BULETINUL INSTITUTULUI POLITEHNIC DIN IAȘI Publicat de Universitatea Tehnică "Gheorghe Asachi" din Iași Tomul LXI (LXV), Fasc. 4, 2015 Secția CONSTRUCȚII. ARHITECTURĂ

# COMPUTATION OF DISPLACEMENTS FOR WOOD BEAMS WITH HYBRID AREAS (WOOD - COMPOSITE STRIPS)

ΒY

## ALEXANDRU SECU and ANA-RALUCA $\mathrm{ROŞU}^*$

"Gheorghe Asachi" Technical University of Iaşi Faculty of Civil Engineering and Building Services

Received: December 10, 2015 Accepted for publication: December 280, 2015

Abstract. The major disadvantage of wood is the coefficient (modulus) of elasticity.

One way to mitigate this disadvantage is hybridisation of some areas from the structural elements through the use of strips, made of composite multi-layers.

The effectiveness of this method is measured by the difference between the displacements of the un-hybridised elements and those who are hybridised.

Finding the displacements for elements with hybrid zones, elements with bysteps inertia moments, is the subject of this paper.

The analysed elements are wood beams belonging to a spatial framework (SF) with continuity in nodes.

Three cases are highlighted by the upgrade of beams through hybridisation over large areas with multi-layers made up of composite lamellas with unidirectional reinforcement.

For the three cases, we determine the expressions of the displacements at the middle of the opening.

Expressions of displacements are a quick pre-dimensioning of elements from a timber spatial framework with continuity in nodes and wood beams with hybrid areas.

**Key words:** hybridization; strips; wood beams; timber spatial framework with continuity in nodes; composites; structural elements.

<sup>\*</sup>Corresponding author: *e-mail:* anaralucarosu@yahoo.com

### **1. Introduction**

The reduced density, the increased resistance, the easiness of processing and the pleasant appearance recommend wood as a structural material, for many types of constructions.

The use of wood is, most of the times, the result of comparison between its advantages, the ones previously mentioned, and the disadvantages of such a material.

From a structural point of view, the main disadvantage of wood is represented by the reduced value of the coefficient of elasticity along the fibres, approximately 100.000 daN/cm<sup>2</sup>. The negative effects of this disadvantage are highlighted at the beam-type elements and are translated in vertical displacement which usually lead to the re-engineering of the element within stiffness condition.

A modern solution for elimination of the effects of this disadvantage consists in hybridisation of some areas of the beam by passing from the transversal section of wood to the strip - wood of composite multi-layer.

As a result of these interventions, the beam is transformed from the element with a constant moment of inertia in an element with by-steps inertia moments.

The rapid quantification of the effects of hybridisation for some areas of the beams implies the determination of the displacement status of this element.

The current work proposes rapid relationships for the determination of these displacements.

## 2. Considerations Regarding Strips, Made of Composite Multi-Layers, Used for the Hybridisation of the Wood Sections

The axis system, at which a wood beam with hybrid portions is reported, is the one in Fig 1.



Fig. 1

It results that the elasticity modulus taken into consideration for determination of the vertical displacements,  $v_z$ , is  $E_x = E_l$  (the wood fibres are parallel with the x axis).

Consequently,  $E_{pc,x}$ , the elasticity modulus of the composite strips following the *x* direction, must be maximum (the lamellas within the composite multi-layer which form the strip is disposed in such a manner so as to a maximum modulus of elasticity will be obtained following the *x* direction).

This thing implies the fact that the multi-layer would be made up of lamellas with long fibres and the direction of the fibres in each lamella identical to x. In this manner,  $E_{1S}$ , the modulus following the 1st direction of the multilayer, is  $E_{1S} = E_{pc,x} = E_1$ .

The price of the matrix-type composites – long fibres being increased, the thickness of the strips of the multi-layers will be reduced (max. 4 mm).

The reduced thickness of the multi-layers implies that the strips will be disposed on the large areas of the beam. In this manner, the risk of the buckling occurrence of the composite strips is avoided.

# 3. Specific Cases for Computation of the $v_Z$ Displacement for Wood Beams with Hybrid Areas

For the exhaustive resolution of the problem, the wood beams are considered as belonging to a spatial framework (SF) with continuity in nodes. In this way, a beam which is removed from the frame is loaded as indicated in Fig. 2.



