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ADAPTATION TO EURONORMS OF THE CALCULUS OF POLYGONAL HOLLOW GIRDERS

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Abstract. Polygonal hollow girders (named also castellated girders) represent an alternative for plain girders, mainly in average size spanning where net loads do not have high values and the condition for selecting the section is imposed by observing the requirements related to limit elastic deformations. In the paper, the calculus of polygonal hollow girders is adapted to the European design norms (SR EN 1993-1-1: 2006; SR EN 1993-1-5: 2008). The numerical example shown facilitates the understanding of the calculus base.

Key words: polygonal hollow girders; European design norms.

1. Introduction

After the 1950’s, structural engineering aimed at an as substantial as possible reduction of the costs related to metal constructions, by diminishing the consumption of steel as this material was, and still remains, at a high price.

Because of restrictive conditions related to admissible deformation, higher steel strength values could not always be properly valorised, so that verification s have been carried out to find structural shapes where the member stiffness increases without adding a higher amount of steel; one of the solutions

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envisaged was using to a larger extent hollow web girders, also called cellular beams.

Several advantages and disadvantages of hollow girders are enumerated below:

Advantages:

a) increased girder stiffness in vertical plane;

b) high strength/weight ratio;

c) smaller maintenance costs;

d) possibility of strengthening or replacing the girders;

e) hollow girders can also be used for allowing pipes, cables and similar to pass through, so that an efficient use of the space between the floor and ceiling is possible and an increase in the net space as well;

f) better appearance (beneficial from an architectural point of view).

Disadvantages:

a) different structural behaviour compared to plain girders, because of the holes present in the web;

b) many failure possibilities, as inside they are non-determined static structures which require a complex structural analysis, in some cases;

c) shear related deformations are important, the deflection ”exact” calculus being more complex than for plain girders;

d) a significant resistance to shear reduction because of present holes, so that where high shear force areas, holes are filled in with material or plain girder segments are added;

e) hollow corners lead to stress concentrations that constraint the application of polygonal hollow girders for potential dynamic loads;

f) the need to take measures to provide the stability of the members during mounting phase.

Historically, the first polygonal hollow girders appeared around year 1910, then were produced by Boyd the 1930’s (also called *Boyd girders*), and became widely produced in Great Britain at the beginning of the 1940’s, with various forms of hollows in the web (Dalban *et al.*, 1983).

The manufacturing technology was developed by the company Litzka Stahlbau from Bavaria, Germany, which also introduced the hollow octagonal shape by interposing spacing plates between the two components of the girder, implicitly leading to an increased stiffness in the members (important increase of the inertia moment), Fig. 1.

The structural metal members with hollows in the web (girders, columns, framework piles, etc.) are made from double T or H sections, cut longitudinally along a polygonal line (in zigzag) or a mixed path, made of straight line and semi-circular arches, the two resulting halves being then welded together after a relative translation; sometimes, spacing elements are added to increase the height of the resulting section (Fig. 2).

Cutting and welding operations are performed with high productivity automated systems, the additional cost of metal manufacturing being thus smaller as compared to the economy in steel (Fig. 2).

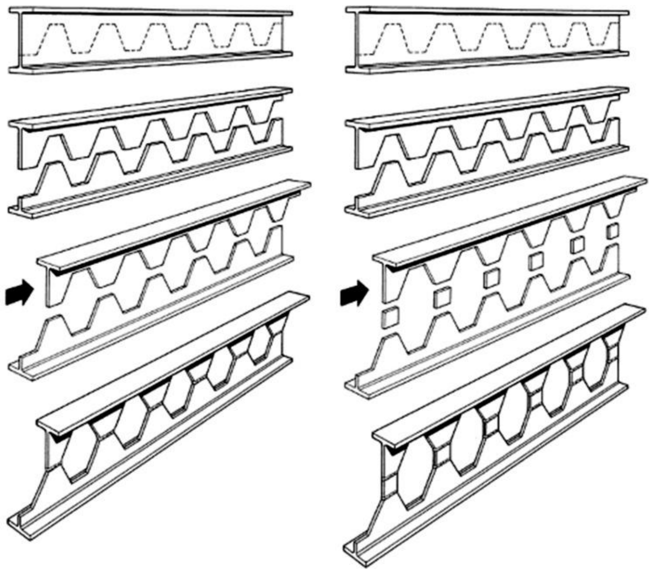


Fig. 1 – Cellular beams with hexagonal and octagonal openings (Castellated girders).

