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THE EVALUATION OF LATERAL BUCKLING ON A BENDING BEAM USING THE FINITE ELEMENT METHOD

BY

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Abstract. This paper aims to determine the lateral displacement response of a beam which is acted upon by a bending force in the direction of maxim moment of inertia. The initial beam eccentricities are taken into consideration, also a non-linear geometric analysis can be done.

In accordance with Eurocode 3, the verification is performed using the next ratio $MEd/M_{b,RD} < 1$.

If the ratio yields a result smaller than 1, then we can say that the beam does not lose its later stability.

In the proposed study, by using the finite element method, we aim to obtain the vertical force-lateral displacement ratio of a beam which is acted upon by an incrementaly static load.

The structural response to the load can be analized in comparison with $M_{b,Rd}$.

Keywords: critical moment; bending; lateral buckling.

1. The Verification of Buckling by Sideway Discharge Using Eurocode 3

In the first phase we shall determine the critical moment as well as the capable moment of the element while taking into consideration the possibility of side buckling. The beam supports are considered as fork types which implies

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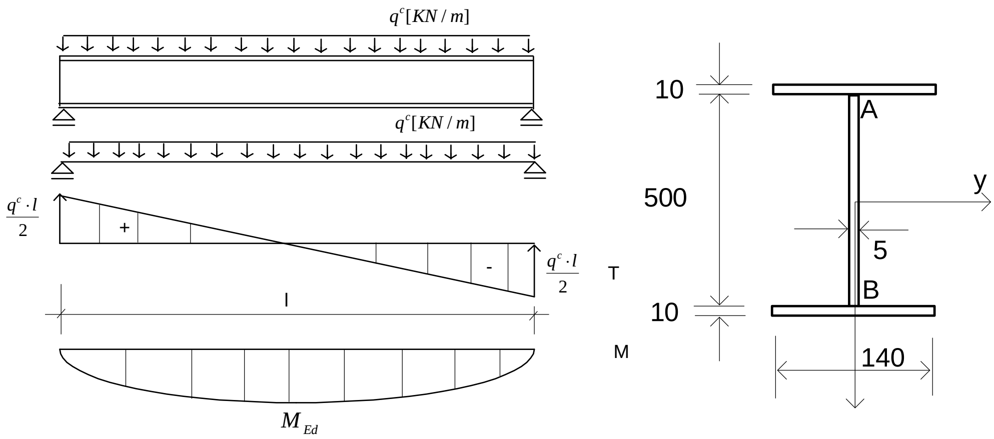
that free buckling can occur. The beam has a length of 12 m and the section from Fig. 1 *b*.

Geometrical and material characteristics:

Steel S355

I_y = 23,415 cm^4; W_y = 900 cm^3; I_z = 457 cm^4.



a *b*

Fig. 1 – *a* – Static and strain diagrams; *b* – beam section.

Torsional inertia momentum:

$$I_T = \frac{1}{3}\sum t^3 b = \frac{1}{3}(2\times 1^3 \times 14 + 0.5^3 \times 50) = 11.4 \text{ cm}^4. \quad (1)$$

Sectorial inertia momentum:

$$I_\omega = I_z\left(\frac{h_w + t_f}{2}\right)^2 = 457\left(\frac{50+1}{2}\right)^2 = 97{,}164 \text{ cm}^6. \quad (2)$$

Critical momentum calculation:

Double symmetrical section

$$\beta_f = 0.5 \rightarrow z_j = 0 \rightarrow c_3 z_j = 0,$$

$$z_g = \frac{h_w}{2} + t_f = \frac{500}{2} + 10 = 260 \text{ mm}, \quad (3)$$

$$M_{\mathrm{cr}} = c_1 \frac{\pi^2 EI_z}{L_{cr,z}{}^2} \left[\sqrt{\left(\frac{K_z}{K_w}\right)^2 \frac{I_\omega}{I_z} + \frac{L_{\mathrm{cr},z}{}^2 GI_T}{\pi^2 EI_z} + (c_2 z_g)^2} - c_2 z_g \right]. \tag{4}$$

The c_1, c_2, c_3 coefficients for the bent structural elements acted upon with direct loads are as follows:

c_1 = 1.127; c_2 = 0,454; c_3 = 0.525.

Resulting M_{cr} = 26 KN.m.

