THEORETICAL MODELS IN THE STUDY OF TEMPERATURE EFFECT ON STEEL MECHANICAL PROPERTIES

BY

ANA-DIANA ANCAȘ and D. GORBĂNESCU

The governing laws adopted by two high qualified scientific organizations (D.T.U. and Eurocode) for the variation of steel mechanical properties at high temperatures are presented.

There are no significant differences between them, but generally, the items advanced by Eurocode 3, part 1.2, are adopted in the most works that approach the thermical-mechanical analysis of steel structures.

1. Introduction

The effect of temperature on the mechanical properties of steel is well studied in literature [1]. From the great amount of results presented in different works regarding the laws adopted in the analysis of steel elements and structures, thermical-mechanical response at high temperatures, in this paper are studied only those according to D.T.U. [2] and Eurocode 3, part 1.2. [3].

2. Governing Laws for Temperature Effect on Steel Mechanical Properties

When the steel elements and structures are subjected to high temperatures (fire) they progressively lose their stiffness and carrying capacity because Young’s $E$ modulus and the elasticity limit, $\sigma_e$, are decreasing.

2.1. Governing Equations According to D.T.U.

D.T.U. proposes a set of relations referring to steel behaviour in the elastic range, at high temperatures.

a) Effect of temperature on Young’s modulus

According to D.T.U., the variation of Young’s modulus must be considered only up to $1,000^\circ C$, because for greater temperatures, the steel has no mechanical resistance.

Young’s modulus variation (Fig. 1) is defined by the relations

\[
\frac{E(\theta)}{E(20)} = 1.0 + \frac{\theta}{2,000 \ln \left( \frac{\theta}{1,100} \right)}, \quad \text{for} \quad 20^\circ C < \theta \leq 600^\circ C,
\]
\[ \frac{E(\theta)}{E(20)} = \frac{690 - 0.69\theta}{\theta - 53.5}, \text{ for } 600^\circ C < \theta \leq 1,000^\circ C, \]

where \( E(\theta) \) is Young's modulus at temperature \( \theta \) and \( E(20) \) is Young's modulus at 20\(^\circ \)C.

Fig. 1: Dependence of Young modulus vs. temperature, proposed by D.E.U.

b) **Effect of temperature on the elastic limit**

The variation of steel elastic limit (Fig. 2) is described by the following relations, according to D.T.U.:

\[ \frac{\sigma_e(\theta)}{\sigma_e(20)} = 1.0 + \frac{\theta}{900 \ln \left( \frac{\theta}{1,750} \right)}, \text{ for } 20^\circ C < \theta \leq 600^\circ C, \]

Fig. 2: Dependence of steel elastic limit vs. temperature, proposed by D.E.U.

\[ \frac{\sigma_e(\theta)}{\sigma_e(20)} = \frac{340 - 0.34\theta}{\theta - 240}, \text{ for } 600^\circ C < \theta \leq 1,000^\circ C, \]
where \( \sigma_e(\theta) \) is the elastic limit at temperature \( \theta \) and \( \sigma_e(20) \) – the elastic limit at \( 20^\circ C \).

3. Governing Equations According to Eurocode 3

Eurocode 3, part 1.2. [3], defines the variation of steel mechanical properties at high temperatures for a heating rate situated between \( 20^\circ C/min \) and \( 50^\circ C/min \), and has been determined for the temperatures between \( 20^\circ C \) and \( 1,200^\circ C \).

3.1. Stress - Strain Relations

The graphical representation of the stress - strain relation proposed by Eurocode 3, part 1.2., has a linear-elliptic shape (Fig. 3) and is described by the relations contained in Table 1.

![Graph showing stress-strain dependence proposed by Eurocode 3, part 1.2.](image)

**Fig. 3.** Stress-strain dependence proposed by Eurocode 3, part 1.2.

<table>
<thead>
<tr>
<th>Elastic zone</th>
<th>( \sigma(\varepsilon, \theta) = E(\theta)\varepsilon )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_I(\varepsilon, \theta) = E(\theta) )</td>
</tr>
</tbody>
</table>

**Elliptical zone**

\[
\sigma(\varepsilon, \theta) = \frac{b}{\sqrt{a^2 - [\varepsilon_e(\theta) - \varepsilon]^2}} + \sigma_p(\theta) - c,
\]

\[
E_I(\varepsilon, \theta) = \frac{b[\varepsilon_e(\theta) - \varepsilon]}{\sqrt{a^2 - [\varepsilon_e(\theta) - \varepsilon]^2}}.
\]

where

\[
a^2 = \frac{E(\theta)[\varepsilon_e(\theta) - \varepsilon_p(\theta)]^2 + c[\varepsilon_e(\theta) + \varepsilon_p(\theta)]^2}{E(\theta)},
\]

\[
b^2 = \frac{E(\theta)[\varepsilon_e(\theta) - \varepsilon_p(\theta)]^2 + c^2}{[\varepsilon_e(\theta) - \varepsilon_p(\theta)]^2},
\]

\[
c = \frac{2(\varepsilon_p(\theta) - \sigma_e(\theta) + E(\theta)[\varepsilon_e(\theta) - \varepsilon_p(\theta)])}{E(\theta)}.
\]

<table>
<thead>
<tr>
<th>Plastic zone</th>
<th>( \sigma(\varepsilon, \theta) = \sigma_e(\theta) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( E_I(\varepsilon, \theta) = 0 )</td>
</tr>
</tbody>
</table>

\[
\varepsilon_p(\theta) = \frac{\sigma_p(\theta)}{E(\theta)}, \quad \varepsilon_e(\theta) = 0.02, \quad \varepsilon_p(\theta) = 0.2.
\]

**Table 1**

Models for Stress-Strain Relations at High Temperature
3.2. Effect of Temperature on Steel Elastic and Proportional Limits

The elastic limit, $\sigma_e(\theta)$, the proportional limit, $\sigma_p(\theta)$, and the shape of the elastic range, $p(\theta)$, have been determined for temperatures situated between 20°C and 1,000°C.

