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CONSTRUCŢII. ARHITECTURĂ

ON THE DESIGN OF SUSPENDED ROOFS WITH PARABOLOIDAL SURFACES

BY

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Some considerations concerning the design of the paraboloidal suspended roofs are made. The main geometric aspects are first time presented. For the roofs we propose, as pattern, the equivalent continuum membranes, and the efforts in the cable are determined by using the membrane efforts and their equations. Two examples are analysed: eliptic paraboloide and hyperbolic paraboloide, with horizontal projection under the form of an ellipse.

1. Concept Aspects

We begin by briefly discussing about the roofs made of thin curved plates which have considerably extended during the 20th century and still continue the tendency. Thin curved plates are efficient structures for the roofs, especially if they work in the membrane state. This type of structure allows covering big surfaces without the use of interior intermediate elements. As support systems, curved plates transmit the effects of the actions applied directly to the leaning and afterwards to the foundation.

Curved plates used as structures for roofs allow especially architectural shapes and they are also very performant from an engineers point of view. Their development is due to the architects and the engineers which operate in this field, the development being sustained also by the evolution of the calculating systems and of the analysis methods.

Creating the membrane state brings some practical difficulties and that is why the adopted systems present deviations, meaning there will be bending and torsion effects, which make the analysis more complex and more difficult for the intuition and the system will be less economical.

The inverted curved plates made on cables have special advantages when subjected to vertical actions, being solicited at stretching efforts which can be taken over by cabling systems. Concave surfaces are less affected by the wind pressure, which has reduced vertical components.

Eliminating the water from precipitations, rain, snow can be made nowadays through special installations having an automatic functioning and reliable means of assurance. Suctions can be diminished by the roof's weight, especially through the covering which ballasts the cabling system and through the conception way it can take over slidings' and some local compressions which can appear in some of the roofs areas. Through the insertion of initial stretching efforts in the cable, the tensions in the compression are taken over by diminishing the pretensionning efforts. The way in which the cable system and the cover are made can create a structural system rigid enough with a high stability which introduces a smaller risk for the flutter and the resonance, generated by the wind and possibly by the earthquake – important dynamic effects for these types of structures. These structural systems are mainly indicated as solutions in the case of public constructions, which are remarkable through their aesthetics, performance, safety and durability.

Of all the suspended roofs we now retain those having parabole-shape and for some of these, we will make a certain development on their design.

2. Synthetic Elements Concerning the Paraboloide Surfaces' Geometry

The paraboloide is a second order surface: its equation in the triorthogonal axis system can be written as

$$z = \tfrac{1}{2}(rx^2 + ty^2), \tag{1}$$

where r and t are constants, having as dimensions the inverse of a length (geometrically, r and t are inverse of the curvature rays). We immediately see that the origin of the reference system belongs to the surface.

The sign of rt product indicates the type of roof namely

a) $rt > 0$ – convex surface, elliptical paraboloide;

b) $rt < 0$ – concave surface, hyperbolical paraboloide.

In conclusion, the surface is described by a parabola, invariable as shape and dimensions, which moves parallel to herself and whose vertex remains on another parabola. The two parabolas have steering curves and their role can be interchanged. If both parabolas have the concavities in the same direction as z, the paraboloide is elliptical and if the concavities have opposite ways, positive and negative on z-axis, the paraboloide is hyperbolic.

The significance of r and t also result from

$$\frac{\partial^2 z}{\partial x^2} = r, \;\; \frac{\partial^2 z}{\partial y^2} = t, \;\; \frac{\partial^2 z}{\partial x \partial y} = s = 0. \tag{2}$$

So, r and t are main curves and the torsion curve is zero, which means the surfaces distortion is null.

By relating the surface to its concave form, given by the equations (1) and (2), two conditions are sufficient to determine the coefficients r and t.

