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SOME CONSIDERATIONS REGARDING THE ASSESSMENT OF DYNAMIC CHARACTERISTICS

BY

OCTAVIAN VICTOR ROŞCA[*]

“Gheorghe Asachi” Technical University of Iaşi,
Faculty of Civil Engineering and Building Services

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Abstract. This paperwork deals with the estimation methods for the dynamic behavior and natural characteristics of structures.

A structural case study of a 3 DOFS structure tested on the master shaking table of the Laboratory of Earthquake Engineering from the Faculty of Civil Engineering and Building Services from Iaşi it is presented.

The specimen is a 3 storey steel frame model with additional masses.

The testing program consisted of low level sweep sine tests (*OX* & *OY* directions). Next, earthquake type excitations were applied, at several degrees of intensity (real and synthetic time-histories).

The verification and validation of modeling assumptions of the finite element models for structural analysis were carried on by the means of dedicated software FEM tools. The conclusions are focused on the accuracy of the models and the dynamic model matrices.

Keywords: structural analysis; dynamic tests; modal assurance criterion.

1. Introduction

In this paper there are presented the steps of the model callibration of a steel structure with 3 floors. The main aim was to adjust the analytical model

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[*] Corresponding author: *e-mail:* victor_rosca@yahoo.com

characteristics concerning the materials (Young modullus, Poisson's ratio) and some rigid zones of the beam – column assemblages, in order to match the natural frequencies of vibration, the real- measured quantitites vs. the calculated values from the eigenvalues of the analytical models.

The analytical models were carried out with the programs SAP 2000, Robot Millenium and FEM tools while the model callibration was carried out using some system identification techniques but in a very convenient way using the FEM tools program.

These tests were part of a larger program and some results and interpretations are presented here after. These results are a continuation of Roşca *et al.*, (2014).

Most of the engineering problems in vibrations (Humar,1990) are leading to the simple or generalized eigenvalue problem. It is commonly accepted and convenient to use the real eigenvalues; therefore the matrices of the dynamic system are symmetric and positive defined. Moreover, in the case of the time history analysis the eq. of motion are decoupled, based on the assumption that the damping is proportional (Rayleigh type) to the mass or stiffness or both. The damping matrix C is then written as a linear combination of M and K ($C = \alpha M + \beta K$). Under these circumstances the time responses obtained separately can be superimposed using the modal participations.

In the structural analysis there are situations when non- proportional damped structures are considered. The hypothesis of proportional damping is advantageous from the numerical point of view and the experimental tests show that this approximation leads in most cases to acceptable results. However it is not demonstrated yet the proportional behavior of the damping.

2. The Physical Model

One of the aims of the experimental program was to study the behavior and to callibrate the numerical models for a steel frame structure with three storeys. This structure was used afterwards for other tests to check some bracing system behavior, base isolation systems etc.

The physical model is a 3 storey steel frame one DDOF granted at each level and on both two main orthogonal directions in plane that coincide with the governing directions of the shaking table. The geometrical scale of the frame model is 1/4.

The tests were carried out on the 3 DOFs shaking table from the Earthquake Research Laboratory of the Faculty of Civil Engineering and Building Services of Iaşi, Romania. An image during tests is presented in the Fig. 1.

Fig. 1 – The physical model of the steel structure.

In the Fig. 2 there are presented the geometric characteristics of the real model. The DDOFs are numbered in ascending order for each level.

The columns are made Romanian steel profiles (steel J235) I80. The assemblages at each storey level are designed to accomplish a rigid plate behavior and therefore to obtain a 2-D "shear frame" behavior along each direction in plane. The span on *OX* is of 1.40 m, on *OY* is of 1.20 m and the storey height is 1.10 m.

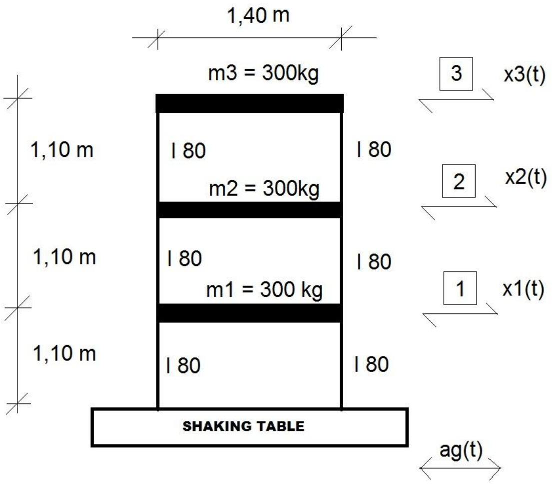


Fig. 2 – The dynamic 2-D model on each direction.

