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ABOUT THE GENERAL STABILITY OF PRESTRESSED BEAMS (III)

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The modification of the design characteristics for I cross-sections with double beams of a prestressed beam, function of the prestressing degree, is analysed. One arrives to the conclusion that the general stability depends on the prestressing degree: the more it increases, the more the stability decreases. However, the reduction of the general stability is independent with respect to the slenderness coefficient of the section webs.

1. Introduction

Nowadays, more and more engineers rely on prestressed elements for structures with large spans or sustaining great service loads. Some of the advantages of such elements are the reduction of their own weight and the possibility to use the entire strength capacity of the non-homogeneous sections. This last advantage is especially seen in case of beams. For such elements, subjected to plane bending, it is possible to obtain the desired stress both in the tie-rod and in the beam at the same time. This is the basic assumption which lies at the basis of a new computation method, called *the equivalent section method.*

Since the prestressed elements are much more slender than their equivalent non-prestressed counterparts, their stability becomes very important. In many cases, the stability condition becomes much more restrictive than the strength condition.

In the previous papers [2], [4] it was shown that the total critical bending moment which corresponds to any position of equilibrium of the beam can be written as

$$M_{\mathrm{cr,\,tot}} = \frac{K}{l}\sqrt{k}, \tag{1}$$

where: K is a numerical coefficient depending on the static model of the beam, l – the length between two successive fixed points, k – the compound rigidity of the prestressed section, defined as

$$k = EI_yGI_t, \tag{2}$$

with: EI_y – the flexural rigidity in the lateral plane, GI_t – the free torsion rigidity.

From the relation (2) it follows that the magnitude of the critical force depends on the compound rigidity, k, which, in there turn, depends on the prestressing degree of the section and, implicitly, on the shape of the section. In [3] there were established some important properties for the I section concerning the ratio between the slenderness coefficient of the section web and the degree of stability of the prestressed beam. This conclusion has been analysed for I sections, and I sections with double webs.

This paper will focus on closed thin-walled sections, and will try to demonstrate that the principle remains valid for this kind of sections, as well (Fig. 1).

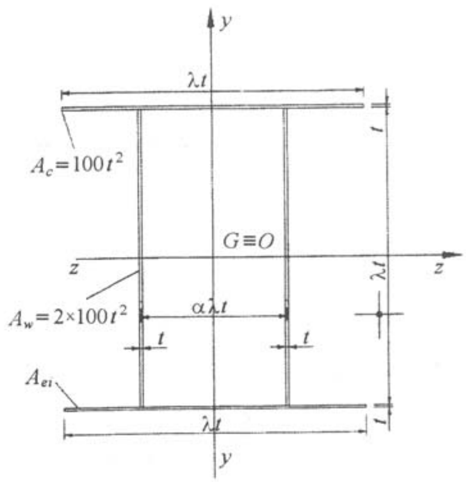

Fig. 1

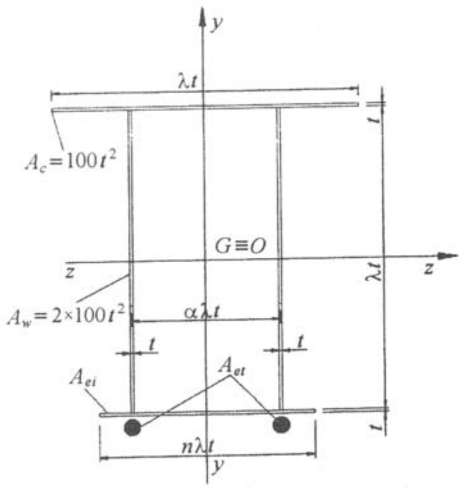

Fig. 2

2. Calculus Assumptions

In what follows, we admit the following assumptions:

a) The tie-rod and the beam materials have linear-elastic behavior.

b) The pre-stressing is realized by straight tie-bars fixed on the tensed flange of the section.

c) The tie-bar is anchored to the beam.

d) The geometrical characteristics are evaluated in the middle line of the section.

e) The pre-stressing is totally realized before the service forces are applied.

f) The torsion rigidity of the section is evaluated as the sum of rigidities of the closed section and of the open flanges.

3. Dependence Between the Critical Force and the Prestressing Degree

By pre-stressing degree we mean (in the sense of the equivalent section method) the ratio between the equivalent area of the pre-stressing tie-rod, A_{lt}, and the tensed flange of the equivalent section, A_{li}.

We shall consider a caisson section with extended top and bottom sides, as presented in Fig. 1.

The equivalent section for section in Fig. 1 has the areas: a) the compressed flange, $A_c = \lambda t^2$; b) the webs, $A_w = 2\lambda t^2$, c) the tensed flange, $A_{ei} = \lambda t^2$.

For a pre-stressing degree $d_p = p$, the pre-stressed section has the areas: a) the compressed flange, $A_c = \lambda t^2$, b) the webs, $A_w = 2\lambda t^2$, c) the tensed flange, $A_{ei} = n\lambda t^2$, d) the equivalent area of the tie-rod, $A_{et} = (1-n)\lambda t^2$, where: λ is the slenderness; α – the ratio between the distance between webs and the total length of the flange; $p = A_{et}/A_{ei}$ – the pre-stressing degree, $n = 1 - p$.

