BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Publicat de
Universitatea Tehnică „Gheorghe Asachi" din Iaşi
Tomul LXI (LXV), Fasc. 2, 2015
Secţia
CONSTRUCŢII. ARHITECTURĂ

THE INFLUENCE OF THE STACKING SEQUENCE ON STRESS AND STRAIN DISTRIBUTIONS FOR QUASI-ISOTROPIC LAMINATES

BY

IULIANA DUPIR (HUDIŞTEANU)* and NICOLAE ŢĂRANU

"Gheorghe Asachi" Technical University of Iaşi
Faculty of Civil Engineering and Building Services

Received: June 10, 2015
Accepted for publication: June 19, 2015

Abstract. A comparative analysis to study the influence of the stacking sequences for quasi-isotropic laminates subjected to axial in-plane load and bending moment is presented in this paper. Two types of quasi-isotropic composite laminates are chosen: they have identical fibre orientation angles, the same layer thicknesses and the number of plies, but different stacking sequences. The stresses and strains are evaluated for each layer of the laminates and their distributions are plotted on diagrams. Other aspects of quasi-isotropic laminates, based on the Classical Lamination Theory (CLT), such as the complex coupling effects that may occur in laminates are also discussed.

Key words: stacking sequence; stress and strain distributions; coupling effects; quasi-isotropic laminate; stiffness matrix.

1. Introduction

The unidirectional fibre reinforced lamina alone is rarely utilised as a structural element, because of its low transverse properties compared to the longitudinal ones. Composite structures are more convenient to be in the form

*Corresponding author: *e-mail*: iulianahudisteanu@ce.tuiasi.ro

of laminates, consisting of multiple layers or plies, oriented in the desired directions and bonded together in a structural unit (Gibson, 2012).

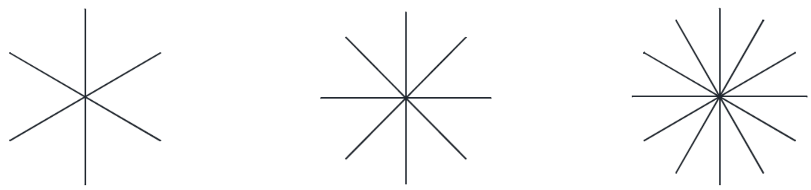
The most important benefit that composite laminates can offer is the design flexibility by virtually limitless combinations of ply composite material constituents, fibre orientation angles, stacking sequences, ply thickness, fibre volume fractions, adopted manufacturing method (Herakovich, 1998).

In Barbero, (2011), the author has a particular opinion, namely that "quasi-isotropic laminates are constructed in an attempt to create a composite laminate that behaves like an isotropic plate" and that "the in-plane behaviour of quasi-isotropic laminates is similar to that of isotropic plates, but the bending behaviour of quasi-isotropic laminates is quite different than the bending behaviour of isotropic plates."

A laminate is called quasi-isotropic when the individual plies are oriented in such a manner as its extensional stiffness matrix [A] behaves like an isotropic matrix. This implies not only that the terms need to satisfy the identities $A_{11} = A_{22}$, $A_{16} = A_{26} = 0$ and $A_{66} = (A_{11} - A_{12})/2$, but also that these stiffness coefficients are independent of the angle of rotation of the laminate. The conditions of isotropic response only apply to the [A] matrix. That is why the reason for calling such a laminate quasi-isotropic and not isotropic is that the other stiffness matrices, [B] and [D], may or may not be fully populated, therefore may not behave like isotropic materials (Staab, 1999; Kaw, 2006).

Generally, the quasi-isotropic laminates need to satisfy several rules, such as: the total number of plies must be $n \geq 3$, all individual layers should have the same thickness and identical orthotropic elastic constants; in addition, it should be observed the Eq. (1) requirement, referring to the $\Delta\theta$ angles between the fibre orientations of the n-layer laminate (Herakovich, 1998; Staab, 1999):

$$\Delta\theta = \frac{\pi}{n} \tag{1}$$



$n = 3,\ \Delta\theta = 60^o$ $n = 4,\ \Delta\theta = 45^o$ $n = 6,\ \Delta\theta = 30^o$

Fig. 1 – Quasi-isotropic fibre orientations.