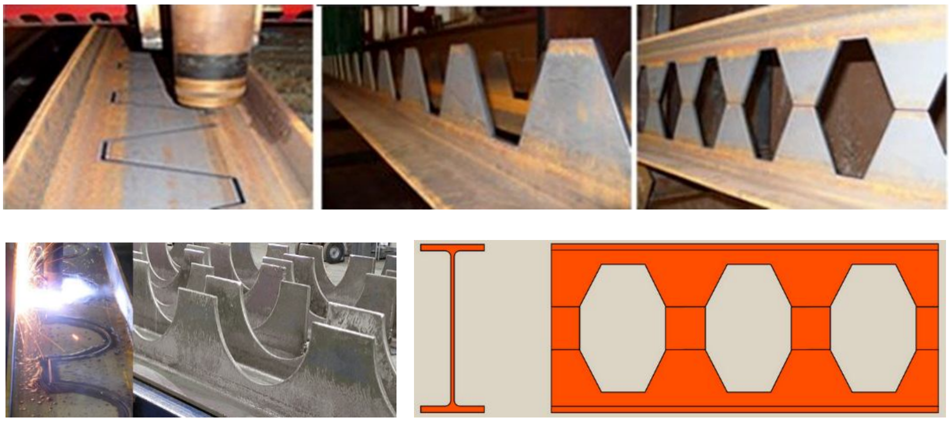


Fig. 2 – Cellular beams fabrication.

Two types of such hollow web girders are produced in this way:

a) girders with polygonal (rectangular) holes;

b) girders with circular holes.

Hollow girders are usually used for producing (simply supported or continuous) beams and for metal columns and frames, specific members and structures for civil and industrial constructions (Fig. 3).

In static systems, where in the same sections occur both bending moments and shear forces with high values (in the continuous beams – the area of intermediate supports, in frames in the area of joints), adjacent area holes are filled with metal sheet.

A solution to increase stiffness can be found by cutting from double **T** or **H**, a rolled section in two equal parts and interposing several welded plates, so that rectangular hollow girders are formed. This solution was used for a metal warehouse (Fig. 3), where the customer had a large amount of rolled section in stock.

Fig. 3 – Warehouse in the Prahova county.

2. Behaviour and Calculus of Polygonal Hollow Girders

The constructive configuration of the girder by including web holes leads to additional stresses coming from the shear force, the girder behaviour being similar to that of a Vierendeel girder, with very stiff posts and flexible flange.

The analysis and calculus of a hollow web girders requires certain verifications for strength, local and general stability (SLU) and verifications of the elastic deformations, and girder stiffness (SLS), respectively:

a) verification of girder chord strength to axial bending stress;
b) verification of girder strength to pure shear, at the hollow area;
c) verification of the support area (post and web);
d) verification of web shear buckling, in the plain area;
e) verification of concentrated forced action (if necessary);
f) verification of cross stiffening bars, where concentrated forces exist;
g) verification of general stability;
h) verification of elastic deformation.

The paper presents the verifications specific for hollow girders, the rest of verifications being common to those of plain girders.

2.1. Girder Stresses in the Hole Area

By admitting the hypothesis that inflexion points are formed in the middle of a hole and by uniformly distributing the shear force at the two chords, one can develop a calculus approach of a simplified form to calculate stresses and verification the girder section.

In the chord, an axial $N_{0.Ed}$ is developed, from the general girder bending stress, while at the ends of the hole bending moments of an opposite sign $M_{0.Ed}$ develop, while produced by shear (Fig. 4).

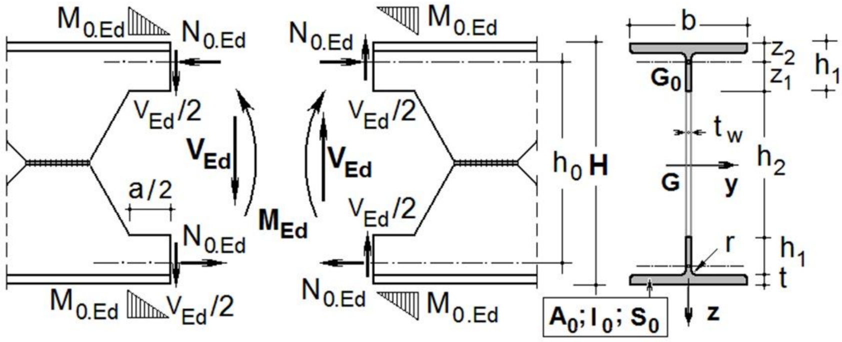


Fig. 4 – Loading around the girder openings.

In a current girder section where a bending moment M_{Ed} and a shear force V_{Ed} act, the values of the stresses $N_{0.Ed}$ and $M_{0.Ed}$, in the hole corner are given by the relationships:

$$N_{0.Ed} = \frac{M_{Ed}}{h_0}; \qquad M_{0.Ed} = \frac{V_{Ed}a}{4}. \tag{1}$$

The normal and shear stresses in the hole corner are:

$$\sigma_{0.Ed} = \frac{N_{0.Ed}}{A_0} \pm \frac{M_{0.Ed}}{I_0} z_i ; \quad \tau_{0.Ed} = \frac{(V_{Ed}/2)S_0}{I_0 t_w} . \tag{2}$$

where: A_0, I_0, S_0 represent the section area, the elastic inertia moment, the static moment of a T shaped chord, respectively; h_0 – the distance between the chord weight centres of T shaped section girder; a – the length of the hole horizontal right side; M_{Ed}, V_{Ed} – the girder stresses in the hole middle part.

The welding seams joining the two parts of the girder are calculated so as to be able to take over the sliding force developed on the distance between two consecutive holes (pitch p).

The connection (shear) force between the two holes can be found from the equilibrium condition of the joint (it is supposed that static stresses M_{Ed} and V_{Ed} remain constant along the distance between two consecutive holes), Fig. 5.

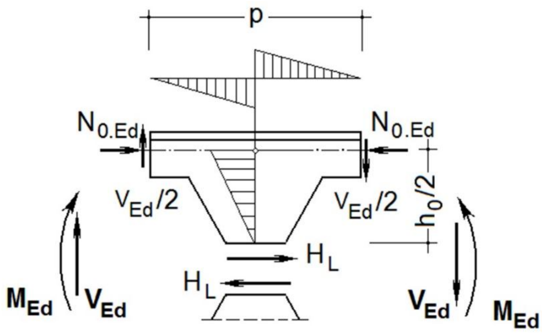


Fig. 5 – Shear evaluation.

The equilibrium relationship is:

$$H_L \frac{h_0}{2} = 2\frac{V_{Ed}}{2} \cdot \frac{p}{2} \Rightarrow \; H_L = \frac{V_{Ed}\, p}{h_0} . \tag{3}$$

It yields:

$$H_L = \frac{V_{Ed}\, p}{h_0} = \ell_s t_w \tau_{II} \Rightarrow \; \ell_s = \frac{V_{Ed}\, p}{h_0 t_w \tau_{II}}; \;\; b = \ell_s + 2t_w . \tag{4}$$

As the chord has a non-symmetrical form at the hole (T section), in the hole corner occur high normal unit stresses from the overlapping of stresses from the bending moment (produced by shear) and axial force (produced by the external bending moment), so that the stress can reach the elastic-plastic domain.

In Fig. 6 are given the stress state and its dynamics with increasing external forces of bending and shear.

If stresses M_{Ed} and V_{Ed} do not have maximum values in the same section of the girder, it is advisable to find that section which has the most detrimental normal stresses verification relationship or, as a variant, several sections are verified to get significant values in the static magnitude.

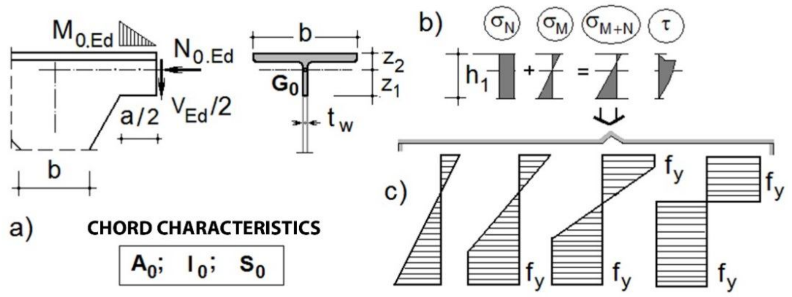


Fig. 6 – State of stresses and their evolution in the top flange.

For a simply supported beam of opening L, loaded with a uniformly distributed force q_{Ed} one can analytically determine the *critical* section of the normal stresses as follows:

$$\sigma_x = \frac{M_{Ed.x}}{A_0 h_0} + \frac{V_{Ed.x} a}{4W_{0.\min}}; \quad M_x = \frac{q_{Ed} L^2}{2}\left(\frac{x}{L} - \frac{x^2}{L^2}\right); \quad V_x = \frac{q_{Ed} L}{2}\left(1 - \frac{2x}{L}\right).$$

From condition, it yields:

$$\frac{\mathrm{d}\sigma_x}{\mathrm{d}x} = 0 \ \Rightarrow x$$

2.2. Verification of the Section Relative to the Hole, According to SR EN 1993-1 (EC 3)

Verification of the flange compression with bending

Classes 1 and 2 cross sections

It is verified whether the class of the compressed flange is 1 or 2 to perform the calculations in the plastic range.

The most unfavourable situation is taken into consideration, where the whole section **T** of the flange is uniformly compressed (Fig. 7).

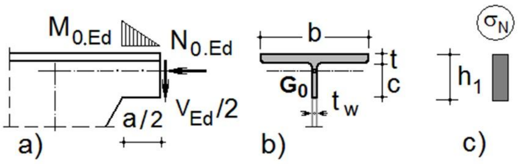


Fig. 7 – Loading of the compression flange.

The condition to be fulfilled for the web slenderness in the cantilever is:

$$\frac{c}{t_w}=\frac{h_1-(t+r)}{t_w}\leq 10\varepsilon;\ \ \varepsilon=\sqrt{\frac{235}{f_y}}=\begin{cases}1.00-S\,235;\\0.92-S\,275;\\0.81-S\,355.\end{cases} \tag{5}$$

If condition (5) is satisfied (the section belongs to Class 1 or Class 2), the flange resistant plastic moment will be:

$$M_{0.pl.Rd}=\frac{W_{0.pl}f_y}{\gamma_{M0}}, \tag{6}$$

where: $W_{0.pl}$ is the chord plastic resistance modulus.

The presence of the shear force reduces the design bending girder strength, but for lower values, respectively under 50% of the value of shear plastic strength, this reduction is insignificant and hence, negligible.

If $V_{Ed}\geq 0.5V_{pl.Rd}$, the bending design strength of the girder will decrease, by evaluating it with a reduced unit design stress in the shear area, where:

$$f_y^{'}=(1-\rho)\ f_y, \tag{7}$$

where: $\rho=\left(\frac{2V_{Ed}}{V_{pl.Rd}}-1\right)^2$

$$V_{pl.Rd}=\frac{A_v\left(f_y/\sqrt{3}\right)}{\gamma_{M0}}, \tag{8}$$

where: $A_v = 2(h_1 - t)t_w$.