In order to determine the capable momentum we need a reduced slenderness ratio:

$$\overline{\lambda_{\mathrm{LT}}} = \sqrt{\frac{w_y f_y}{M_{\mathrm{cr}}}} = \sqrt{\frac{900 \times 10^3 \times 355}{26 \times 10^6}} = 3.505. \tag{5}$$

The buckling reduction factor calculation:

$$\phi_{\mathrm{LT}} = 0.5\left[1 + \alpha\left(\overline{\lambda}_{\mathrm{LT}} - \overline{\lambda}_{\mathrm{LT},0}\right) + \beta \overline{\lambda}_{\mathrm{LT}}^2\right] = 5.868, \tag{6}$$

where: $\overline{\lambda}_{\mathrm{LT},0} = 0.4$; $\beta = 0.75$; $\alpha = 0.49$,

$$\chi_{\mathrm{LT}} = \frac{1.0}{\varphi_{\mathrm{LT}} + \sqrt{\varphi_{\mathrm{LT}} - \beta \cdot \overline{\lambda}_{\mathrm{LT}}^2}} = \frac{1}{5.868 + \sqrt{5.868^2 - 0.75 \times 3.505^2}} = 0.092, \tag{7}$$

$$\frac{1}{\overline{\lambda}_{\mathrm{LT}}^2} = \frac{1}{3.505^2} = 0.081, \tag{8}$$

$$\chi_{\mathrm{LT}} = \min(0.092; 0.081) = 0.081, \tag{9}$$

$$M_{b,\mathrm{RD}} = \chi_{\mathrm{LT}} \frac{W_y f_y}{\gamma_{M1}} = 0.081 \frac{900 \times 10^3 \times 355}{1.0} = 25.88 \text{ KN.m}. \tag{10}$$

Initially the distributed load was not defined, which resulted in the next criterion:

$$M_{\mathrm{ED}} < M_{b,\mathrm{RD}}, \tag{11}$$

$$M_{\mathrm{ED}} = \frac{q^c l^2}{8} \to q^c = 1.437 \text{ KN/m}. \tag{12}$$

This is the maximum load, in accordance with Eurocode 3, under which the studied beam does not lose its stability to lateral buckling.

2. The Verification of Sideway Discharge Buckling Using the Finite Element Method

When a bent bar is acted upon with a load in the direction of its maximum inertia axis, the bar tends to lose its stability by buckling, this implies a lateral displacement(bending in the direction of its minimum inertia axis) as well as a sectional rotation(twisting along the direction of the longitudinal axis)

The element used in the analysis is Plane 3-D, which has the advantage of being able to model the bending of the beam, by using higher grade interpolation functions, thus producing nodes in the middle of the side of the element.

In order to obtain more realistic results, we operate with geometric nonlinearity because when the element loses its lateral stability the displacements are quite considerable, thus deeming necessary the reconstruction of the rigidity matrices for each step of the calculation.

The supports at each end of the beam were modeled as fork type and assure free buckling. The applied load is a uniformly distributed one and was incrementally applied, a maximum distributed load of 10 KN/m was divided in 100 calculation steps.

The imperfection used in the analysis is in accordance with the shape and amplitudes presented in Appendage VIII and sub-chapter VII.1

In Appendage VIII for the initial local imperfections in arch e_0 of the beams (initial curvatures) for the the buckling due to the bending, which are defined using the ratio e_0/l, yield the value of $l/300$.

To the finite element model, an initial imperfection of 4 cm (a curvature of the beam in the arch) was applied. The analysis was performed according to Appendage C of the SR EN 1993-1-5 standard.

Table 1

Hypotheses Used when Choosing the Method Based on Finite Elements

No.	Material behavior	Geometric behavior	Imperfections	Usage example
1	Linear	Non-linear	Yes	Elastic buckling resistance

3. Conclusions

The finite element method with geometric nonlinearity is used to determine the real value of the load on the beam at which the lateral stability is lost. This is done by interpreting the force-displacement ratios in Figs. 3 and 4.

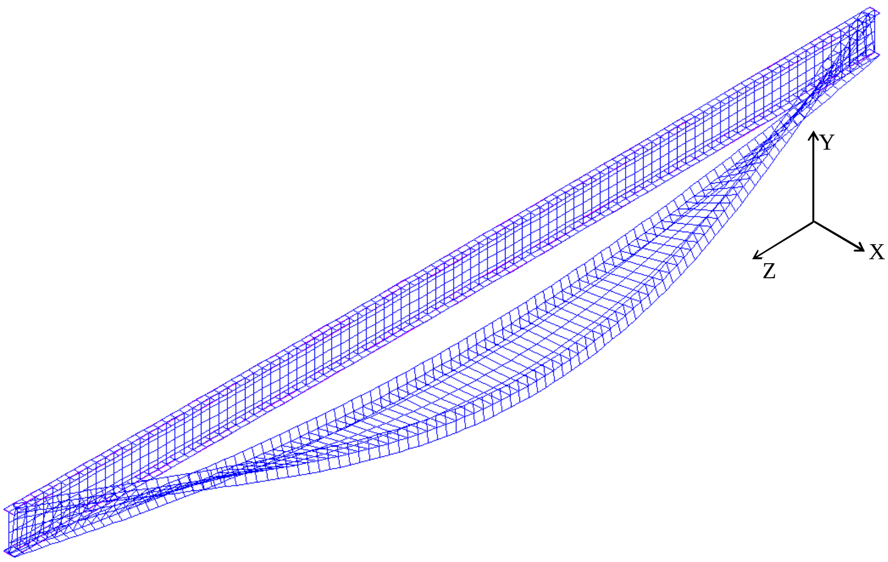


Fig. 2 – Deformed shape of imperfect beam acted on with a bending force.