Examples of quasi-isotropic laminates include the configurations: $[0/\pm 60]_s$, $[0/\pm 45/90]_s$, illustrated in Fig. 2.

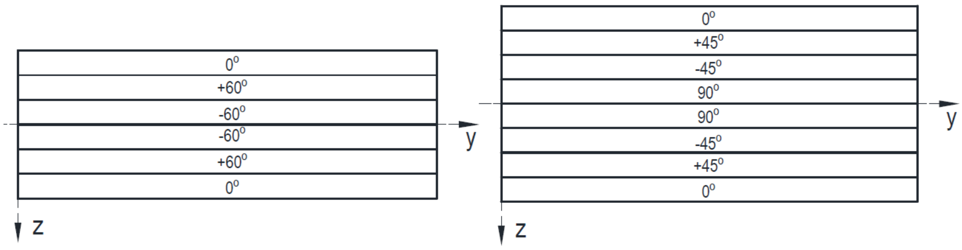


Fig. 2 – Quasi-isotropic laminates: $[0/\pm60]_s$, $[0/\pm45/90]_s$.

2. Evaluation of Stresses and Strains Along the Thickness of the Laminate

Constitutive equations provide the relationship between stresses and strains; therefore, the equations which describe the linear-elastic response of the composite laminate, acted by in-plane forces and moments, are developed.

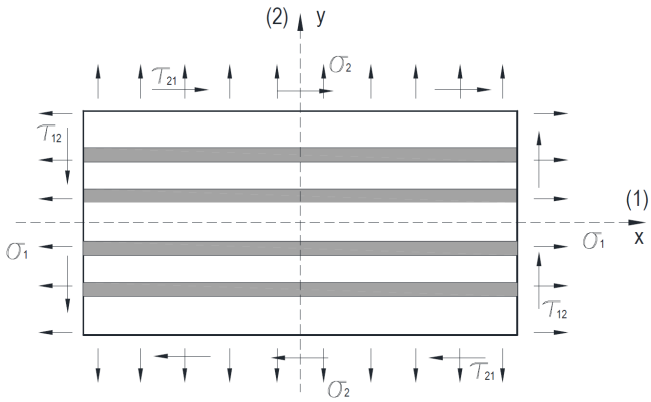


Fig. 3 – The specially orthotropic lamina.

For an individual layer *k* in a multidirectional laminate, the stress-strain relations referred to its principal material axes (1, 2), namely for a specially orthotropic lamina, Fig. 3, are given in Eq. (1) (Herakovich, 1998; Daniel & Ishai, 2006):

$$\{\sigma\}_k = [Q]_k \{\varepsilon\}_k \,, \tag{1a}$$

which can be expanded as:

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix}_k = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix}_k \,, \tag{1b}$$

where: $[Q]$ is the reduced stiffness matrix; σ_1, σ_2, σ_{12} – in-plane stress components along the principal material axes; ε_1, ε_2, γ_{12} – in-plane strain components along the principal material axes.

The coefficients Q_{ij} of the reduced stiffness matrix can be expressed in terms of elastic engineering constants, as it follows (Ţăranu *et al*., 2013):

$$Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}},$$
$$Q_{12} = \frac{\nu_{21}E_1}{1-\nu_{12}\nu_{21}} = \frac{\nu_{12}E_2}{1-\nu_{12}\nu_{21}}, \tag{2}$$
$$Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}},$$
$$Q_{66} = G_{12},$$

where: E_1, E_2, $G_{12}$are the axial and shear moduli along the principal material axes; ν_{12}, ν_{21} – the Poisson's ratios.

The elastic constants are determined with respect to the material characteristics of the constituents (Jones, 1999), with Eq. (3):

$$E_1 = E_f V_f + E_m V_m; \qquad E_2 = \frac{E_f E_m}{V_f E_m + V_m E_f};$$
$$G_{12} = \frac{G_f G_m}{V_f G_m + V_m G_f}; \tag{3}$$
$$\nu_{12} = \nu_f V_f + \nu_m V_m; \qquad \nu_{21} = \nu_{12}\frac{E_2}{E_1},$$

where: E_f, E_m are the longitudinal Young's modulus of the fiber and matrix respectively; G_f, G_m – the shear modulus of the fiber and matrix; ν_{12}, ν_{21} – the Poisson's ratios; V_f, V_m – the fiber and matrix volume fractions.

