces can be written:

$$M_x = -D\left(\frac{\partial^2 w}{\partial x^2} + \nu\frac{\partial^2 w}{\partial y^2}\right); \quad M_y = -D\left(\frac{\partial^2 w}{\partial y^2} + \nu\frac{\partial^2 w}{\partial x^2}\right);$$

$$M_{xy} = M_{yx} = T = -D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}; \tag{10}$$

$$V_x = -D\frac{\partial}{\partial x}\nabla^2 w; \quad V_y = -D\frac{\partial}{\partial y}\nabla^2 w.$$

2.2. First Order Shear Deformation Theory (FSDT) for Sandwich Plates

Consider a general polygonal shaped (Fig. 3 *a*) sandwich plate. The facings of thickness h_f and the core of thickness h_c (Fig. 3 *b*) are made of isotropic materials, having the following elastic constants: E – Young's modulus of elasticity, ν – Poisson's ratio, and shear modulus, G, identified by the subscript index "*f* " for the facings and "*c*" for the core.

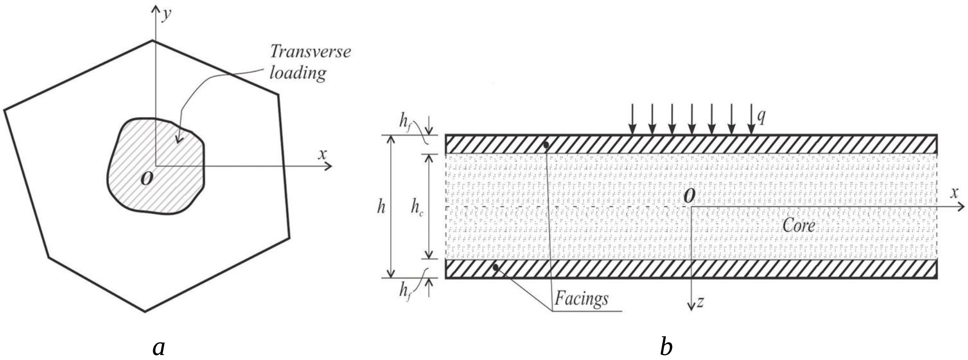


Fig. 3 – Geometry and loading of a sandwich plate: *a* – general polygonal shape; *b* – plate cross-section.

In FSDT (Mindlin), the transverse shear deformation is considered to be continuous and constant on the thickness. As a result, a linear and normal segment on the middle plane remains linear and inextensible after deformation, but it will rotate after deformation with the angles φ_x and φ_y from the initial position. Considering the relative small thickness of the two facings, the hypothesis of equal rotation of the core and of the facings is accepted. Under these conditions, for the polygonal shaped sandwich plates the moment-displacement relations are given by (Wang *et. al.*, 2000):

$$
\begin{aligned}
M_x &= (D_c + D_f)\frac{\partial \varphi_x}{\partial x} + (\upsilon_c D_c + \upsilon_f D_f)\frac{\partial \varphi_y}{\partial y};\\
M_y &= (D_c + D_f)\frac{\partial \varphi_y}{\partial y} + (\upsilon_c D_c + \upsilon_f D_f)\frac{\partial \varphi_x}{\partial x};\\
M_{xy} &= \frac{1}{2}\left[(1-\upsilon_c)D_c + (1-\upsilon_f)D_f\right]\left(\frac{\partial \varphi_x}{\partial y} + \frac{\partial \varphi_y}{\partial x}\right);\\
V_x &= K_s\left(G_c h_c + 2G_f h_f\right)\left(\varphi_x + \frac{\partial w}{\partial x}\right);\\
V_y &= K_s\left(G_c h_c + 2G_f h_f\right)\left(\varphi_y + \frac{\partial w}{\partial y}\right),
\end{aligned}
\tag{11}
$$

where the flexural stiffness of the core, D_c, and of the facings, D_f, are:

$$
D_c = \frac{E_c h_c^3}{12(1-\upsilon_c^2)};\quad D_f = \frac{2E_f h_f\left(\dfrac{3h_c^2}{4} + \dfrac{3h_c h_f}{2} + h_f^2\right)}{3(1-\upsilon_f^2)},
\tag{12}
$$