In the case of the hyperbolic paraboloide, if the substitutions

$$r = \frac{1}{g_1}, \quad t = \frac{1}{g_2} \tag{3}$$

are performed, where $g_1g_2 > 0$, the equation (1) can be expressed as

$$\left(\frac{x}{\sqrt{g_1}} + \frac{y}{\sqrt{g_2}}\right)\left(\frac{x}{\sqrt{g_1}} - \frac{y}{\sqrt{g_2}}\right) = 2z. \tag{4}$$

We observe that on the surface of the hyperbolic paraboloide there are two systems of rectilinear generatrices, whose equations are

$$\frac{x}{\sqrt{g_1}} \pm \frac{y}{\sqrt{g_2}} = \frac{z}{\alpha_{1,2}}, \quad \frac{x}{\sqrt{g_1}} \mp \frac{y}{\sqrt{g_2}} = 2\alpha_{1,2}, \tag{5}$$

where α_1 and α_2 represent the parameters of the two generatrices systems, with the aid of which we also can express the parametric equations of the hyperbolic paraboloide namely

$$x = \frac{\alpha_1 + \alpha_2}{\sqrt{g_1}}, \quad y = \frac{\alpha_1 - \alpha_2}{\sqrt{g_2}}, \quad z = 2\alpha_1\alpha_2. \tag{6}$$

Each system of generators are parallel with the fixed plane, being called steering planes having the equation

$$\frac{x}{\sqrt{g_1}} - \frac{y}{\sqrt{g_2}} = 0, \quad \frac{x}{\sqrt{g_1}} + \frac{y}{\sqrt{g_2}} = 0. \tag{7}$$

3. Generation of the Hyperbolic Paraboloide Surface

If we consider two generatrices in a system, then any generatrix from other system intersects the two generatrices and is parallel to the corresponding steering plane. This characteristic suggests a way to generate the hyperbolic paraboloide by a variable straight line parallel with a matronless plane which intersects two given straight lines. Being a surface which can be generated by a straight lines movement, the hyperbolic paraboloide is a slided surface. From the point of view of the suspended systems, it results that the hyperbolic paraboloide can be achieved through a cabling system rectilinear stretched. We can trace these straight lines on the surfaces outline and afterwards by using the proper installation we can tense the cables so that they become rectilinear and the knots on the generators be on a straight line.

The generatrices in the two systems make an angle, Ω, which in the Oxy plane is projected into ω. The angle ω can be calculated using the relation

$$\operatorname{tg}\omega = \frac{2\sqrt{g_1g_2}}{g_1 - g_2}, \quad \operatorname{tg}\frac{\omega}{2} = \pm\sqrt{\frac{g_2}{g_1}}. \tag{8}$$

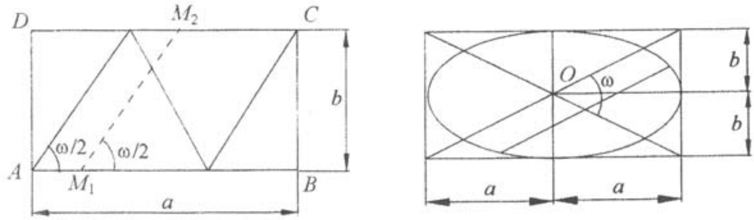
It is important to mention the fact that the two generatrices pass through a point of the outline obtained through the projection of the surface on the Oxy plane.

On a rectangle outline with the sides a and b (Fig. 1 a) the depart condition from M_1 to return to M_1 after a zigzag route, mentioning the ω angle, is equivalent to that leaving from A with $\omega/2$ arrising in C. From geometrical conditions we obtain

$$\text{tg}\,\frac{\omega}{2} = y\frac{b}{a}. \tag{9}$$

On an elliptic outline with the semi-axis a and b (Fig. 1 b), the rectilinear generatrices projections must be parallel with the diagonals of the rectangle circumscribed to the ellipse, which create the ω angle

$$\text{tg}\,\frac{\omega}{2} = \frac{b}{a}. \tag{10}$$



a b

Fig. 1.– Generatrices directions.

We distinguish two practical situations concerning the paraboloide surfaces outline and so the roofs as well namely

a) Hyperbolic paraboloide roof with rectilinear sides.

b) Hyperbolic paraboloide having a surface delimited by curved sides also parallel – in particular by a family of steering curves, which carries its own name and hyperbolic paraboloid.

A shape that is more practical from the point of view of its achievement is obtained through reporting of the surface to the rectilinear generatrices which is generally a system of oblique cartesian coordinates, the axis creating an ω angle.

In technical language, the hyperbolic paraboloide delimited by rectilinear generatrices are also called "hipari". The way of calculating, especially the efforts, also depends on the shape of the covered surface outline but also of the shape of the board sides, which can be curved or rectilinear. From the point of view of the studied analysis, there are different cases which need to be analysed separately.

4. Suspended Roofs of the Elliptic Paraboloide Type on Elliptic Outline

The surface's equation is obtained from (1) if $r = f_1/a^2$ and $t = f_2/b^2$, where z-axis has a positive sign; so the surface is turned upside down; a and b are the semi-axis of the roofs outline ellipse, projected on the Oxy plane; f_1 and f_2 are the share of the roofs surface for $x = \pm a$ and $y = \pm b$ (Fig. 2).

If $f_1 = f_2$, the surface sides is a curved plane.

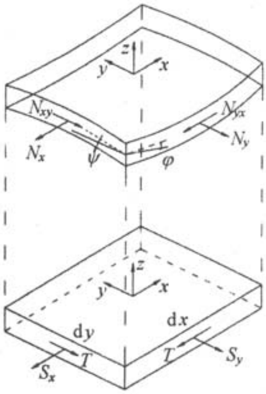


Fig. 2

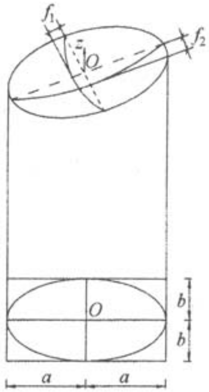


Fig. 3

The surface being turned upside down and leaning on the outline on a marginal structure in a membrane state is solicited in the case of stretching; the projected efforts being of an opposite sign to those determined in the case of the roof facing the concavity downwards (Fig. 3). For an uniform charge in a horizontal plane

$$S_x = \frac{pa^2}{2(f_1 + f_2)}, \quad S_y = \frac{pb^2}{2(f_1 + f_2)}, \quad T = 0. \tag{11}$$

The real efforts are

$$N_x = \frac{S_x}{\lambda}, \quad N_y = \lambda S_y, \quad N_{xy} = N_{yx} = T = 0, \quad \lambda = \frac{\cos\varphi}{\cos\Psi}. \tag{12}$$

The efforts (11) and (12) are determined considering a continuous surface made from a homogeneous material; we also can consider the boards thickness as being constant.

The structural system being static determined the efforts depend on the surface's configuration: the outlines and the f_1 and f_2 are arrows, and them being established it is finally only the charging to determine the efforts' values. In the case of the considered surface, we only have stretching tensions on normal sections at the tribulation axis, which are also symmetry axis.

Since the thickness of the shells and the nature of the material are not relevant, there is a certain freedom in the achievement of this type of roof. If we can imagine, the surface could be made using a uniform net of steel wires and the directions of this wire are parallel on both directions with the planes xz and yz. For very small unitary lengths, like the wire section, each wire will be loaded with tensions S_x and S_y, which in the wire notation are H_x and H_y and these tensions are constant per unit length.

In particular, if the length unit takes a constant distance between the cables in a certain direction, the efforts are H_x and H_y and the tensions will be

$$(13) \qquad T_x = H_x\sqrt{1+\left(\frac{\partial z}{\partial x}\right)^2}, \quad T_y = H_y\sqrt{1+\left(\frac{\partial z}{\partial y}\right)^2}.$$

If the slopes $\partial z/\partial x$ and $\partial z/\partial y$ in the directions of the x- and y-axis are small, their squares can be practically neglected, the tensions in the cables being H_x and H_y which are at the same time N_x and N_y. Obviously, corrections can be made, if is the case.

Determining the cable net assumes establishing the distances between cables on the two directions and the cables type.

We assume that the types of cables have already been chosen so that the maximum efforts in the cables, N_{cx} and N_{cy}, can be determined. From the relation of equivalence it results

$$(14) \qquad N_x d_x = N_{cx} \text{ and } N_y d_y = N_{cy},$$

where d_x and d_y are the distances between the cables

$$(15) \qquad \begin{cases} d_x = \dfrac{N_{cx}}{N_x}, & d_y = \dfrac{N_{cy}}{N_y}, \\ d_x = \dfrac{N_{cx}}{H_x}, & d_y = \dfrac{N_{cy}}{H_y}, \end{cases}$$

respectively

$$(16) \qquad \begin{cases} d_x = \dfrac{\sigma_{\text{cable}} A_{cx}}{H_x} = \dfrac{2\sigma_{\text{cable}} A_{cx}(f_1+f_2)}{pa^2}, \\ d_y = \dfrac{\sigma_{\text{cable}} A_{cy}}{H_y} = \dfrac{2\sigma_{\text{cable}} A_{cy}(f_1+f_2)}{pb^2}, \end{cases}$$

where A_{cx} and A_{cy} are the areas of the cables sections.

If we establish the net, d_x and d_y can be determined, also N_{cx} and N_{cy} and their types in the catalogues or the fabrication prospects, for example

$$(17) \qquad N_{cx} = H_x d_x \quad N_{cy} = H_y d_y.$$

If for the covering a light concrete is used, which can also have a protection role and then the cable pretension through the execution of the high covering of total pretensionning (or a partial pretensionning) the concrete will be precompressed and will be able to take over some of the stretching efforts. Still, the roof's carrying capacity will be retained by the cables.

5. Suspended Roofs: Hyperbolic Paraboloide Type on Elliptic Center

The hyperbolic paraboloide has the surface equation as (1), where $r = f_1/a^2$ and $t = f_2/b^2$, where a, b, f_1, f_2 have similar significance to those in the case of the elliptic paraboloid, with the specification that f_2 has an opposite sign to f_1 and the main curves have opposite directions, having implications on the efforts obtained from those of the elliptic paraboloid, by replacing f_2 with $-f_2$.

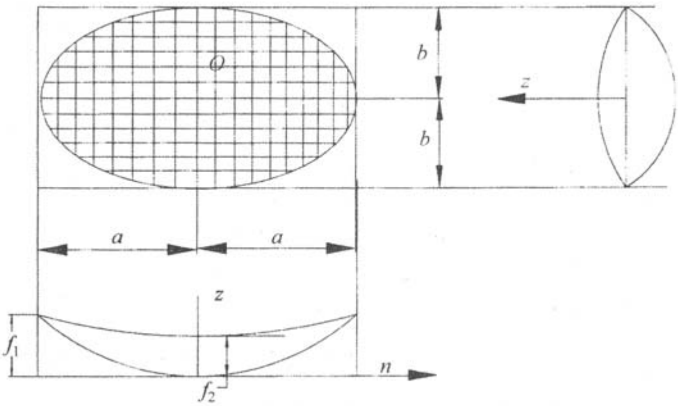


Fig. 4.- Hiperbolic paraboloide suspended roofs.

For the turned surface, the efforts designed at a uniform load have the expressions

$$S_x = H_x = \frac{pa^2}{2(f_1 - f_2)}, \quad S_y = H_y = \frac{pb^2}{2(f_1 - f_2)}, \quad T = 0. \tag{18}$$

We observe that the situation $f_1 = f_2$ or approaching values must be avoided since the efforts can grow very much, going towards infinite.

For determining the efforts in the cables, we proceed in a similar way as in § 4, namely

$$\begin{cases} d_x = \dfrac{N_{cx}}{H_x} = \dfrac{2\sigma_{\text{cable}} A_{cx}(f_1 - f_2)}{pa^2}, \\ d_y = \dfrac{N_{cy}}{H_y} = \dfrac{2\sigma_{\text{cable}} A_{cy}(f_1 - f_2)}{pb^2}. \end{cases} \tag{18}$$

6. Conclusions

For the presented cases, designing suspended roofs is relatively simple and more sensible situations are limited. Among them, we mention the problem of the flutter, which can be important in the case of some strong sustaining effects and in that of a light covering with a reduced rigidity. This is the reason why, for this type of roof, it is recommended to use flagstone coverings set on the old cable-delimited ones with a corresponding stretch of the purpose.

The elliptic shape of the outline of these roofs avoids the perturbation effects of the wind, especially at corners.

Other types of roofs in the case of which we observe specific effects will be presented in a future work.

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ASUPRA PROIECTĂRII ACOPERIŞURILOR SUSPENDATE CU SUPRAFEŢE PARABOLOIDALE

(Rezumat)

Se fac unele consideraţii privind proiectarea acoperişurilor suspendate paraboloidale. Sunt prezentate mai întâi principalele aspecte geometrice. Pentru acoperişuri pe cabluri s-a propus un model de membrană continuă echivalentă, iar eforturile din cablu sunt determinate utilizând eforturile de membrană şi ecuaţiile acestora. Au fost analizate două exemple: paraboloidul eliptic şi paraboloidul hiperbolic cu proiecţie orizontală sub formă de elipsă.