The constitutive equation for an individual layer k of a composite laminate, expressed after transformation to the laminate global coordinate system, namely a generally orthotropic lamina, Fig. 4, (Herakovich, 1998; Daniel & Ishai, 2006), is presented in Eq. (4):

$$\{\sigma\}_k = \left[\overline{Q}\right]_k \{\varepsilon\}_k, \tag{4a}$$

and in the expanded form:

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k \qquad (4b)$$

where: $\left[\overline{Q}\right]$ is the transformed reduced stiffness matrix; σ_x, σ_y, σ_{xy} – in-plane stress components along the global reference axes; ε_x, ε_y, γ_{xy} – in-plane strain components along the global reference axes.

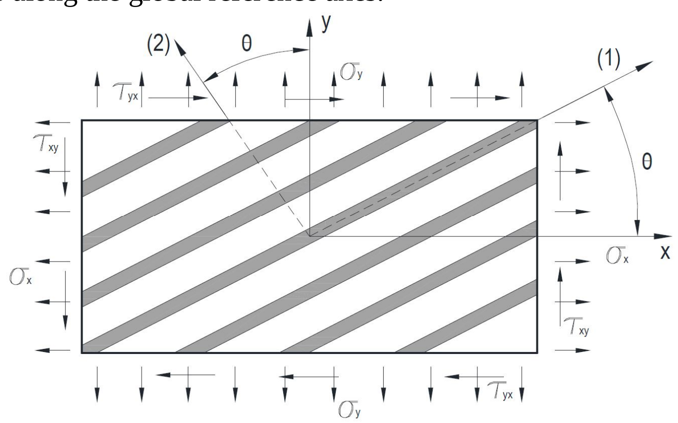


Fig. 4 – The generally orthotropic lamina.

The coefficients of the transformed reduced stiffness matrix are determined with respect to the previous computed values of the stiffness matrix and to the fiber orientation angles, as follows (Ţăranu & Isopescu, 1996):

$$\begin{aligned} \overline{Q}_{11} &= Q_{11}c^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}s^4 \\ \overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})c^2s^2 + Q_{12}(s^4 + c^4) \\ \overline{Q}_{22} &= Q_{11}s^4 + 2(Q_{12} + 2Q_{66})c^2s^2 + Q_{22}c^4 \\ \overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})c^3s + (Q_{12} - Q_{22} + 2Q_{66})s^3c \\ \overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})cs^3 + (Q_{12} - Q_{22} + 2Q_{66})sc^3 \\ \overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})c^2s^2 + Q_{66}(c^4 + s^4), \end{aligned} \qquad (5)$$

where: $c = \cos\theta$; $c^2 = \cos^2\theta$; $c^4 = \cos^4\theta$; $s = \sin\theta$; $s^2 = \sin^2\theta$; $s^2 = \sin^2\theta$.

The geometrical characteristics for a basic composite laminate are shown in Fig. 5.

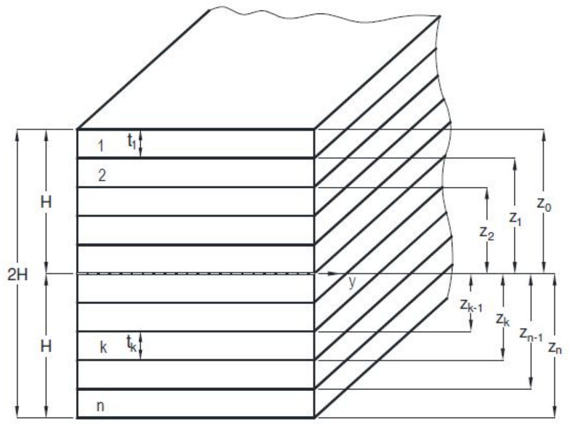


Fig. 5 – Geometrical characteristics of a *n*-layered laminate.

The extensional stiffness matrix or *the in-plane stiffness matrix* [*A*] is defined as the sum of the product of the individual layers $\overline{Q}_{ij}$ and the laminas thicknesses, as shown in Eq. (6*a*) and (6*b*), (Herakovich, 1998). The matrix [*A*] also relates the resultant in-plane forces to the in-plane strains (Jones, 1999; Kaw, 2006).

$$A_{ij} = \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(z_k - z_{k-1}\right), \tag{6a}$$

$$\left[A\right] = \left[\sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \int_{z_{k-1}}^{z_k} \mathrm{d}z \right] = \begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix}, \tag{6b}$$

where: z_k, z_{k-1} are the coordinates to the bottom and to the top of the *k* layer.