The following dimensionless parameters are defined:

\[
K_e(\theta) = \frac{\sigma_e(\theta)}{\sigma_e(20)}, \quad K_p(\theta) = \frac{\sigma_p(\theta)}{\sigma_p(20)}, \quad K_E(\theta) = \frac{E(\theta)}{E(20)}.
\]

In Table 2 there are presented the variations of parameters $K_e(\theta)$, $K_p(\theta)$ and $K_E(\theta)$ caused by temperature increasing, in steps of 100°C. For other temperatures, a linear interpolation is admitted.

<table>
<thead>
<tr>
<th>Temperature, [°C]</th>
<th>20</th>
<th>100</th>
<th>200</th>
<th>300</th>
<th>400</th>
<th>500</th>
<th>600</th>
<th>700</th>
<th>800</th>
<th>900</th>
<th>1,000</th>
<th>1,100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_e(\theta)$</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
<td>0.78</td>
<td>0.47</td>
<td>0.23</td>
<td>0.11</td>
<td>0.06</td>
<td>0.004</td>
<td>0.2</td>
<td></td>
</tr>
<tr>
<td>$K_p(\theta)$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.90</td>
<td>0.80</td>
<td>0.70</td>
<td>0.60</td>
<td>0.31</td>
<td>0.18</td>
<td>0.09</td>
<td>0.065</td>
<td>0.0045</td>
<td>0.00225</td>
</tr>
<tr>
<td>$K_E(\theta)$</td>
<td>1.00</td>
<td>1.00</td>
<td>0.807</td>
<td>0.613</td>
<td>0.42</td>
<td>0.36</td>
<td>0.13</td>
<td>0.075</td>
<td>0.05</td>
<td>0.0375</td>
<td>0.0025</td>
<td>0.0125</td>
</tr>
</tbody>
</table>

The corresponding graph is represented in Fig. 4.

![Figure 4](image)

Fig. 4.– Dependence of parameters $K_e(\theta)$, $K_p(\theta)$ and $K_E(\theta)$ vs. temperature, proposed by Eurocode 3, part 1.2.

3.3. Effect of Temperature on the Tangent Modulus

The tangent modulus, $E_t(\theta)$, represents the shape of the characteristic diagram (relation stress-strain), when it has a non-linear shape, at temperature $\theta$.

Its variation, produced by the temperature increasing, is expressed by the following relations that are graphically represented in Fig. 5:

\[
\frac{E_t(\theta)}{E(20)} = 5 \times 10^{-5}\theta + 10^{-2}, \quad \text{for } 0 \leq \theta \leq 300^\circ\text{C},
\]
\[
\frac{E_i(\theta)}{E(20)} = -7 \times 10^{-5} \theta + 10^{-2}, \text{ for } 300^\circ C \leq \theta \leq 600^\circ C;
\]

\[
\frac{E_i(\theta)}{E(20)} = 4 \times 10^{-2}, \text{ for } \theta > 600^\circ C.
\]

![Graph](image)

Fig. 5.- The dependence between the tangent modulus vs. temperature.

### 3.4. Effect of Temperature on Poisson’s Ratio

The variation of Poisson’s ratio ($\gamma$) is defined by relations

\[
\gamma(\theta) = 3.78 \times 10^{-8} \theta + 0.283, \text{ for } 0 \leq \theta \leq 450^\circ C,
\]

\[
\gamma(\theta) = 9.20 \times 10^{-8} \theta + 0.259, \text{ for } \theta > 450^\circ C,
\]

and is represented in Fig. 6.

![Graph](image)

Fig. 6.- Dependence of Poisson’s ratio vs. temperature.
3.5. Residual Stress at High Temperatures

As concerns the residual stress, $\sigma_r$, the effect of temperature upon their values is the same as in case of elastic limit, so that, the following relation:

\[
\sigma_r(\theta) = 0.3\sigma_e(\theta).
\]

is adopted.

4. Conclusions

1. The comparative presentation of the laws that govern the temperature effect on steel mechanical properties shows no significant differences.
2. The theoretical models have been chosen in function of their simplicity.
3. In case of steel, the simple models are needed in the plastic design, which provides the quick and accurate evaluation of fire resistance.

Received, January 26, 2007  
"Gh. Asachi" Technical University, Jassy,  
Department of Structural Mechanics

REFERENCES


MODELE TEORETICE IN STUDIUL EFECTULUI TEMPERATURII
ASUPRA PROPRIETĂŢILOR MECANICE ALE OŢELULUI

(Rezumat)

Se prezintă comparativ legile adoptate de foruri cu mare competență (D.T.U. și Eurocode) pentru variația caracteristicilor elastice și mecanice ale oțelului la temperaturi înalte. Fără a sublinia diferențieri foarte însemnate, se poate aprecia faptul că propunerile din Eurocode 3, parte 1.2, au o prezență evazi-generală în lucrările axate pe analiza termo-mecanică a structurilor mecanice.