The cross-section has been built based on the optimum shape design, which states that the total area of the webs should be equal to the total area of the flanges,

$$A_f = 2\lambda t^2, \qquad A_w = 2\lambda t^2, \tag{3}$$

3.1. Calculus Procedure

a) For the non-prestressed case

$$\begin{gathered} I_y = \frac{\lambda^3 t^4}{6}\left(1 + \frac{1}{\lambda^2} + 3\alpha^2\right), \\ I_t = I_{t1} + I_{t2}, \end{gathered} \tag{4}$$

where: I_{t1} is the torsion inertial moment of the closed cross-section, I_{t2} – the cumulated inertial torsion moment of the open cross-section, namely

$$\begin{gathered} I_{t1} = 2\frac{\alpha^2\lambda^2(\lambda+1)^2 t^4}{\lambda + 1 + \alpha\lambda}, \\ I_{t2} = 4\frac{\lambda t^4(1-\alpha)}{6}, \\ I_t = \lambda t^4\left[\frac{2\lambda\alpha^2(\alpha+1)^2}{\lambda+1+\alpha\lambda} + \frac{2(1-\alpha)}{3}\right]. \end{gathered} \tag{5}$$

b) For the case with $ap = 1 - n$ degree of prestressing

$$I_{yn} = \frac{\lambda^3 t^4}{6}\left(1 + n^3 + \frac{2}{\lambda^2} + 6\alpha^2\right), \tag{6}$$

$$\begin{gathered} I_{t1} = 2\frac{\alpha^2\lambda^2(\lambda+1)^2 t^4}{\lambda + 1 + \alpha\lambda}, \\ I_{t2} = 2\frac{(\lambda t - \alpha\lambda t)t^3}{6} + 2\frac{(\lambda t n - \alpha\lambda t)t^3}{6}, \\ I_t = \lambda t^4\left[\frac{2\lambda\alpha^2(\lambda+1)^2}{\lambda+1+\alpha\lambda} + \frac{1+n-2\alpha}{3}\right]. \end{gathered} \tag{7}$$

The reduction of the critical force is

$$d_s = \sqrt{\frac{k_{0n}}{k_e}} = \sqrt{\frac{I_{yn} I_{tn}}{I_y I_t}}. \tag{8}$$

Performing the calculus, it results

$$d_s = \sqrt{\frac{\left(1 + n^3 + \frac{2}{\lambda^2} + 6\alpha^2\right)\left[\frac{2\alpha\lambda^2(\lambda+1)^2}{\lambda+1+\alpha\lambda} + \frac{1+n-2\alpha}{3}\right]}{2\left(1 + \frac{1}{\lambda^2} + 3\alpha^2\right)\left[\frac{2\alpha^2\lambda(\lambda+1)^2}{\lambda+1+\alpha\lambda} + \frac{2(1-\alpha)}{3}\right]}}. \tag{9}$$

This is the general formula that can be used to compute the reduction of the critical force for any slenderness, any distance between webs and any prestressing degree.

Performing the calculus for several slenderness ratios, the data presented in the Table 1 are obtained.

Table 1

	λ	n	α	d_s
1	100	0.8	0.8	0.92766503
2	150			0.92766771
3	175			0.92766829
4	200			0.92766866
5	250			0.9276691

By modifying the distance between bars, the data presented in Table 2 are obtained.

Table 2

	λ	n	α	d_s
1	100	0.8	0.2	0.88435573
2			0.4	0.91385202
3			0.5	0.92766771
4			0.6	0.9395132
5			0.8	0.95730494

The prestressing degree also influences the results. By modifying the value of p, we obtain the data from Table 3.

Table 3

	λ	n	α	d_s
1	100	0.2	0.5	0.92766503
2		0.4		0.88089850
3		0.5		0.86601229
4		0.6		0.85588746
5		0.8		0.84648158

4. Conclusions

The data presented in Tables 1,....3 permit to establish the following conclusions:

1. The obtained results lead to the conclusion that the general stability of the prestressed beam, having the closed thin walls section, depends on the prestressing degree: the more it increases, the more the stability decreases.

2. However, the reduction of the general stability is independent with respect to the slenderness coefficient of the web. In the same time, the general stability is better if the distance between the webs is greater; this fact being limited by the local stability of the flanges.

3. These remarks are convergent with those established for the open thin walls section, but the effective values of the critical stability force are greater for the first category of sections.

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CU PRIVIRE LA STABILITATEA GENERALĂ A GRINZILOR PRETENSIONATE (III)

(Rezumat)

Se studiază modificarea caracteristicilor de concepţie pentru secţiunea tip cheson cu perete dublu a unei grinzi pretensionate în funcţie de gradul de pretensionare. Se ajunge la concluzia că stabilitatea generală a unui astfel de element depinde de gradul de pretensionare şi anume cu cât acest grad este mai mare, cu atât stabilitatea generală a elementului se reduce. În acelaşi timp se menţionează că stabilitatea elementului nu depinde de coeficientul de svelteţe al inimii secţiunii, ceea ce reprezintă un aspect deosebit de important pentru concepţia raţională a acestor elemente structurale.