The evaluation of the *bending-stretching coupling matrix* [*B*] is done with Eq. (7*a*) or (7*b*), (Herakovich, 1998). The importance of [*B*] matrix is referred to the coupling effect between the force and moment terms to the middle plane strains and middle plane curvatures (Kaw, 2006).

$$B_{ij} = \frac{1}{2} \sum_{k=1}^{n} \left(\overline{Q}_{ij}\right)_k \left(z_k^2 - z_{k-1}^2\right), \tag{7a}$$

$$\left[B\right] = \left[\sum_{k=1}^{n} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \int_{z_{k-1}}^{z_k} z\,\mathrm{d}z \right] = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix}. \tag{7b}$$

The bending stiffness matrix $[D]$ relates the resultant bending moments to the plate curvatures (Kaw, 2006; Herakovich, 1998):

$$D_{ij}=\frac{1}{3}\sum_{k=1}^{n}\left(\bar{Q}_{ij}\right)_k\left(z_k^3-z_{k-1}^3\right), \tag{8a}$$

$$[D]=\left[\sum_{k=1}^{n}\begin{bmatrix}\bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16}\\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26}\\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66}\end{bmatrix}_k \int_{z_{k-1}}^{z_k} z^2\mathrm{d}z\right]=\begin{bmatrix}D_{11} & D_{12} & D_{16}\\ D_{12} & D_{22} & D_{26}\\ D_{16} & D_{26} & D_{66}\end{bmatrix}. \tag{8b}$$

The $\begin{bmatrix}A & B\\ B & D\end{bmatrix}$ matrix is defined as the laminate *assembled stiffness matrix* and it is useful in characterizing and predicting the laminate behaviour subjected to various loading schemes.

The resultant in-plane forces (N_x, N_y, N_{xy}) and moments (M_x, M_y, M_{xy}) respectively, are shown in Fig. 6.

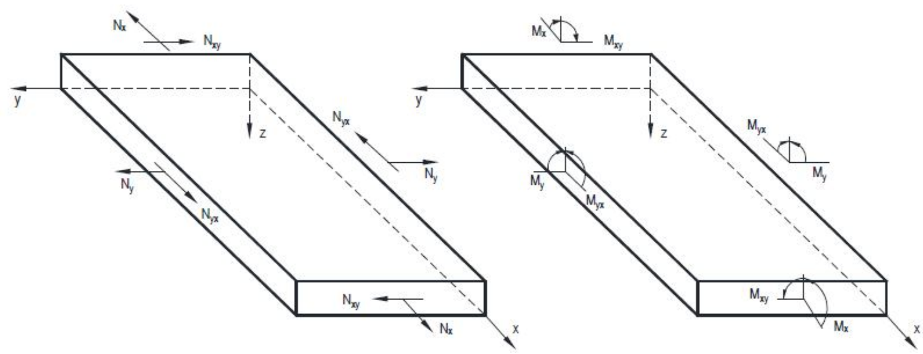


Fig. 6 – In-plane forces and moments per unit width.

The six force and moment resultants form a system that is statically equivalent to the stress system on the laminate, with respect to the middle plane of the multi-layered composite.

In order to evaluate the reference plane strains and curvatures, the force-deformation and moment-deformation relations can be combined, as it follows (Gibson, 2012):

$$\begin{Bmatrix}N_x\\ N_y\\ N_{xy}\end{Bmatrix}=\begin{bmatrix}A_{11} & A_{12} & A_{16}\\ A_{12} & A_{22} & A_{26}\\ A_{16} & A_{26} & A_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_x^0\\ \varepsilon_y^0\\ \gamma_{xy}^0\end{Bmatrix}+\begin{bmatrix}B_{11} & B_{12} & B_{16}\\ B_{12} & B_{22} & B_{26}\\ B_{16} & B_{26} & B_{66}\end{bmatrix}\begin{Bmatrix}k_x\\ k_y\\ k_{xy}\end{Bmatrix}, \tag{9a}$$

$$\begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}, \tag{9b}$$

where: $(\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0)$ represents the laminate mid-plane strains; (k_x, k_y, k_{xy}) represents the laminate curvatures.