Fig. 2

The particular cases, when one or both of the nodes are not continuous, they are obtained through the annulment of one or both moments ( $m_i = 0$ ,  $m_j = 0$  or  $m_i = 0$  and  $m_i = 0$ ).

Starting from the beams in Fig. 2 the specific cases are the ones indicated in Figs. 3,...,5.



Fig. 3 – Case I. The wood beam does not have a sufficient bending rigidity and is hybridised on the (1-2a) area.



Fig. 4 – Case 2. The wood beam cannot undertake the  $m_i$  and  $m_j$  moment and is hybridised on the areas  $a_1$  and  $b_1$ .



Fig. 5 – Case 3. The wood beam cannot undertake the mi and mj moment and in the same time, the hybrid beam on the areas  $a_2$  and  $b_2$  does not have a sufficient stiffness and is additionally hybridised on the 2c area (this case can also be applied for the situation in which Mc (the mid – span Moment) cannot be taken over by the wood beam).

In Figs. 1,...,5 the following relations are made: l – computation length of the beam; a – length of each area of the beam (two areas), in case 1, on which the transversal sections are made only of wood; (1 - 2a) – length of the area of the beam, in case 1, on which the transversal sections are hybrids (wood – composite strip at the lower part);  $a_1$  and  $b_1$  – the lengths on the portions of the beam, in case 2, on which the transversal sections are hybrids (wood – composite strip at the upper part);  $[l - (a_1 + b_1)]$  – the lengths on the portions of the beam, in case 2, on which the transversal sections are made only of wood;  $a_2$  and  $b_2$  – the lengths on the areas of the beam, in case 3 on which the transversal

sections are hybrids (wood – composite strip at the upper part); 2c – the lengths on the areas of the beam, in case 3, on which the transversal sections are hybrids (wood – composite strip at the lower part);  $a_3 = [1/2 - (a_2 + c)]$  and  $b_3 = [1/2 - (b_2 + c)]$  – the lengths on the areas of the beam, in case 3, on which the transversal sections are made only of wood; I – the inertia moment of the transversal section made only of wood, for all cases (1; 2 and 3);  $I_1$  – the inertia moment equivalent in wood, of the hybrid section, irrespective of the position of the composite strip for all cases (1; 2 and 3);  $m_i$  and  $m_j$  – moments on the *i* and *j* bar edges, resulted from the structural computation of the spatial framework (SF).

### 4. Computation Method Used for Case 1.

The computation method used in this work, for determination of  $v_Z$  displacements is Mohr Maxwell.

For case 1 the steps for applying the method are the following:

a) the acceptance of the reference displacement  $v_Z(l/2)$ ;

b) application of the principle related overlapping of effects:

$$v_{Z}(l/2) = v_{Z,m_{i}}(l/2) + v_{Z,p}(l/2) + v_{Z,m_{j}}(l/2),$$
(1)

where:  $v_z(l/2)$  – the displacement at the middle of the opening, produced by all loads in case 1 ( $m_i$ ,  $m_j$  and p);  $v_Z(l/2)_{F_i}$  – the displacement at the middle of the opening, produced by a Fi load, Fi being either  $m_i$ ,  $m_j$  or p on the beam in case 1.

The displacement  $v_z(l/2)_{mi}$  was determined on the static scheme indicated in Fig. 6.



Fig. 6 – The static scheme and the load for the determination of the displacement  $v_z$  (1.2),  $m_i$ .

The expression of the moment provided by  $m_i$  in a x coordinate is:

$$M_{y,m_i}(x) = -m_i + \frac{m_i}{l}x.$$
 (2)

The expressions of the moment  $\overline{M}_{y,T}(x)$  is determined on a static scheme in Fig. 7 and are the following:

$$\overline{M}_{y,\overline{1}}(x) = \frac{x}{2} \text{ on the interval } \left[0, \frac{l}{2}\right],$$

$$\overline{M}_{y,\overline{1}}(x) = \frac{l-x}{2} \text{ on the interval } \left[\frac{l}{2}, l\right].$$

$$(3)$$

$$\overbrace{l}^{\overline{M}_{y,\overline{1}}} \underbrace{l/2}_{\overline{2}} \underbrace{l/2} \underbrace{l/2}_{\overline{2}} \underbrace{l/2}_{\overline{2}} \underbrace{l/2}_{\overline{2}} \underbrace{l/2}_{\overline{2}} \underbrace{l/2} \underbrace{l/2$$

Fig. 7 – The static scheme and the load for the determination of the  $\overline{M}_{y,\overline{1}}(x)$  moment.

The integration was performed on the intervals: 0-a; a-l/2; l/2 - (l-a); (l-a) - l.

The following expressions for the displacements were obtained:

$$\begin{aligned} v_{Z,0-a}\left(\frac{l}{2}\right) &= -\frac{m_{i}a^{2}}{12EII}(3l-2a), \\ v_{Z,a-l}\left(\frac{l}{2}\right) &= -\frac{m_{i}a^{2}}{24EI_{1}l}\left[l^{3}-2a^{2}\left(3l-2a\right)\right], \\ v_{Z,l/2-(l-a)}\left(\frac{l}{2}\right) &= -\frac{m_{i}}{48EII}\left(l^{3}-8a^{3}\right), \\ v_{Z,(l-a)-l}\left(\frac{l}{2}\right) &= -\frac{m_{i}a^{3}}{6EIl}(3l-2a). \end{aligned}$$

$$(4)$$

Through the summing of the displacements on the intervals, the expressions (4), the following was obtained

$$v_{Z(l/2),m_i} = -\frac{m_i}{48EI} \left( \frac{12a^2l}{I} + \frac{3l^3 - 12a^2l}{I_1} \right).$$
(5)

Starting from the symmetry of the beam and from the expression (5)  $v_Z(l/2)$  will be obtained

$$v_{Z(l/2),m_j} = -\frac{m_j}{48EI} \left( \frac{12a^2l}{I} + \frac{3l^3 - 12a^2l}{I_1} \right).$$
(6)

The displacement  $v_{Z(l/2), p}$  was determined on the static scheme in Fig. 8.



The expression of the moment provided by *p* in a *x* coordinate, on the interval 0 - l/2, is:

$$M_{y,p}(x) = \frac{p}{2} \left( l_x - x^2 \right).$$
(7)

Using the symmetry and the relation (3) we obtain

$$v_{Z,0-a}(l/2) = \frac{p}{24EI} (4la^3 - 3a^4), \tag{8}$$

$$v_{Z,a-\frac{l}{2}}(l/2) = \frac{p}{384EI_1} (5l^4 - 64la^3 + 48a^4).$$
(9)

Through the summing of the two displacements and the use of symmetry  $v_{Z(l/2), p}$  is obtained:

$$v_{Z,p}(l/2) = \frac{5pl^4}{384EI_1} + \frac{pa^3(4l-3a)(I_1-I)}{24EI_1I}.$$
 (10)

The expression of the displacement  $v_Z(l/2)$  is obtained through the summing of the expressions (5,) (6) and (10).

$$v_{Z}(l/2) = \frac{5pl^{4}}{384EI_{1}} + \frac{pa^{3}(4l-3a)(I_{1}-I)}{24EI_{1}I} - \frac{1}{48EI} \left(\frac{12a^{2}l}{I} + \frac{3l^{3}-12a^{2}l}{I_{1}}\right) (m_{i}+m_{j}).$$
(11)

For the usual situation, a = l/4, the following is obtained

$$v_{Z}(l/2)\left(a=\frac{l}{4}\right) = \frac{pl^{4}(13I_{1}+67I)}{6,144EII_{1}} - \frac{\left(m_{i}+m_{j}\right)l^{2}\left(I_{1}+3I\right)}{64EII_{1}}.$$
 (12)

# 5. The Results Obtained for Cases 2 and 3

The displacements from the middle of the span of the beams from fig. 4 and fig. 5, case 2 (c<sub>2</sub>) and case 3 (c<sub>3</sub>), is determined in a similar way with  $v_{Z,c_1}(l/2)$  with the expressions:

$$v_{Z,c_{2}}(l/2) = \frac{5pl^{4}}{384EI} - \frac{p(I_{1}-I)}{48EI_{1}I} \Big[ a^{3} (4l-3a_{1}) + b_{1}^{3} (4l-3b_{1}) \Big] - \frac{(m_{i}+m_{j})l^{2}}{16EI} + \frac{I_{1}-I}{12EI_{1}Il} \Big\{ m_{i} \Big[ a_{1}^{2} (3l-2a_{1}) + 2b_{3} \Big] + m_{j} \Big[ b_{1}^{2} (3l-2b_{1}) + 2a_{1}^{3} \Big] \Big\},$$
(13)