As the chord is subjected to bending compression, the verification criterion (in the absence of the shear force or when $V_{Ed}\leq 0.5V_{pl.Rd}$) will be:

$$M_{0.Ed}\leq M_{0.N.Rd} \tag{9}$$

where: $M_{0.N.Rd}$ is the design strength bending moment, when the axial force is present:

$$M_{0.N.Rd} = M_{0.pl.Rd}\left[1 - \left(\frac{N_{0.Ed}}{N_{0.pl.Rd}}\right)^2\right], \tag{10}$$

where: $N_{0.pl.Rd} = \dfrac{A_0 \cdot f_y}{\gamma_{M0}}$

and the verification criterion becomes:

$$\frac{M_{0.Ed}}{M_{0.pl.Rd}} + \left(\frac{N_{0.Ed}}{N_{0.pl.Rd}}\right)^2 \leq 1.0\,. \tag{11}$$

Class 3 cross sections

The condition is verified:

$$\sigma_{0.x.Ed} \leq \frac{\sigma_{\lim}}{\gamma_{M0}} \leq \frac{f_y}{\gamma_{M0}}\,. \tag{12}$$

In section *x*, the normal stresses $\sigma_{0.x.Ed}$ in the chord are calculated from the bending moment and axial force with the relationship:

$$\sigma_{0.x.Ed} = \frac{M_{0.Ed.x}}{A_0 h_0} + \frac{V_{0.Ed.x}a}{4W_{0.el.\min}}\,. \tag{13}$$

Section resistance to shear

The design shear force should satisfy in very cross section the relationship:

$$\frac{V_{Ed}}{V_{c.Rd}} \leq 1.0\,, \tag{14}$$

where $V_{c.Rd}$ is the design shear strength, considered as follows:

$$V_{c.Rd} = \begin{cases} V_{pl.Rd} = \dfrac{A_v\left(f_y/\sqrt{3}\right)}{\gamma_{M0}} \text{ – the plastic design resistance to shear,} \\ \text{– the elastic design resistance to shear (in elastic calculus).} \end{cases} \tag{15}$$

To verifying the elastic design resistance to shear, the relationship below is applied:

$$\frac{\tau_{Ed}}{f_y/(\sqrt{3}\gamma_{M0})} \leq 1.0 . \tag{16}$$

The verification in the elastic stage covers requirements and excludes partial plastifying coming from shear.

Verification of the section in the full web area

The verification occurs according to SR EN 1993-1-1: 2006. Eurocode 3:

Design of steel structures.

Verification of the general stability

The general stability of the girder between two cross connections (or the lateral buckling) can be verified in a simplified form by calculating the stabile length of a bar sector as defined by the relationship:

$$L_{\text{stabile}} = \begin{cases} 35\varepsilon i_{0.z} & -\text{for:} \quad 0.625 \leq \psi \leq 1, \\ (60-40\psi)\varepsilon i_{0.z} & -\text{for:} \quad -1 \leq \psi \leq 0.625, \end{cases} \tag{17}$$

where: ψ is the ratio of bending moments at the ends of bar sector; $i_{0.z}$ – the radius of gyration of the compressed chord in relation with the weak axis of the sector:

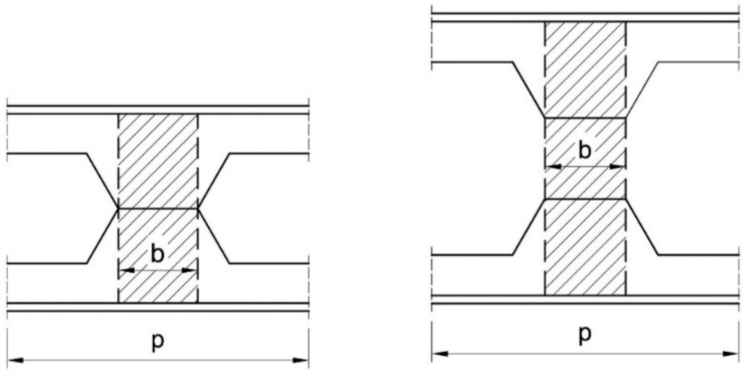
$$i_{0.z} = \sqrt{\frac{I_{0.z}}{A_0}} , \tag{18}$$

where: $I_{0.z}$ is the compressed chord inertia moment in relation with the weak axis of the sector; A_0 – the compressed flange area.