In the condensed form, the corresponding inverted force-deformation relationship is given by Eq. (10), such as (Herakovich, 1998):

$$\begin{Bmatrix} \varepsilon^0 \\ k \end{Bmatrix} = \begin{bmatrix} A' & B' \\ C' & D' \end{bmatrix} \begin{Bmatrix} N \\ M \end{Bmatrix}, \tag{10}$$

where:

$$\begin{gathered} \left[A'\right] = \left[A^*\right] - \left[B^*\right]\left[D^*\right]^{-1}\left[C^*\right] \\ \left[B'\right] = \left[B^*\right]\left[D^*\right]^{-1} \\ \left[C'\right] = -\left[D^*\right]^{-1}\left[C^*\right] \\ \left[D'\right] = \left[D^*\right]^{-1} \end{gathered} \tag{11}$$

and:

$$\begin{gathered} \left[A^*\right] = [A]^{-1} \\ \left[B^*\right] = -[A]^{-1}[B] \\ \left[C^*\right] = [B][A]^{-1} \\ \left[D^*\right] = [D] - [B][A]^{-1}[B] \end{gathered} \tag{12}$$

The evaluation of in-plane strain components at a specified distance z is related to the laminate mid-plane strains and laminate curvature, as follows (Gibson, 2012; Herakovich, 1998):

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix}_k = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}, \tag{13}$$

The stresses in any k layer can be computed by substituting the expressions for the strains from Eq. (13) into the plane stress constitutive equation (Gibson, 2012):

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}_k \begin{Bmatrix} k_x \\ k_y \\ k_{xy} \end{Bmatrix}, \qquad (14)$$

3. Analysis and Results

In order to analyse the stacking sequence influence on the stress and strain distribution of multi-layered composite elements, two different types of quasi-isotropic laminates have been selected. The laminates are made of S glass fibre with V_f = 0.6 and polyester resin; the characteristics of the composite material constituents are given in Table 1. The in-plane forces and moments acting on the laminate are: N_x = 1 kN, M_x = 10 kN.mm.

Table 1
Composite Materials' Characteristics for Quasi-Isotropic Laminates

Materials for the composite laminate	E, [GPa]	ν	G, [GPa]	ρ, [kg/m^3]
S glass fibre	85.5	0.22	26.72	2,500
Polyester resin	4	0.39	1.44	1,200

The comparative analysis is done based on Classical Lamination Theory and the obtained results are centralized in Table 2. The quasi-isotropic laminates have identical characteristics (Table 1 and Table 2), the only variable parameter being the stacking sequence.

Table 2
Comparison of Quasi-Isotropic Laminates with Different Stacking Sequences

<table>
<tr><td colspan="2">Quasi-isotropic laminates - $\Delta\theta = 60^o$
Assumptions for analysis:
- equal number of layers (n = 3)
- equal thickness layers (t = 0.4mm)
- same fibre orientation angles ($\theta_1 = 0^o, \theta_2 = 60^o, \theta_3 = -60^o$)
- different stacking sequences</td></tr>
<tr><td>$[+60/0/-60]$
Anti-symmetric quasi-isotropic laminate</td><td>$[0/\pm 60]$
Asymmetric quasi-isotropic laminate</td></tr>
</table>

Table 2

Continuation

Assembly of the ABD matrix type

$$\left[\begin{array}{ccc|ccc} A_{11} & A_{12} & 0 & 0 & 0 & B_{16} \\ A_{12} & A_{22} & 0 & 0 & 0 & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & 0 \\ \hline 0 & 0 & B_{16} & D_{11} & D_{12} & 0 \\ 0 & 0 & B_{26} & D_{12} & D_{22} & 0 \\ B_{16} & B_{26} & 0 & 0 & 0 & D_{66} \end{array}\right] \qquad \left[\begin{array}{ccc|ccc} A_{11} & A_{12} & 0 & B_{11} & B_{12} & B_{16} \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & B_{26} \\ 0 & 0 & A_{66} & B_{16} & B_{26} & B_{66} \\ \hline B_{11} & B_{12} & B_{16} & D_{11} & D_{12} & D_{16} \\ B_{12} & B_{22} & B_{26} & D_{12} & D_{22} & D_{26} \\ B_{16} & B_{26} & B_{66} & D_{16} & D_{26} & D_{66} \end{array}\right]$$