$$v_{Z,c_{3}}(l/2) = \frac{5pl^{4}}{384EI_{1}} - \frac{p(I_{1}-I)}{48EII_{1}} \Big[ 2d^{3}(4l-3d) - a_{2}^{3}(4l-3a_{2}) - b_{2}^{3}(4l-3b_{2}) \Big] - \frac{(m_{i}+m_{j})l^{2}}{16EI_{1}} - \frac{m_{i}(I_{1}-I)}{12EI_{1}Il} \Big[ 3ld^{2} - a_{2}^{2}(3l-2a_{2}) - 2b_{2}^{3} \Big] - (14) - \frac{m_{j}(I_{1}-I)}{12EI_{1}Il} \Big[ 3ld^{2} - b_{2}^{2}(3l-2b_{2}) - 2a_{2}^{3} \Big].$$

with d = (l/2) - c.

## 6. Conclusions

The expressions (12), (13) and (14) are rapid instruments for the verification of the displacement of the wood beam with hybrid areas.

They are applied after the performance of the spatial framework (SF) computation with no wood hybrid areas, in order to perform hybrid areas caused by the local lack of fulfilment of the stiffness verifications.

## REFERENCES

Secu Al., Roșu A.R., *Structuri în cadre din lemn cu porțiuni hibride (lemn – platbande compozite)*. Ed. Societății Academice "Matei–Teiu Botez", Iași, 2013.

Secu Al., Structures en materiaux composites. Ed. Document, Iași, 1997.

Rouby D., Materiaux composites. I.N.S.A. de Lyon, 1975.

- Secu Al., Isopescu D., Țăranu N., Metodologie modern pentru determinarea caracteristicilor necesare proiectării materialelor composite cu matrice polimerică. Revista Materiale de construcții, 3, 206-209 (1997).
- Secu Al., Isopescu D., Enţuc I., Dinga F. Structural Rehabilitation of Wooden Structural Members Using Multi – Layered Composite Materials. Proc. Internat. Conf. VSU 22-23 may, Sofia, Bulgaria, I, pag. II205-II210, 2006.
- Secu Al., Isopescu D., Enţuc I., Dinga F., Structural Rehabilitation of Wooden Structural Bars Subjected to Compression and Bending Using Multi – Layered Composite Jackets. Proc. of the National Symp. with Internat. Participation Dedicated to the day of Faculty of Civil Engineering of Iaşi, Ed. Societății Academice "Matei Teiu-Botez", Iaşi, pag.203-210, May 19, 2006.
- Secu Al., Isopescu D., Enţuc I., Dinga F., Structural Rehabilitation of Wooden Beams Using Multi – Layered Composite Jackets. Proc. of the National Symp. with Internat. Participation Dedicated to the day of Faculty of Civil Engineering of Iaşi, Ed. Societății Academice "Matei Teiu-Botez", Iaşi, May 19, 2006.

## CALCULUL DEPLASĂRILOR GRINZILOR DIN LEMN CU PORȚIUNI HIBRIDE (LEMN – PLATBANDE COMPOZITE)

#### (Rezumat)

Dezavantajul major al lemnului este valoarea scăzută a modulului de elasticitate longitudinal.

Una din metodele de atenuare a acestui dezavantaj este hibridarea unor zone din elementele structurale prin utilizarea platbandelor, realizate din stratificate compozite.

Eficiența acestei metode se măsoară prin diferența dintre deplasările elementelor nehibridate și deplasările elementelor hibridate.

Aflarea deplasărilor pe elemente cu zone hibride, elemente cu moment de inerție în trepte, constituie subiectul prezentei lucrări.

Elementele analizate sunt grinzile unui cadru spațial cu continuitate in noduri.

Sunt evidențiate trei cazuri de upgradare a grinzilor prin hibridare pe zonele întinse cu stratificate realizate din lamele compozite ce au faza de armare unidirecționala.

Pentru cele trei cazuri sunt determinate expresiile săgeților la mijlocul deschiderii.

Expresiile săgeților reprezintă un instrument rapid de predimensionare a elementelor structurilor în cadre din lemn, cu continuitate în noduri și grinzi cu porțiuni hibride.