The specimen was mounted with additional masses manufactured of RC plates at each storey with the weight of 300 kg. The additional masses respected the similitude criteria in order to model the variable loads acting on the structure.

The model was instrumented with accelometers at each floor plate and the displacents were measured with LVDTs. Thus, absolute displacements and accelerations were obtained on both directions along each DOF. The shaking table is equipped with it's own measurement transducer set at the plate level for action and control.

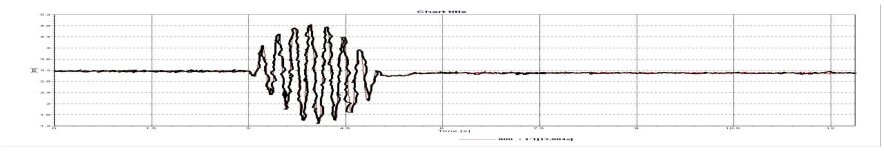
The testing program started with sweep sine tests at a low level of intensity. There were obtained the frequencies of the real model on the shaking table and there are emphasized in the Table 1.

Table 1

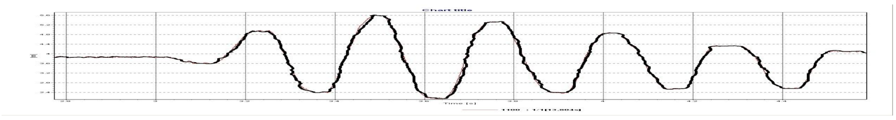
Modal Quantities (Experimental) OX Direction

Natural mode of vibration	Frequency Hz	Modal shapes (normalized)		
1	3.8	0.0128	0.0319	0.0447
2	8.2	0.0361	0.0293	–0.0317
3	11.7	–0.0410	0.0357	–0.0319

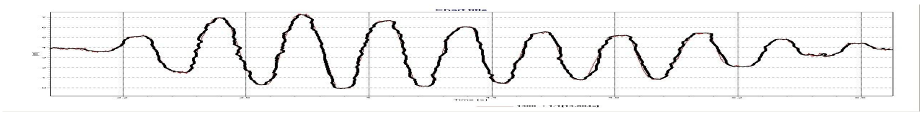
In the next step the shaking table was drived with sine motions. The modal quantities were obtained (physical frequancies, modal shapes and damping characteristics). Natural frequencies were applied at different amplitudes and input energies. In the next Fig. 3 we depict an example of the measurements when the motion was applied with $f_1 = 3.8$ Hz.



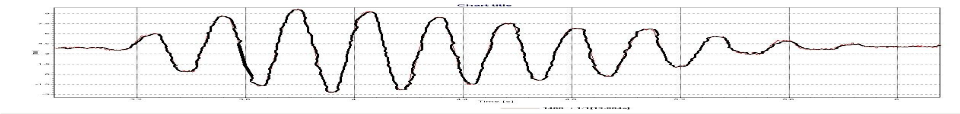
Absolute displacement at the base (sweep sine *OX* direction)



displacement at the first floor level (*OX*)



displacement at the second floor level (*OX*)



displacement at the 3rd floor level (*OX*)

Fig. 3 – Second phase-tests-displacement control *OX* direction.

The last step of the testing program with the simple steel structure consisted of seismic actions induced to the shaking table at several degrees of intensity. There were used recorded and filtered accelerograms of the "Bucureşti 1986" earthquake and the "Vrancea 1986 " earthquake. The level of intensities and accelerations was increased up to the level of 0.6 g for the peak ground accelerations.

3. The analytical model

The analytical dynamic models consisted of 2D models with 3DDOFs for each direction and computer programs like SAP2000 and Robot Millenium were used to build the analytical models. Taking into account this simple geometry the models were built with the above mentioned programs and the same modal quantities were obtained. Also the calibration specialized software, FEMtools was used as it has an "in built" dynamic modeler. The classic static and modal analysis were performed. Later on the seismic analysis were carried on in order to check the behavior in the case of earthquake action.

In [4] there are presented more results obtained with the SAP2000 computer program which provided also an input for the analysis with the FEMtools software.

The modal frequencies were obtained, for the Mode 1 - f_1 = 3.46 Hz, for the second Mode, f_2 = 10.44 Hz and for the third one, f_3 = 16.11 Hz (in the case of ox main direction). These frequencies were little shifted from those of the physical model.

