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EFFECTIVE WIDTH OF STEEL FLANGE GIRDERS RELATED TO SHEAR LAG PHENOMENON

BY

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Abstract. In the conventional theory of bending, shear strains are neglected and the stress distribution can be determined from the Navier's equations.

The term shear lag is related to some of the discrepancies between this approximate theory of the bending of beams and their real behaviour and refers to the increases of the bending stresses near the flange-to-web junctions, and the corresponding decreases in the flange stresses away from these junctions.

In case of wide flanges of plated structures, shear lag may be taken into account by a reduced flange width concentrated along the webs in the direction of the action.

When designing plated structures, the effects of shear lag, plate buckling and interaction of both effects should be taken into account at the ultimate, serviceability or fatigue limit states.

In EN 1993-1-5, the concept of taking shear lag into account is based on effective^s width of the flange which is defined in order to have the same total normal force in the gross flange subjected to the real transverse stress distribution as the effective flange subjected to a uniform stress equal to the maximum stress of the real transverse distribution.

Aspects concerning the shear lag phenomena and two design examples of effective^s width of the flange calculation and of the shear lag effects are presented in this paper.

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Keywords: plate buckling; shear lag effects; effective width; orthotropic deck; Eurocodes.

1. Introduction

In the conventional theory of bending, shear strains are neglected so that it can be assumed that plane sections remain plane after loading. From this assumption follow the simple linear distributions of the bending strains and stresses, the distribution resulting from linear elastic theory using Bernouilli's hypothesis.

The stress distribution can be determined from the Navier's equations but, for stiffened plates, this is limited to plates where the longitudinal and transverse stiffeners are closely spaced, symmetrical to both sides of the plate, and produce equal stiffness in the longitudinal and transverse direction. This configuration leads to an isotropic behavior but in practice this way of stiffening is not practical and therefore not commonly used (*Europen Steel Design Education Programme. ESDEP)*.

The term shear lag is related to some of the discrepancies between this approximate theory of the bending of beams and their real behaviour, and in particular, refers to the increases of the bending stresses near the flange-to-web junctions, and the corresponding decreases in the flange stresses away from these junctions (Trahair *et al.*, 2008).

In case of wide flanges of plated structures, shear lag may be taken into account by a reduced flange width concentrated along the webs in the direction of the action, Fig. 1.

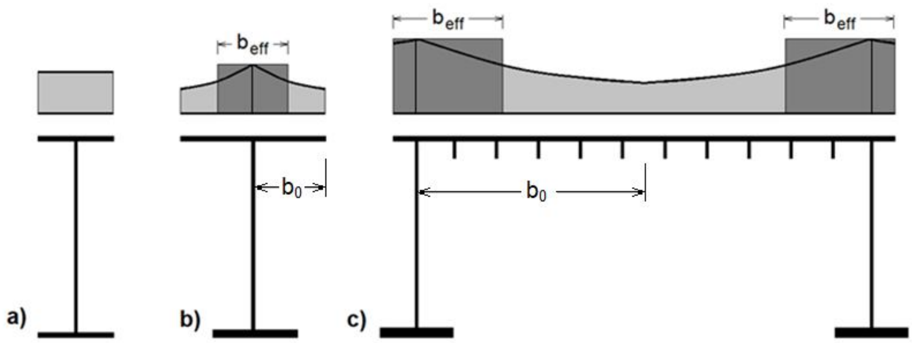

Fig. 1 – Effective width due to shear lag: a) thin wide flange; b) wide flange; very wide flange.

In very wide flanges, shear lag effects have to be taken into account for the verification of stresses, especially for short spans, because it causes the longitudinal stresses at a flange/web intersection to exceed the average stresses in the flange.

The shear stresses calculated by the conventional theory are as shown in Fig. 2 a, and these induce the warping (longitudinal) displacements shown in Fig. 2 b. The warping displacements of the web are almost linear, and these are responsible for the shear deflections which are usually neglected when calculating the beam's deflections. The warping displacements of the flanges vary parabolically, and it is these which are responsible for most of the shear lag effect (Trahair *et al*., 2008).

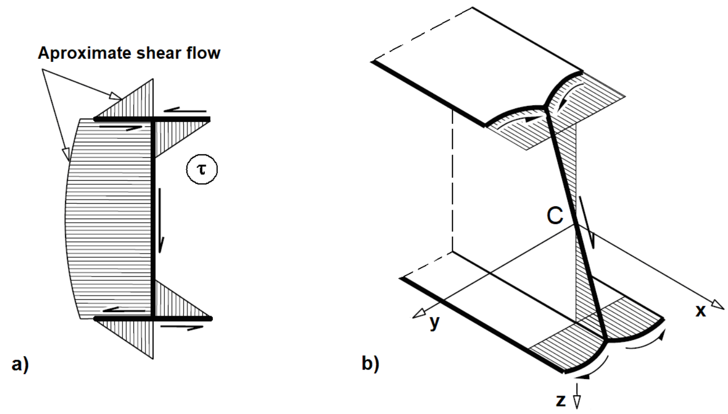

Fig. 2 – a) Shear flow due to a vertical shear V_z; b) Warping displacement due to shear. (Trahair *et al*., 2008).

Shear lag effects are usually very small except near points of high concentrated load or at reaction points in short-span beams with thin wide flanges. In particular, shear lag effects may be significant in light-gauge, cold-formed sections and in stiffened box girders. Shear lag has no serious consequences in a ductile structure in which any premature local yielding leads to a favourable redistribution of stress. The increased stresses due to shear lag may be of consequence in a tension flange which is liable to brittle fracture or fatigue damage, or in a compression flange whose strength is controlled by its resistance to local buckling (Trahair *et al*., 2008).

An approximate method of dealing with shear lag is to use an effective width concept, in which the actual width b of a flange is replaced by a reduced width b_{eff} given by:

$$\frac{b_{\text{eff}}}{b_0} = \beta. \tag{1}$$

This is equivalent to replacing the actual flange bending stresses by constant stresses which are equal to the actual maximum stress and distributed over the effective flange area $b_{\mathrm{eff}} \times t$. This approach is similar to that used to allow for the redistribution of stress which takes place in a thin compression flange after local buckling but, the two effects of shear lag and local buckling are quite distinct, and should not be confused (Trahair *et al.*, 2008).

In relation to the effective width method, SR EN 1993-1-5. §3.3 introduces three different designations for three types of effective width (SR EN 1993-1-5: 2006. Eurocod 3 (EC3-1-5); Beg *et al*., 2010; Johansson *et al*., 2007):

- Effective^{s} width – shear lag effects;
- Effective^{p} width – local buckling of plates;
- Effective width – interaction of shear lag and local buckling.

When designing plated structures, the effects of shear lag, plate buckling and interaction of both effects should be taken into account at the ultimate, service ability or fatigue limit states.

Shear lag and plate buckling reduce the stiffness of plated structures and should be accounted for in the global analysis.

Shear lag can occur in the tension flange as well as in the compression flange, when interaction should be considered between shear lag and pale buckling effects.

For simplicity the effective^{s} width may be taken as constant over the length of each span:

$$b_{\mathrm{eff}} = \min\left(b_0; L_i/8\right). \tag{2}$$

2. Effective^{s} Width for Elastic Shear Lag Analysis and Calculation of Stresses

In EN 1993-1-5, the concept of taking shear lag into account is based on effective^{s} width of the flange which is defined in order to have the same total normal force in the gross flange subjected to the real transverse stress distribution as the effective flange subjected to a uniform stress equal to the maximum stress of the real transverse distribution:

$$\int_0^b \sigma_x(y) t_f \mathrm{d}y = b_{\mathrm{eff}} t_f \sigma_{x\max}, \tag{3}$$

where: $b_{\mathrm{eff}} = \beta b_0$; β is an efficiency factor, Table 1.

Table 1
Coefficient β Evaluation

Location for verification	β – value	k
	$\beta = 1.0$	≤ 0.02
Regions of positive (sagging) bending	$\beta = \beta_1 = \dfrac{1}{1+6.4k^2}$	0.02,...,0.70
	$\beta = \beta_1 = \dfrac{1}{5.9k}$	> 0.70
Regions of negative (hogging) bending	$\beta = \beta_2 = \dfrac{1}{1+6.0\left(k-\dfrac{1}{2{,}500k}\right)+1.6k^2}$	0.02,...,0.70
	$\beta = \beta_2 = \dfrac{1}{8.6k}$	> 0.70
Linear bending on end supports	$\beta_0 = \left(0.55+\dfrac{0.025}{k}\right)\beta_1$; but $\beta_0 < \beta_1$	all k values
Cantilever	$\beta = \beta_2$ at support and at the end	all k values

where:

$$k = \frac{\alpha_0 b_0}{L_e} \text{ – factor related to the stiffening ratio } \alpha_0 \tag{4}$$

$$\alpha_0 = \sqrt{1+\frac{A_{sl}}{b_0 t}} \text{ – orthotropic plate factor} \tag{5}$$

A_{sl} – longitudinal stiffeners area; $t = t_f$ – plate thickness; b_0 – full width; L_e – effective length.

Provided adjacent spans do not differ more than 50% and any cantilever span is not larger than half the adjacent span the effective lengths L_e may be determined from Fig. 3. For all other cases L_e should be taken as the distance between adjacent points of zero bending moment.

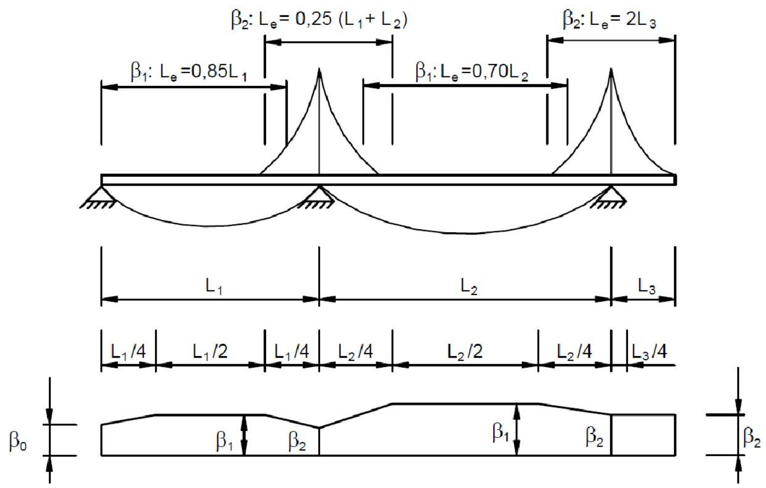

Fig. 3 – Definition of the effective length L_e (SR EN 1993-1-5: 2006).

The transverse bending stresses distribution due to shear lag, Fig. 4, is given by the relations:

If:

$$\beta > 0.2: \begin{cases} \sigma_2 = 1.25(\beta - 0.20)\sigma_1 \\ \sigma(y) = \sigma_2 + (\sigma_1 - \sigma_2)\left(1 - \dfrac{y}{b_0}\right)^4 \end{cases} \tag{6.a}$$

If:

$$\beta \le 0.2: \begin{cases} \sigma_2 = 0 \\ \sigma(y) = \sigma_1\left(1 - \dfrac{y}{b_1}\right)^4 \end{cases} \tag{6.b}$$

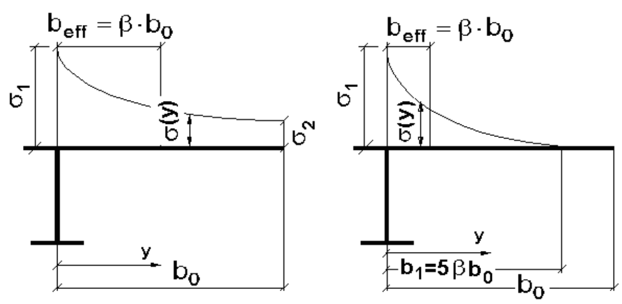

Fig. 4 – Distribution of stresses due to shear lag (SR EN 1993-1-5: 2006).

According to EN 1993-1-5, the shear lag in the flanges may be neglected if $b_0 < L_e/50$.

It should be noticed that, the more stiffened the flange is, the smaller its effective^s width is, but this influence is not so important in hogging bending region (Beg *et al*., 2010).

3. Interaction Between Shear Lag and Plate Buckling at ULS

In case of a flange in compression at ULS (ultimate limit state) verification, the plate buckling effects which results in an effective^p area of the flange may occur in addition to the shear lag effects.

At the ultimate limit state shear lag effects may be determined as follows:

a) elastic shear lag effects as determined for serviceability and fatigue limit states;

b) combined effects of shear lag and of plate buckling;

c) elastic-plastic shear lag effects allowing for limited plastic strains.

EN 1993-1-5 proposes two models of steps for interaction between shear lag and plate buckling:

*Model 1 (method **b**):*

– calculate the effective^p area to plate buckling;

– define an effectivep stiffening ratio $k = (\alpha_0 b_0)/L_e$ to be used instead of the stiffened ratio α_0 and calculating the reduction factor β_{ult} (using Table 1) instead of β, where:

$$\alpha_0^* = \sqrt{\frac{A_{c\ \mathrm{eff}}}{b_0 t}}\,, \tag{7}$$

– calculate the effective area A_{eff} for taking shear lag and plate buckling effects into account as follows:

$$A_{\mathrm{eff}} = \beta_{\mathrm{ult}} A_{c\ \mathrm{eff}}\,. \tag{8}$$

*Model 2 (method **c** – recommended in EN 1993-1-5):*

– an elastoplastic reduction factor $\beta^k \geq \beta$ is directly applied to the effective^p area of the compression flange, where k is based on α_0:

$$A_{\mathrm{eff}} = \beta^k A_{c\ \mathrm{eff}} \geq A_{c\ \mathrm{eff}}\beta\,. \tag{9}$$

It is recommended to apply the reduction factor of the area to the thickness of the plate, and not to the width (Beg *et al*., 2010).

4. Design Examples

4.1. Example 1: Orthotropic Deck of a Steel Road Bridge

It is evaluated the effective[s] width and the shear lag at serviceability limit state (SLS) for an orthotropic plate, as being the top flange of a steel road bridge girder with 3 spans: 40 m + 60 m + 40 m. The statically system and the cross–section of the steel girder are presented in Fig. 5, an open section with two main girders and an orthotropic steel deck (Jantea *et al*., 2000).

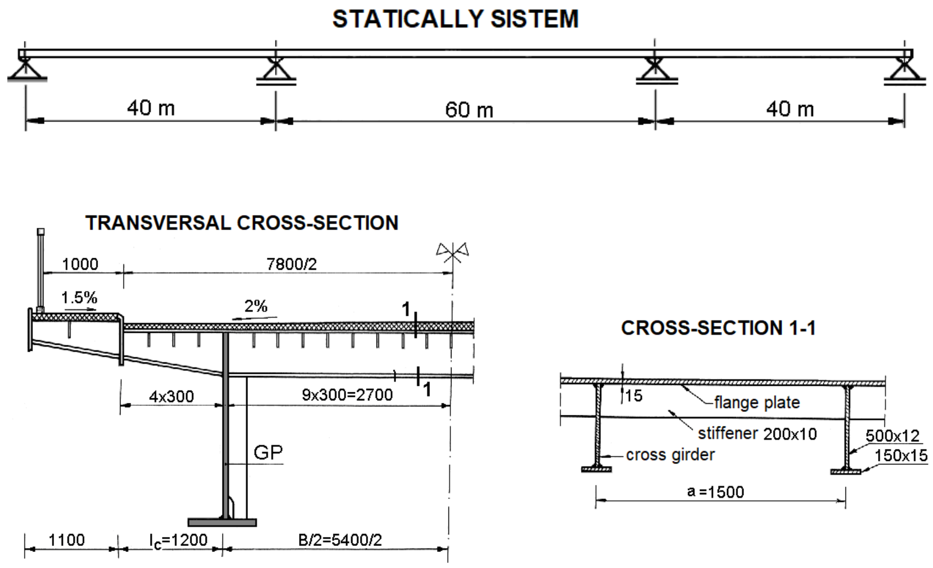

Fig. 5 – