Abstract. This paper presents the analytical methods for beam-to-column joints performed with angle cleats and bolts. The analytical results are compared to reliable numerical results obtained through calibrating experimental data and extended using sensitivity analysis. Two different analytical methods namely the Frye & Morris and Kishi & Chen approaches were applied. The paper analyses the effectiveness of these models to evaluate the semi-rigid connections behaviour using lower / upper and two angle cleats on the beam’s web.

Keywords: angle cleat joints; semi-rigid joints; numerical analysis; FEM.

1. Introduction

It is well known that beam-to-column connections have stiffness, which is theoretical between rigid extremes and pinned (Csebfalvi, 2007). In practice of structural analysis, frame structures are considered traditionally either pinned or rigid. Actually connections defined as pinned possess some stiffness and those defined with rigid behaviour has some flexibility. Including the semi-rigid
joint behaviour in the frame analysis leads to a more realistic, reliable and economical results. The experimental researches are directed to evaluate the bending moment-rotation relationship of the connections (Alexa & Moldovan, 2005).

Although large amount of work, extensively documented is available in this research area, regarding semi-rigid connection behavior, they are not really used in practical design. Despite de fact that considerations of semi-rigid connection behaviour in steel frame design received special attention from researchers in the last decades, only a few frame design computational tools are applied the actual knowledge in common design practice (White & Chen, 1996). Moreover, the connection moment-rotation characteristics are available for a reduced number of joint typologies and design parameters.

This paper focuses on the joint typologies using angle cleats with bolted connections. Due to several advantages provided by such joints (reduced cost and reduced complexity of execution), their behavior were extensively studied by researchers through experimental, numerical and analytical assessments. Compared to welded joints, bolted connections are more ductile, have relatively high capacity to dissipate energy and can be fitted relatively easy.

The objective of this paper is to study the efficiency of two analytical methods, by comparing the results with numerical analysis calibrated on experimental tests (Reynosa, 2015). The numerical models were developed in previous study by the author, obtaining 4 calibrated numerical models (Ghindea et al., 2015).

Further using the reference model a sensitivity analysis was performed obtaining other results by virtual testing that are considered equivalent to the experimental results. The first 4 models represent a beam-to-column joint with angle cleat on the upper flange and 2 angle cleats on the web, and the other 4 models are joints with angle cleats only on the flanges. The difference between the both sets of 4 different numerical models is the angle cleats thickness used on the beam flanges (Table 1).

The purpose of numerical experimental and numerical research is to cover the results with analytical models. The abundance of the experimental results, and more recently of the numerical ones, cannot be predicted and covered by a single analytical model (Alexa & Moldovan, 2005). Moreover, for the most joint typologies were proposed numerous analytical models.

In these paper two analytical models results are presented, in order to evaluate both joint configurations proposed to analysis and presented above. The Fig. 1 below show the deformed shapes and stresses distribution for the two beam-to-column configuration using angle cleats obtained by numerical simulations.
Fig. 1 – Numerical results obtained on the two joint typologies.

Table 1
Joint Configuration Proposed to Analytical Evaluation

<table>
<thead>
<tr>
<th>Joint</th>
<th>Column</th>
<th>Beam</th>
<th>Flanges angle cleat</th>
<th>Web angle cleat</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Angle cleats on flanges and web</td>
<td>TSW-8 HEA 300 IPE 240</td>
<td>L120×90×8</td>
<td>L100×100×10</td>
</tr>
<tr>
<td>2</td>
<td>TSW-9 HEA 300 IPE 240</td>
<td>L120×90×9</td>
<td>L100×100×10</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>TSW-10 HEA 300 IPE 240</td>
<td>L120×90×10</td>
<td>L100×100×10</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>TSW-12 HEA 300 IPE 240</td>
<td>L120×90×12</td>
<td>L100×100×10</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>Angle cleats on flanges</td>
<td>TS-8 HEA 300 IPE 240</td>
<td>L120×90×8</td>
<td>–</td>
</tr>
<tr>
<td>6</td>
<td>TS-9 HEA 300 IPE 240</td>
<td>L120×90×9</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>TS-10 HEA 300 IPE 240</td>
<td>L120×90×10</td>
<td>–</td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>TS-12 HEA 300 IPE 240</td>
<td>L120×90×12</td>
<td>–</td>
<td></td>
</tr>
</tbody>
</table>

2. Analytical Models

This work aims to approach two analytical models for evaluating the semi-rigid behaviour of the presented joints configurations. In first step, the bending moment-rotation behaviour curve is modelled using the polynomial model developed by Frye and Morris (1985). This can be associated with the most classic nodes typologies used in structural engineering. The model does not include all the parameters of the beam-to-column connection. For example, the model developed by Frye and Morris, does not include the properties of the steel material which is an important factor for the joint capacity of distributing the bending moment. The constitutive relationship for polynomial model is:

\[
\theta_r = C_1 (K \times M)^1 + C_2 (K \times M)^3 + C_3 (K \times M)^5. \tag{1}
\]
The relative rotation is evaluated taking into consideration different values for bending moment. The bending moment-rotation behaviour curve is a multi-linear one, and its precision is determined by the number of increments chosen for different bending moment values. The $C_1$, $C_2$, $C_3$ parameters are determined depending on the connections type and parameter $K$ is determined based on geometrical parameters of the joint connecting elements. On the other side the equation for the coefficient $K$ varies depending on the joint typology. For the connection using two angle-cleat on the flanges and also two angle cleats on the web, the $C_1$, $C_2$, $C_3$ constants and $K$ parameter are defined as described by:

$$C_1 = 2.23 \times 10^{-5},$$
$$C_2 = 1.85 \times 10^{-8},$$
$$C_3 = 3.19 \times 10^{-12},$$
$$K = d^{-1.297} t^{-1.129} t_c^{-0.415} t_a^{-0.694} (g - d_b/l)^{1.350},$$

where the parameters for determining the $K$ coefficient are mentioned in Fig. 2.

![Diagram](image)

Fig. 2 – Parameters for evaluating the bending moment-rotation curve for a joint with angle cleat on flanges and web (Frye & Morris, 1975).

In case of connections with two angle cleats only on the flanges the coefficients $C_1$, $C_2$, $C_3$ and $K$ are calculated as follows:

$$C_1 = 2.59 \times 10^{-1},$$
$$C_2 = 2.88 \times 10^{3},$$
\[ C_3 = 3.31 \times 10^4, \quad (8) \]
\[ K = d^{-1.5} t^{-0.5} t^{-0.7} d^{-1.1}. \quad (9) \]

The parameters for determining the \( K \) coefficient are mentioned in Fig. 3.

![Diagram](image.png)

**Fig. 3** – Parameters for evaluating the bending moment-rotation curve for a joint with angle cleat on flanges (Frye & Morris, 1975).

The second analytical model for evaluating the beam-to-column joint is approached by using the method proposed by Kishi and Chen in 1990 (Chen & Lui, 1991; Kishi, et al., 1987a; Kishi, et al., 1987b). Bending moment-rotation relationship is in exponential form, where the exponent depends on the type of joint.

\[ \theta = \frac{M}{R \sqrt{1 - (M/M_u)^n}}, \quad (10) \]

where: \( M_u \) represents the capable bending moment of the joint section, \( n \) – the exponent depending on joint type.

### 3. Analytical Results

The analytical evaluations for all 8 beam-to-column joints using the Frye and Morris method and also the Kishi and Chen method are represented by the bending moment-rotation curve compared with the results obtained by numerical simulations (Figs. 4 to 11).
Fig. 4 – The comparison of numerical and analytical results for joints with angle cleat on flanges and web having 8 mm thickness.

Fig. 5 – The comparison of numerical and analytical results for joints with angle cleat on flanges and web having 9 mm thickness.

Fig. 6 – The comparison of numerical and analytical results for joints with angle cleat on flanges and web having 10 mm thickness.
Fig. 7 – The comparison of numerical and analytical results for joints with angle cleat on flanges and web having 12 mm thickness.

Fig. 8 – The comparison of numerical and analytical results for joints with angle cleat only on flanges having 8 mm thickness.

Fig. 9 – The comparison of numerical and analytical results for joints with angle cleat only on flanges having 9 mm thickness.
Fig. 10 – The comparison of numerical and analytical results for joints with angle cleat only on flanges having 10mm thickness.

Fig. 11 – The comparison of numerical and analytical results for joints with angle cleat only on flanges having 12mm thickness.

In case of beam-to-column joints having the angle cleats on flanges and web, both analytical models approached in this paper have the behaviour curves with the initial stiffness a higher value than the numerical models. Comparing both analytical models results that Frye-Morris model is more rigid and the Kishi-Chen model is closer to the curve obtained by numerical model. It is noted that for the first four joints (TSW-8,9,10,12), the numerical results are obtained after the experimental tests calibration available in the specialized literature with a minimum deviation of the real situation.

Comparing the behaviour curves for the joint type having angle cleats only on flanges can be notice that the initial stiffness for numerical models are closer to numerical results. The best correlation between the behaviour curves obtained analytical and numerical was obtained for joint with angle cleat on both flanges and also on the web with a 9 mm thickness using the Kishi-Chen model.

3. Conclusion

The paper highlights the semi-rigid behaviour of beam-to-column bolted connections with angle cleats using several arrangements.
In this paper were evaluated by analytical models 8 beam-to-column joints divided in two sets, representing two joints typologies and the results analytical obtained were compared to numerical results.

For joint configuration having angle cleats on flanges and web the Kishi-Chen model is more reliable also from the initial stiffness point of view. In case of joints without angle cleats on the web, both models approximate with better precision the initial stiffness.

The paper provides future research having the objective of extending the study to other joints configurations, respectively by applying other existing mathematical models proposed by scientific literature. Another interesting objective is to evaluate the studied joints in this paper using the components method based on SR EN 1993 1-8.

Further study will be achieved in order to conduct experimental investigations, and develop appropriate design methods, for the studied beam-to-column connections.

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STUDIUL COMPORTĂRII ÎMBINĂRILOR METALICE SEMIRIGIDE REALIZATE CU CORNIERE ŞI ŞURUBURI
Partea 2. Evaluări analitice şi numerice

(Rezumat)