K_s is the shear correction factor (coefficient), typically taken at 5/6 (Timoshenko & Woinowski-Krieger, 1968; Wang *et al.*, 2000; Qatu, 2004). The analysis presented in the (Birman&Bert, 2002) paper results in the conclusion that K_s should be taken equal to unity, as a first approximation, for both two-skin as well as for multi-skin sandwich structures.

The moment sum of the Mindlin plate theory from eq. (6) becomes:

$$
M = \frac{M_x + M_y}{(1+\upsilon_c)D_c + (1+\upsilon_f)D_f} = \frac{\partial \varphi_x}{\partial x} + \frac{\partial \varphi_y}{\partial y}.
\tag{13}
$$

The equilibrium equations of a sandwich plate according to FSDT are the same as those of CPT (Timoshenko & Woinowski-Krieger, 1968; Wang *et al.*, 2000; Szilard, 2004):

$$
\begin{gathered}
\frac{\partial V_x}{\partial x} + \frac{\partial V_y}{\partial y} = -q \quad a);\quad \frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} - V_x = 0 \quad b);\\
\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} - V_y = 0 \quad c).
\end{gathered}
\tag{14}
$$

By substituting into eqs. (14) the moments and shear forces from eqs. (11), using the moment sum from eq. (13), and after some processing, the following relationships are obtained:

$$K_s\left(G_c h_c + 2G_f h_f\right)\left(\phi_x + \frac{\partial w}{\partial x}\right) = \left(D_c + D_f\right)\frac{\partial^2\phi_x}{\partial x^2} +$$
$$+\left(\upsilon_c D_c + \upsilon_f D_f\right)\frac{\partial^2\phi_y}{\partial x\partial y} + \frac{1}{2}\left[\left(1-\upsilon_c\right)D_c + \left(1-\upsilon_f\right)D_f\right]\left(\frac{\partial^2\phi_x}{\partial y^2} + \frac{\partial^2\phi_y}{\partial x\partial y}\right);$$
$$K_s\left(G_c h_c + 2G_f h_f\right)\left(\phi_y + \frac{\partial w}{\partial y}\right) = \left(D_c + D_f\right)\frac{\partial^2\phi_y}{\partial y^2} + \tag{15}$$
$$+\left(\upsilon_c D_c + \upsilon_f D_f\right)\frac{\partial^2\phi_x}{\partial x\partial y} + \frac{1}{2}\left[\left(1-\upsilon_c\right)D_c + \left(1-\upsilon_f\right)D_f\right]\left(\frac{\partial^2\phi_y}{\partial x^2} + \frac{\partial^2\phi_x}{\partial x\partial y}\right);$$
$$K_s\left(G_c h_c + 2G_f h_f\right)\left(M + \nabla^2 w\right) = -q.$$

If eq. (14 *b*) is derived by *x*, eq. (14 *c*) by *y* and then by summing them up considering eq. (14 *a*), the following equation is obtained:

$$\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x\partial y} + \frac{\partial^2 M_y}{\partial y^2} = -q. \tag{16}$$

The moments M_x, M_y and M_{xy} from eqs. (11) are replaced in eq. (16) and after some convenient processing, considering eq. (13), an extremely compact form of eq. (16) is obtained:

$$\nabla^2 M = -\frac{q}{D_c + D_f}. \tag{17}$$

The moment sum from eq. (15 *c*), *M*, can be expressed as:

$$M = -\nabla^2 w - \frac{q}{K_s\left(G_c h_c + 2G_f h_f\right)}. \tag{18}$$

Consequently eq. (17) can also be written:

$$\nabla^2 M = \nabla^2\left[-\nabla^2 w - \frac{q}{K_s\left(G_c h_c + 2G_f h_f\right)}\right] = -\frac{q}{D_c + D_f}. \tag{19}$$