This approach is used in a similar form for plain girder (where $H/t_f \leq$ $\leq 40\varepsilon$).The verification of general stability with compressed chord buckling is detailed by Moga (2013).

Verification of the Web to Local Buckling

Due the fact that the web of the section is weaker because of the holes, it is necessary to verifying it in the area of full web, similar to a compressed post of section bxt_w and buckling length equal to web height (Fig. 8).



a) b)

Fig. 8 – Model for the web verification.

The compression force is considered equal to the highest value of the shear force V_{Ed} or concentrated force, if the local load is present.

Relative to high concentrated forces, cross stiffening bars can be provided.

Verification of the Girder Stiffness

Girder stiffness are verified by means of the sag, with the simplified relationship:

$$\delta = \frac{5}{48} \cdot \frac{M_{Ed.\max} L^2}{EI_y} \le \delta_a\,, \tag{19}$$

where: I_y is the average inertia moment of the girder.

A more exact calculation can be made by means of the Mohr-Maxwel method, with the relationship:

$$\delta = \int \frac{Mm}{EI} \mathrm{d}x + \int \frac{Nn}{EA} \mathrm{d}x + \int \frac{Vv}{GA} \mathrm{d}x, \tag{20}$$

where: *M, N, V* represent the bending moment, the axial force in the chords, the shear force respectively, produced by the actual load of the girder; *m, n, v* – represent the bending moment, the axial force in the chords, the shear force respectively, produced by the imaginary unit load; $P = \bar{1}$, applied in the point where the sag is calculated and according to its direction.

The octagonal hollow girders are verified similarly to the girders with hexagonal holes.

3. Numerical Example

The regular hexagonal hollow girder is verified for the strength of the compressed flange and the web shear stress.

The following design data are known:

– design stresses in the verified region: M_{Ed} = 3,400 kNm; V_{Ed} = = 1,400 kN;

– the basic girder: HE 1000 A; Steel: S 355;

– castellated girders, see Fig. 9;

– critical lengths: $L_{cr.y} = L_{cr.x} = L_{cr.T}$ = 4.00 m.

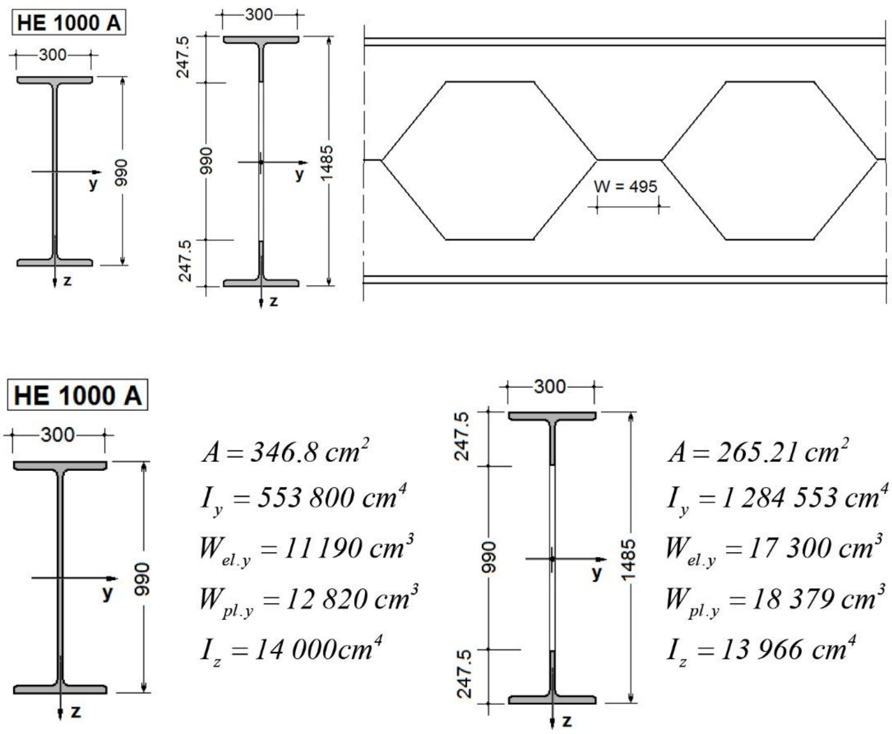


Fig. 9 – Characteristics of the castellated girders.