4. The Modal Assurance Criterion

4.1. Model validation

The modal assurance criterion (MAC) was first advocated by Allemang and Brown in 1982 and is now widely accepted as an index and a technique involved to measure the correlation between the measured modal quantities, the analytical dynamic modes of vibration and in between both of this information.

In what concerns the terms that follows, we denominate by $x(t)$ the physical displacements of the model measured on the shaking table, $\dot{x}(t)$ is the vector of velocities and $\ddot{x}(t)$ is the acceleration vector of the horizontal DDOFs.

The dynamic quantities are denominated with X_{mj} for the measured mode no. j while an analytical mode k is written as X_{ak}. The analytical modes are computed using usual dynamic procedures or are extracted from a specific software. The eigenvalue corresponding to the i-th mode of vibration is called Λ_i (square of the cirqular frequency) and the diagonal matrix of the eigenvalues is simply denoted by Λ.

The desired requirement in the experimental research is to match as "good" as possible the experimental model with the analytical one. This leads to good prototipes and further structural optimisations. Model updating is therefore demanded but this leads to several problems regarding the increase of the degrees of freedom in the analytical model and the limited numbers of transducers to be mounted in practice. Then it is very difficult to assess the damping effects.

A strong and effective criterion is the MAC. It is often used to couple the experimental (measured) mode shapes and the mathematicaly eigenvectors. It is easy to apply and doesn't need the estimation of the system matrices. The MAC is defined by:

$$\mathrm{MAC}_{jk} = \frac{\left| X_{mj}^{T} X_{ak} \right|^{2}}{\left(X_{ak}^{T} X_{ak} \right)\left(X_{mj}^{T} X_{mj} \right)}. \tag{1}$$

As it can be easily seen, the MAC index has values in between 0 and 1. The value of 1 means that one modal shape is multiple or "identical" to the other one (exprimental vs. analytical). The indexes must be consistent in the above relationship, i.e. the measurement points must be of the same number with the granted DDOFs and must coincide at positions. If the measurement points are selected at the joints of the finite element model, then MC implies the selection of the elements in the analytical modal eigenvectors to be connect the measured locations.

An important observation is that in the structural analysis there are situations when non- proportional damped structures are encountered. The hypothesis of proportional damping leads to real modal matrix and spectral matrix. If the damping is non- proportional, then the complex matrices [1] are obtained. The complex mode shapes can be correlated using the MAC criterion as long as the transpose generalized matrix is taken to be the conjugate transpose.

The MAC is quite difficult to be observed if the modes are quite close in frequency. A higher order/second order of MAC called co- ordinate Mac or COMAC is used instead. The eq. (1) establishes the correlation between two modes for all the measurement points. The COMAC is using the same relathionship but it takes into account the pair measurement - analytical for all the eigenmodes.

4.2. The Orthogonality Test

The natural modes of vibration for a regular structure are orthogonal, that is, paying respect to the mass matrix:

$$[X]^{T}[M][X] = [I_{N}] \tag{2}$$

and

$$[X]^T[K][X]=[\Lambda]. \tag{3}$$

where: [X] is the matrix of the eigenvectors, [I_N] is the identity matrix and [Λ] is the modal matrix. If the experimental modes are replaced into eq. (3) then the weight of the modal shapes can be evaluated. If experiments are replaced for the analytical modes in eq. (3) then orthogonality is no longer relevant. Instead, the pre- or post- product with the measured vector acts as a weight. One may notice the exact match of MAC in case of the analytical model (Fig. 4).

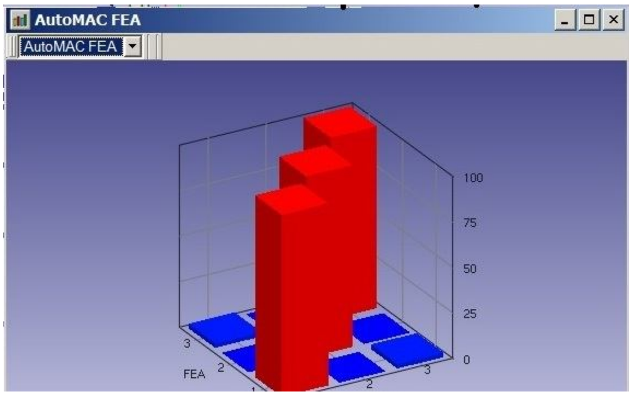


Fig. 4 – Model validation – MAC criterion for analytical eigenvalues.