Micromechanics of lamina

Elastic engineering constants (principal material directions); $V_f = 0.6$

$E_1 = 52.9$ [GPa]; $E_2 = 9.34$ [GPa]; $G_{12} = 3.33$ [GPa]; $\nu_{12} = 0.288$; $\nu_{21} = 0.051$

Macromechanics of the lamina

Coefficients of the reduced stiffness matrix, Q_{ij} [GPa]

$$[Q] = \begin{bmatrix} 53.69 & 2.74 & 0 \\ 2.74 & 9.48 & 0 \\ 0 & 0 & 3.33 \end{bmatrix}$$

Coefficients of the transformed reduced stiffness matrix, $\overline{Q}_{ij}$ [GPa]

$\theta = 0^o$

$$[\overline{Q}] = \begin{bmatrix} 53.69 & 2.74 & 0 \\ 2.74 & 9.48 & 0 \\ 0 & 0 & 3.33 \end{bmatrix};$$

$\theta = 60^o$

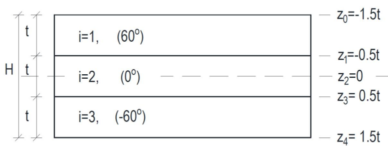
$$[\overline{Q}] = \begin{bmatrix} 12.21 & 11.06 & 4.77 \\ 11.06 & 34.32 & 14.38 \\ 4.77 & 14.38 & 11.65 \end{bmatrix};$$

$\theta = -60^o$

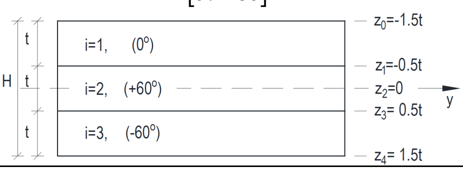
$$[\overline{Q}] = \begin{bmatrix} 12.21 & 11.06 & 4.77 \\ 11.06 & 34.32 & 14.38 \\ 4.77 & 14.38 & 11.65 \end{bmatrix}$$

Macromechanics of the laminate

$[+60/0/-60]$



$[0/\pm 60]$



The extensional stiffness matrix [A], [kN/mm]

$$[A] = \begin{bmatrix} 31.25 & 9.94 & 0 \\ 9.94 & 31.25 & 0 \\ 0 & 0 & 10.65 \end{bmatrix}$$

Table 2
Continuation

The bending-stretching coupling matrix [*B*], [kN]

$[B]=\begin{bmatrix} 0 & 0 & -1.53 \\ 0 & 0 & -4.6 \\ -1.53 & -4.6 & 0 \end{bmatrix}$	$[B]=\begin{bmatrix} -6.64 & 1.33 & -0.76 \\ 1.33 & 3.97 & -2.3 \\ -0.76 & -2.3 & 1.33 \end{bmatrix}$

The bending stiffness matrix [D], [kN·mm]

$[D]=\begin{bmatrix} 1.98 & 1.55 & 0 \\ 1.55 & 4.81 & 0 \\ 0 & 0 & 1.63 \end{bmatrix}$	$[D]=\begin{bmatrix} 4.63 & 1.02 & -0.31 \\ 1.02 & 3.22 & -0.92 \\ -0.31 & -0.92 & 1.10 \end{bmatrix}$

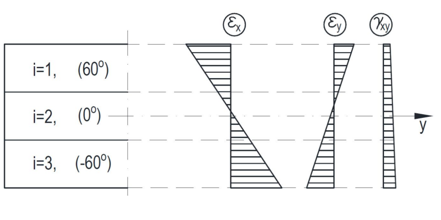
The mid-plane strains and curvatures of the laminate $\left(\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0, k_x, k_y, k_{xy}\right)$

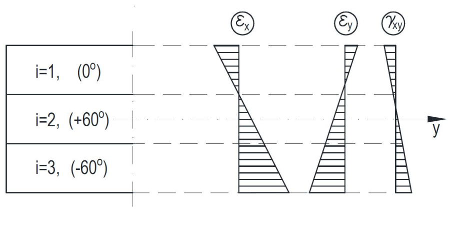
(0.036; –0.011; 0.049; 6.75; –2.126; 2.32×10^{-3})	(1.058; –0.343; 8.14×10^{-3}; 4.055; –1.266; 0.072)

Strains $\left(\varepsilon_x, \varepsilon_y, \gamma_{xy}\right)$ and distribution of strains on laminate thickness

$[+60/0/-60]$		ε_x	ε_y	γ_{xy}
i = 1	sup.	–4.014	1.265	0.048
	inf.	–1.314	0.414	0.049
i = 2	sup.			
	z = 0	0.036	–0.011	0.049
	inf.	1.386	–0.436	0.049
i = 3	sup.			
	inf.	4.086	–1.287	0.05

$[0/\pm60]$		ε_x	ε_y	γ_{xy}
i = 1	sup.	–1.375	0.417	–0.035
	inf.	0.247	–0.09	–0.006
i = 2	sup.			
	z=0	1.058	–0.343	0.008
	inf.	1.869	–0.596	0.023
i = 3	sup.			
	inf.	3.491	–1.103	0.051



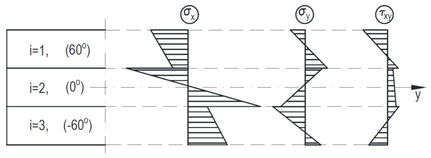


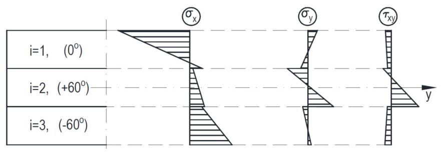
Stresses $\left(\sigma_x, \sigma_y, \tau_{xy}\right)$ and distribution of stresses on laminate thickness, [kN/mm²]

$[+60/0/-60]$		σ_x	σ_y	τ_{xy}
i = 1	sup.	–34.808	–0.305	–0.406
	inf.	–11.235	0.382	0.254
i = 2	sup.	–69.413	0.328	0.162
	z = 0	1.903	–0.006	0.163
	inf.	73.219	–0.34	0.165
i = 3	sup.	11.866	–0.354	0.238
	inf.	35.431	0.306	–0.4

$[0/\pm60]$		σ_x	σ_y	τ_{xy}
i = 1	sup.	–72.682	0.184	–0.117
	inf.	13.015	–0.175	–0.021
i = 2	sup.	1.993	–0.44	–0.186
	z=0	9.166	0.046	0.209
	inf.	16.339	0.532	0.604
i = 3	sup.	16.124	0.116	–0.078
	inf.	30.195	0.028	–0.197

Table 2
Continuation





Significant differences between the two quasi-isotropic laminates can be observed according to the results presented in Table 2, even if the only variable parameter is the stacking sequence. The stack of the plies through the thickness of the laminate is determinant for its symmetry. Therefore, in case of this analysis, two different types of quasi-isotropic laminates resulted when choosing different stacking sequences: an anti-symmetric laminate and an asymmetric one. A laminate is considered anti-symmetric when the material and the thickness of the layers are the same with respect to the middle surface, but the ply orientation has opposite sign related to the mid-plane. Asymmetric laminates suggest that there is no symmetry with respect to the middle surface.

As a consequence of the resulted laminates, significant differences are noticed between the two types of the assembled *ABD* matrices, which are useful in predicting the behaviour of multi-layered composites.

The stacking sequence is not involved in the micro and macro mechanics of lamina, where the only influencing parameters are the properties of the constituents of the composite material and fibre orientation angles.

The extensional stiffness matrix [*A*] is a function of the layer thicknesses $\left(z_k - z_{k-1}\right)$ and of the transformed reduced stiffness matrix $\left[\overline{Q}\right]$, as shown in Eq. (6a), but it is independent of the stacking sequence. Therefore, matrix [*A*] is the same for both quasi-isotropic laminates. The terms $A_{16} = A_{26} = 0$, meaning that the matrix [*A*] has an isotropic response and that the extension and the shear are uncoupled.