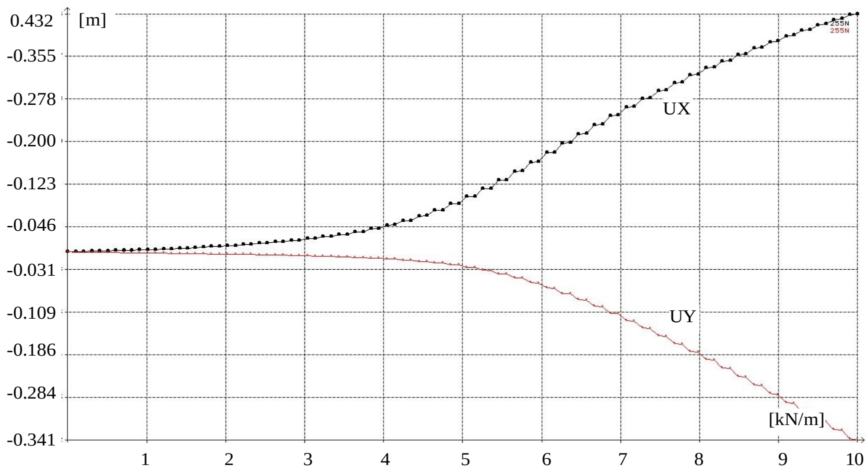


Fig. 3 – Force-displacement ratio for node A in Fig. 1 *b*; UX displacement curve in the direction of the eccentricities in the middle of the beam; UY vertical displacement curve.

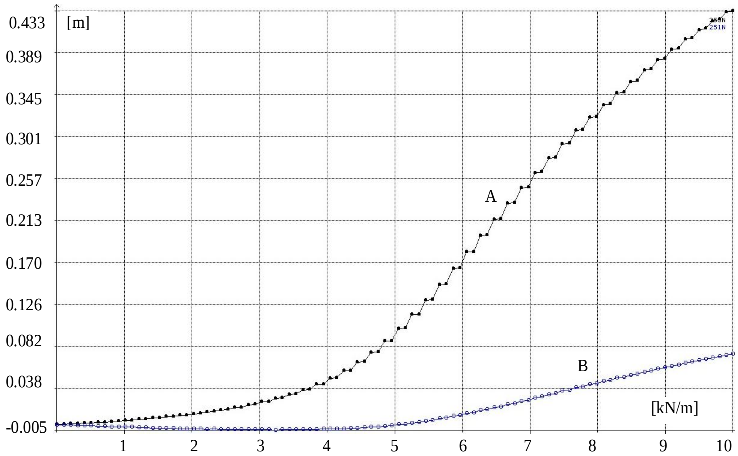


Fig. 4 – Force-displacement ratio in the direction of the eccentricity for nodes A and B.

In the analytical study if the bent beam, by using Eurocode 3, the beam loses its stability by sideway discharge buckling at a load of 1.43 KN/m.

According to Figs. 3 and 4 it can be said that the force-displacement ratio is linear until it reaches a value of 3–4 KN/m after which an asymptotic expansion can be observed.

The interpretation of these results can be made in the sense of a certain load reserve which the beam can handle up until the moment before the stability is lost.

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EVALUAREA FLAMBAJULUI LATERAL PENTRU O GRINDĂ ÎNCOVOIATĂ FOLOSIND ANALIZA PRIN ELEMENTE FINITE

(Rezumat)

Lucrarea îşi propune să determine răspunsul în deplasări laterale al unei grinzi solicitate la încovoiere pe direcţia momentului de inerţie maxim. Excenticităţile iniţiale

ale grinzii se iau in considerare, deasemenea poate fi realizată şi o analiză geometric neliniară.

Conform Eurocode 3 verificarea se face folosind relația MEd/$M_{b,RD}$ < 1. Dacă relația este îndeplinită grinda nu îşi pierde stabilitatea laterală.

În studiul propus, printr-o analiză cu FEM, pentru grinda încovoiată se doreşte să se obțină relația forță verticală-deplasare laterală pentru o încărcare statică incrementală.

Răspunsul structurii la încărcare poate fi analizat comparativ cu $M_{b,RD}$.