Solution:

Section class

Compressed flange: $\frac{c}{t_w} = \frac{\left[300-\left(16.5+2\times30\right)\right]/2}{31} = 3.6 < 9\varepsilon = 7.3 \Rightarrow$ Class 1

Web: $10\varepsilon = 8.1 < \dfrac{c}{t_w} = \dfrac{248-(31+30)}{16.5} = 11.3 < 14\varepsilon = 14.34 \Rightarrow$ Class 3

Resistance compression with bending

The castellated girder belongs to Class 3, and is verified by the condition:

$$\sigma_{0.Ed} \le \frac{\sigma_{\lim}}{\gamma_{M0}} \le \frac{f_y}{\gamma_{M0}}.$$

In the verified section, the normal unit stresses $\sigma_{0.x.Ed}$ in the chord are calculated from the bending moment and the axial force with the relationship:

$$\sigma_{0.Ed} = \frac{M_{0.Ed}}{A_0 h_0} + \frac{V_{0.Ed}\,a}{4W_{0.el.\min}} = \frac{3{,}400\times 10^4}{132.6\times 138.7} + \frac{1{,}400\times 10^2}{4\times 274.2} = 1{,}976\ \text{daN/cm}^2$$

$$\sigma_{0.Ed} < \frac{f_y}{\gamma_{M0}} = 3{,}550\ \text{daN/cm}^2;\ \ \frac{\sigma_{0.Ed}}{f_y/\gamma_{M0}} = 0.56\,.$$

Section resistance to shear

For the elastic range calculus, the design shear force should fulfil the relationship:

$$\frac{\tau_{Ed}}{f_y/(\sqrt{3}\gamma_{M0})} \le 1.0;$$

$$\tau_{Ed} \approx \frac{V_{Ed}}{A_V} = \frac{1{,}400\times 10^2}{71.6} = 1{,}955\,\text{daN/cm}^2;\ \ A_v \approx 2\cdot 21.7\cdot 1.65 = 71.6\ cm^2$$

It yields: $\dfrac{\tau_{Ed}}{f_y/(\sqrt{3}\gamma_{M0})} = \dfrac{1{,}955}{3{,}550/\sqrt{3}} = 0.95 < 1.0.$

4. Conclusions and Remarks

Castellated girders represent a viable alternative for plain girders, mainly when structures have an average spanning, in which net loads are not high and section selection depends on observing the requirements for ultimate elastic deformations.

The analysis and calculus of hollow web girders needs the performance of the following verification for strength, local and general stability (SLU) and elastic deformations, as well as girder stiffness (SLS):

a) verification of girder chord strength to axial bending stress;

b) verification of girder strength to pure shear, at the hollow area;

c) verification of the support area (post and web);

d) verification of web shear buckling, in the plain area;

e) verification of concentrated forced action (if necessary);

f) verification of cross stiffening bars, where concentrated forces exist;

g) verification of general stability;

h) verification of elastic deformation.

The girder presented in the numerical example satisfies the requirements for strength and stability as verified, the ultimate strengths for the girder being covered as follows:

a) the strength of the compressed chord to compression and bending: 56%;

b) the strength of the girder to pure shear: 95%.

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ADAPTAREA LA EURONORME A CALCULULUI GRINZILOR CU GOLURI POLIGONALE ÎN INIMĂ

(Rezumat)

Grinzile ajurate sunt o alternativă viabilă la grinzilor cu inimă plină, cu deosebire în cazul structurilor cu deschideri medii, la care încărcările utile nu au valori ridicate, iar condiția de alegere a secțiunii este impusă de respectarea cerințelor privind deformațiile elastice limită.

În lucrare se face o adaptare a calculului grinzilor cu goluri poligonale la normele europene de proiectare (SR EN 1993-1-1: 2006; SR EN 1993-1-5: 2008).

Exemplul numeric prezentat facilitează înțelegerea bazei de calcul teoretic.