On a straight simply supported edge (with the normal, "*n*", and the tangential direction "*s*") of a polygonal plate, in *xy* plane, the following boundary conditions can be written:

$$w = 0; \qquad M_n = 0; \qquad \varphi_s = \varphi_x l + \varphi_y m = 0\,, \tag{20}$$

where *l*, *m* are the directional cosines of the normal to the contour. Due to the last two conditions (M_n = φ_s = 0), on the simply supported edge the condition *M* = 0 is also satisfied.

2.3. Relationships Between CPT and FSDT

In order to avoid the confusion between response parameters from the two plate theories, the following superscripts are considered: "*K*" for CPT (Kirchhoff) and "*M*" for FSDT (Mindlin).

A first connection between the response parameters from the two plate theories is obtained by comparing eq. (5 *a*) with eq. (19). By using the notation with superscripts previously described and by replacing *q* from eq. (5 *a*) into eq. (19), the following is obtained:

$$\nabla^2 M^M = \nabla^2\left[\nabla^2\left(-w^M + \frac{M^K}{K_s\left(G_c h_c + 2G_f h_f\right)}\right)\right] = -\frac{q}{D_c + D_f}\,. \tag{21}$$

On the boundary of a simply supported polygonal plate, the synthesis of the governing equations (eqs. (7) and boundary conditions (9)) in CPT leads to the following conditions:

$$w^K = M^K = \nabla^2 w^K = 0\,. \tag{22}$$

In FSDT, the same plate will satisfy the following boundary conditions:

$$w^M = M^M = \nabla^2\left(-w^M + \frac{M^K}{K_s\left(G_c h_c + 2G_f h_f\right)}\right) = 0\,. \tag{23}$$

Conditions (22) and (23) enable the establishment of connections on the boundary of the simply supported polygonal plate, between parameters *w* and *M* from the two plate theories.

Finally, the comparison between Eqs. (1) and (21) leads to a relationship which links the deflection of the sandwich plate, w^M, to the parameters, w^K and M^K, computed for the equivalent homogeneous plate in CPT:

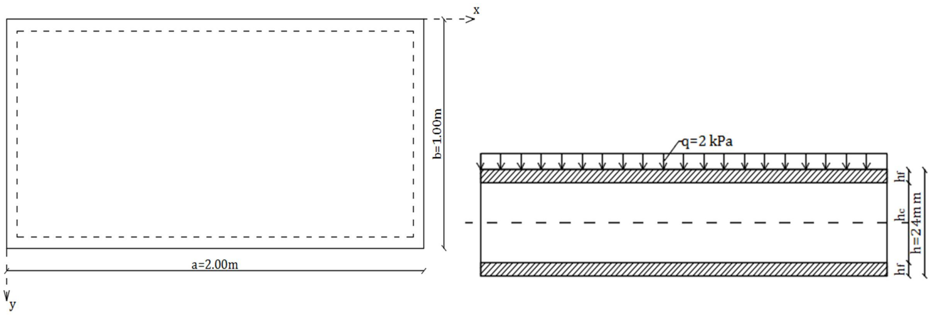
$$w^M = \frac{D}{D_c + D_f} w^K + \frac{M^K}{K_s\left(G_c h_c + 2G_f h_f\right)} \tag{24}$$

Eq. (24) allows the sandwich plate deflection calculation using the deflection and moment sum of the equivalent single layer plate, deducted from the classical plate theory (Kirchhoff).

3. Numerical Investigations

3.1. Geometrical and Mechanical Properties of the Model

The numerical analysis carried out followed the determination of the maximum bending deflection for a sandwich panel, whose geometry, boundary and loading conditions are show in Fig. 4.



a *b*

Fig. 4 – The analyzed plate model: *a* – geometry and boundary conditions; *b* – cross-section and loading.

The elastic characteristics of the constituent materials are shown in Table 1.

Table 1

Typical Elastic Properties of Used Face and Core Materials

Component part	Facings	Core
Elastic constants of material	Aluminum	Extruded polystyrene
Young's modulus	E_f = 71,000 MPa	E_c = 12 MPa
Shear modulus	G_f= 26,700 MPa	G_c = 4.5 MPa
Poisson's ratio	v_f = 0.33	v_c = 0.33

In order to highlight the influence of the layer thickness upon the bending stiffness, the total thickness of the plate was kept constant (h = 24 mm) and the core and facings thickness, h_c and h_f, varied.

The flexural stiffness of the core, D_c, and of the facings, D_f , were computed using eq. (12), and are shown in Table 2, alongside that of the monolayer isotropic homogenous plate (composed of the two facings overlapping, without the polystyrene core between them).

The graphic in Fig. 5 *a* shows how the facings stiffness, D_f, varies with the ratio $2h_f / h$ and the corresponding isotropic plate stiffness, D, depending on the thickness $2h_f$. Fig. 5 *b* shows how the core stiffness varies depending on the ratio h_c / h. It was found that the flexural stiffness of the core, D_c, is very small (practically negligible) compared to that of the facings, which justifies composing the corresponding monolayer isotropic plate from the two facings.

Table 2

Flexural Stiffnesses D_f, D_c, D for Various Ratios $2h_f / h$ and h_c / h

Thicknesses and ratios				Sandwich plate		Monocoque plate	
h_f mm	h_c mm	$2h_f/h$	h_c/h	D_f Nmm	D_c Nmm	$h = 2h_f$ mm	D Nmm
12.0	0.0	1.0	0.0	91,787,678	–	24.0	91,787,678
10.8	2.4	0.9	0.1	91,695,890	15.5	21.6	66,913,217
9.6	4.8	0.8	0.2	91,053,377	124.1	19.2	46,995,291
8.4	7.2	0.7	0.3	89,309,411	418.9	16.8	31,483,174
7.2	9.6	0.6	0.4	85,913,267	992.9	14.4	19,826,138
6.0	12.0	0.5	0.5	80,314,218	1,939.2	12.0	11,473,460
4.8	14.4	0.4	0.6	71,961,540	3,350.9	9.6	5,874,411
3.6	16.8	0.3	0.7	60,304,505	5,321.1	7.2	2,478,267
2.4	19.2	0.2	0.8	44,792,387	7,942.9	4.8	734,301
1.2	21.6	0.1	0.9	24,874,461	11,309.3	2.4	91,788
0.0	24.0	0.0	1.0	–	15,513.4	0.0	–

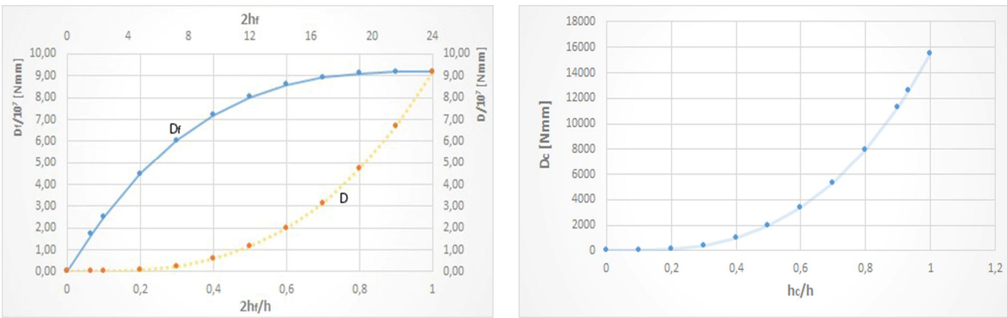


Fig. 5 – Flexural stiffnesses variation: *a* – D_f and D; *b* – D_c.

3.2. Maximum Bending Deflection

Considering the ratio $h/\min(a,b)$ = 24/1,000 = 0.024, the analyzed sandwich plate fits in the thin plates category. As a result, the Kirchhoff theory (CPT) for the monolayer isotropic plate, respectively its extension for the multilayer plates (classical laminate theory – CLT), should be satisfactory for the bending behavior analysis. However, the extremely low flexural stiffness of the core, compared to that of the facings, makes us wonder if the use of a higher order theory is required. In this respect, J. Vinson, in the chapter "Sandwich Analysis" from (Santare & Chajes (Eds.), 2008), states the fact that "*most sandwich structures can be analyzed by using the laminate analysis methods of composite material structures by employing the A, B, and D stiffness matrices. Only in the case of sandwich constructions which use a very flexible core must a higher order sandwich theory be used*".

Hence, the calculation of the maximum transverse deflection were performed using both the classical laminate theory (CLT) and, in particular, CPT (for the monolayer isotropic plate), as well as the first order shear deformation theory (FSDT).

For checks and comparisons, the analysis methods used for the classical theory were the double trigonometric series method (Navier), the finite difference method and the Autodesk Simulation Composite Design software (ASCD). For FSDT, alongside the Navier solution and the finite difference method, an analysis was performed using the finite elements method (ANSYS software), as well as a simulation with a composite laminates analysis software (MSC Structural Mechanics Calculators).

The Navier solution (double trigonometric series method) was applied according to the indications in (Reddy, 2004; Reddy, 2006).

Table 3
Maximum Flexural Displacement of Sandwich Plate, [mm]

Thickness		CLT (CPT)			FSDT			
h_f mm	h_c mm	Navier	MDF	ASCD	Navier	MDF	MEF	MSC
12.0*	0.0	0.221	0.225	–	0.221	0.226	0.208	0.222
10.8	2.4	0.221	0.225	0.221	0.687	0.685	0.696	0.732
9.6	4.8	0.222	0.227	0.225	1.111	1.102	1.129	1.166
8.4	7.2	0.227	0.231	0.227	1.472	1.457	1.497	1.532
7.2	9.6	0.236	0.241	0.236	1.765	1.746	1.796	1.829
6.0	12.0	0.252	0.257	0.252	1.995	1.973	2.032	2.065
4.8	14.4	0.282	0.287	0.282	2.179	2.155	2.221	2.257
3.6	16.8	0.336	0.343	0.336	2.337	2.313	2.386	2.428
2.4	19.2	0.452	0.461	0.452	2.518	2.495	2.575	2.633
1.2	21.6	0.814	0.831	0.814	2.914	2.897	2.987	3.090
0.8**	22.4	1.180	1.204	1.179	3.285	3.276	3.368	3.513

* The first line of the table refers to the homogenous isotropic aluminum plate.
** The last line refers to a real sandwich panel (the $h_f = 0$ line was eliminated, because the given load generated very large deflections).

The finite difference method for the classical theory assumed the Poisson equations, (5), of the monolayer isotropic plate, to be transcribed in finite differences in the eight nodes of a mesh with the step Δ = 250 mm (because of symmetry, the calculations were performed on a quarter of the plate). The values for *M* and *w*, were inserted in (24), in order to determine the maximum deflection according to FSDT.

The maximum deflections, calculated using the two plate theories, through the above mentioned methods, are shown in Table 3.

The variation of the maximum deflection of the plate with the core thickness is presented in Fig. 6, with values computed for both CPT and FSDT.

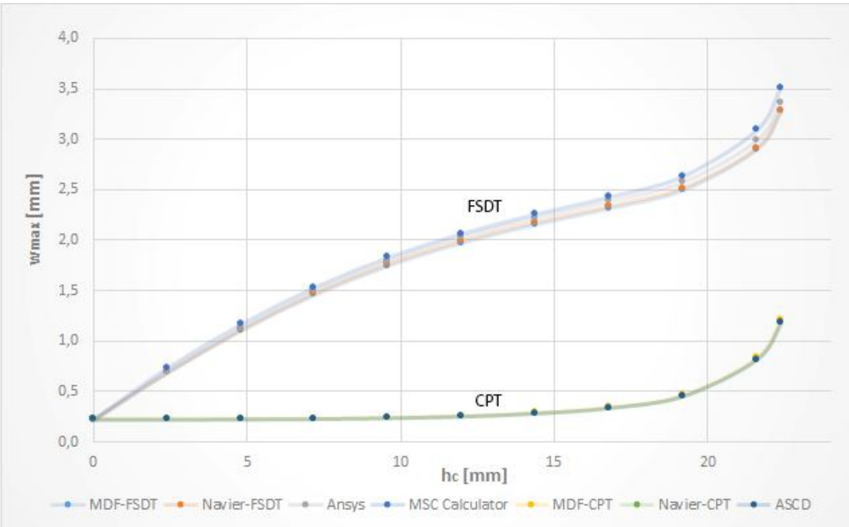


Fig. 6 – Maximum flexural displacement versus core thickness.

4. Conclusions

Sandwich panels are used in preference to conventional composites where the structure is required to have both high flexural strength and low weight. A sandwich plate or panel is a special type of laminated composite, in which the facings are two thin laminas, but with high strength and flexural stiffness, and the core is a “thick” lamina, whose role is to assure the transfer of stresses between the facings and to withstand transverse shear.

In most cases, sandwich plates fit in the thin plate category, for whom the classical laminate theory (CLT) is a widely accepted macromechanical approach for the determination of the mechanical behavior. Only in the case of sandwich constructions which use a very flexible core must a higher order sandwich theory be used and also for the very thick ones.

The paper presents the main theoretical aspects of the classical theory (CPT) for homogenous and isotropic plates, and those of the first order shear deformation theory (FSDT) for sandwich plates. Equation (24) allows calculating the maximum deflection for FSDT as long as the deflections and the Marcus moments are known in CPT for the corresponding homogenous isotropic plate.

The numerical investigations carried out allowed determining the maximum bending deflection, using the two plate theories, for a sandwich panel with a varying core and facings thickness (total thickness was kept constant).

The results show a concordance between the analytical solutions and those obtained from the numerical methods (FDM, FEM), both in CPT (CLT) as well as FSDT.

The classical plate theory (CPT or CLT), in some cases, greatly underpredicts the bending deflections, because its assumptions cannot account for the effect of the transverse shear deformations. In these situations, the use of a higher order theory is mandatory and/or an experimental verification of the results.

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INVESTIGAȚII ANALITICE ŞI NUMERICE PRIVIND COMPORTAREA LA ÎNCOVOIERE A PLĂCILOR SANDVIŞ

(Rezumat)

Se face o trecere în revistă a principalelor teorii utilizate în analiza comportării la încovoiere a plăcilor plane neomogene de tip panou sandviş, menționându-se şi limitele lor de aplicabilitate. Având în vedere că în tehnică se folosesc panouri sandviş alcătuite din diverse materiale şi cu o mare varietate dimensională, sunt necesare investigații numerice şi experimentale pentru a stabili, cu precizie acceptabilă, parametrii de răspuns la solicitarea de încovoiere. Analizele numerice efectuate se referă la săgeata maximă a unui panou sandviş, simplu rezemat pe contur şi încărcat uniform, la care se menține constantă grosimea totală, dar se variază grosimea miezului din polistiren extrudat şi grosimea fețelor din aluminiu. Rezultatele obținute prin metode numerice (MDF, MEF) se compară cu soluțiile analitice existente în literatură şi se fac observații utile în proiectarea unor astfel de structuri.