Further precison can be achieved using the normalized modal vectors with respect to the mass matrix. Normalizing the MAC over the mass matrix leads to the zero off- diagonal terms in the MAC matrix. One can apply instead the modified MAC criterion:

$$\mathrm{modMAC}_{jk}=\frac{\left|X_{mj}^{T}\,M\,X_{ak}\right|^{2}}{\left(X_{ak}^{T}\,M\,X_{ak}\right)\left(X_{mj}^{T}\,M\,X_{mj}\right)}. \tag{4}$$

The overall value of the MAC index shall tiny differ because the mass matrix is diagonal but mode to next mode can be afterwards checked.

4.3. The Model Validation

The FEM Tools software was used for model validation. The parameter validation included the spectral matrix and the stiffness properties. The correlation was achieved mainly on the basis of the MAC (Modal Assurance Criterion). The MAC matrix is depicted in the Fig. 5.

After the FEM updating the new modal quantities are those from the Table 2.

Table 2

Modal Quantities (Numerical, after the model updating)

Natural mode of vibration	Frequency - tests Hz	Frequency - model Hz
1	3.8	3.439
2	8.2	8.433
3	11.7	11.558

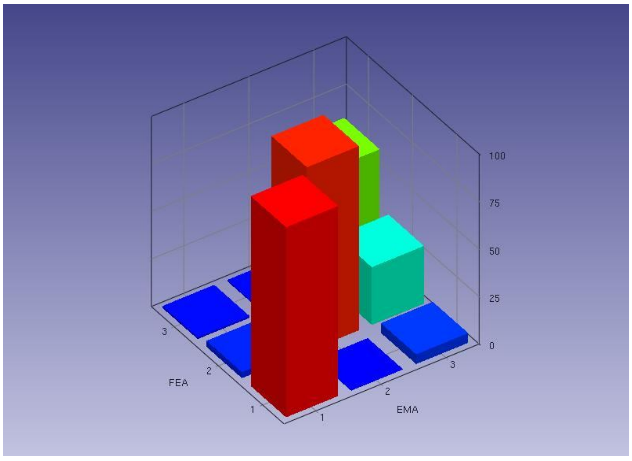


Fig. 5 – Model validation – MAC criterion.

One may notice a very good correlation between the experimental and the analytical modes of vibration in the case of the first mode and the second one. The othogonality check looks fine. But in the case of the 3rd mode the correlation is smaller and a loss of orthononality can be observed in the case of modes 2 and 3. Some further modifications for the stiffness characteristics of the analytical model can be expected; anyway, the model with the properties figured in the Table no. 2 presented a good behavior the time history domain.

5. Conclusions

In this paper there are presented the analytical methods that were used to perform the correlation between the measured modal shapes and the anlytical modal vectors for a structure that was tested on the shaking table from the Laboratory of Earthquake Engineering from the Structural Mechanics Department of the Faculty of Civil Engineering and Building Services from Iaşi.

Although the structural model is not very large, consisting of a 3 storey steel frame, at the first comparison, the real values of the measured frequencies were different from those obtained with computer programs. In order to repect the simillitude criteria, the steel frame was built with additional masses at each level.

The model validation was accomplished using the FEM Tools software. Some changes were made for the material properties of the SAP 2000 model. Then, using the MAC shape correlation, the model parameters were adjusted.

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CONSIDERAȚII ASUPRA EVALUĂRII CARACTERISTICILOR DINAMICE

(Rezumat)

Acest articol prezintă o serie de metode de estimare a comportării dinamice şi a evaluării caracteristicilor diamice ale structurilor.

Studiile sunt bazate pe analiza matematică şi experimentală a unei structuri cu 3 grade de libertate dinamică testată pe platforma seismică Master a Laboratorului de Inginerie Seismică din cadrul Facultății de Construcții şi Instalații din Iaşi.

Cadrul testat are 3 niveluri şi este prevăzut cu mase adiționale.

Programul de testare a fost constituit din acțiuni "sweep sine" la un nivel slab de amplitudine (pe direcțiile OX şi OY). Apoi au fost aplicate acțiuni de tip seismic

corespunzătoare unor diverse cutremure (reale sau obținute sintetic) şi la intensități crescute gradual.

Verificarea şi validarea ipotezelor de modelare au fost conduse cu ajutorul unui program spesializat "FEM Tools". In capitolul "Concluzii" sunt prezentate observații referitoare la precizia modelelor şi în final sunt prezentate matricele dinamice ale modelului analitic.