The bending-stretching coupling matrix [*B*] is dependent on the stacking sequence through the term $\left(z_k^2 - z_{k-1}^2\right)$. For a symmetric laminate, the coefficients $B_{ij} = 0$ and, in contrast, for the $\left[0/\pm 60\right]$ asymmetric quasi-isotropic laminate, $B_{ij} \neq 0$, so B_{16} and B_{16} are nonzero, meaning that the twisting-shearing coupling and the bending-shearing coupling occurs. In case of $\left[+60/0/-60\right]$ laminate, the terms $B_{11} = B_{22} = B_{12} = B_{66} = 0$, but B_{16} and B_{16} have nonzero values, which is true in general for anti-symmetric laminates.

From Eq. (8*a*) it can be observed that the bending matrix [*D*] is dependent on the stacking sequence through the term $\left(z_k^3 - z_{k-1}^3\right)$. The obtained results reveal that there is no bending-twisting coupling in the case of $[+60/0/-60]$ anti-symmetric laminate, in comparison with the $[0/\pm 60]$ asymmetric laminate, where D_{16} and D_{26} are nonzero.

Table 2 shows that the strains vary linearly through the thickness, but they have distinct variation for the two types of quasi-isotropic laminates.

The variation of stresses through the laminate thickness is obtained by determining the stress variations in all layers. Because the transformed reduced stiffness matrix $\left[\overline{Q}\right]$ changes and it is dependent on material and orientation of each ply, the stresses are discontinuous at the interface of two plies and the stress gradient in two adjacent laminas is different.

4. Conclusions

According to the results presented in the case study, changing the lamina orientations while maintaining equal angles between the adjacent plies, means to obtain different [*B*] and [*D*] matrices, but same [*A*] matrix. Moreover, the stacking sequence has a great influence on strain and stress distributions, where significant differences appear between the two different laminates.

Interchanging the stacking sequence while maintaining the same ply thickness for each lamina results in a laminate whose response to axial loads is unchanged, but which may have different response to bending or different coupling effect may occur.

REFERENCES

Barbero E.J., *Introduction to Composite Materials Design*. Second Edition, CRC Press, Taylor & Francis Group, New York, 2011.

Daniel I.M., Ishai O., *Engineering Mechanics of Composite Materials.* Second Edition, Oxford Univ. Press, Oxford, New York, 2006.

Gibson R.F., *Principles of Composite Material Mechanics*. CRP Press, Taylor & Francis Group, Boca Raton, 2012.

Herakovich C.T., *Mechanics of Fibrous Composites*. Univ. of Virginia, John Wiley & Sons, Inc., United States of America, 1998.

Jones R.M., *Mechanics of Composite Materials*. Second Edition, Taylor & Francis, Inc., Philadelphia, 1999.

Kaw A.K., *Mechanics of Composite Materials.* Second Edition, CRC Press, Taylor & Francis Group, New York, 2006.

Staab G.H., *Laminar Composites*. Butterworth-Heinemann, Reed Elsevier Group, 1999.

Ţăranu N., Isopescu D., *Structures Made of Composite Materials*. Edit. Vesper, Iaşi, 1996.

Ţăranu N., Bejan L., *Mecanica mediilor compozite armate cu fibre*. Edit. Cermi, Iaşi, 2005.

Ţăranu N., Bejan L., Cozmanciuc R., Hohan R., *Materiale şi elemente composite*. I. *Prelegeri şi aplicaţii*. Edit. Politehnium, Iaşi, 2013.

INFLUENŢA SUCCESIUNII DE STIVUIRE A LAMELELOR ASUPRA DISTRIBUŢIEI TENSIUNILOR ŞI DEFORMAŢIILOR SPECIFICE LA STRATIFICATELE CVASI-IZOTROPE

(Rezumat)

Se prezintă o analiză comparativă cu privire la modul cum influenţează ordinea de stivuire a lamelelor distribuţia tensiunilor şi deformaţiilor pe grosimea stratificatelor cvasi-izotrope, solicitate în planul acestora. Pentru studiul de caz au fost alese două tipuri de stratificate cvasi-izotrope, care au aceleaşi unghiuri de orientare ale fibrelor, număr egal de straturi şi aceeaşi grosime a lamelelor, dar succesiune de stivuire diferită. Tensiunile şi deformaţiile specifice sunt determinate pentru fiecare strat în parte, obţinându-se distribuţia acestora pe toată grosimea stratificatului. Sunt discutate şi alte aspecte, în conformitate cu teoria stratificatelor, precum efectele de cuplare complexe specifice acestori tipuri de elemente compozite.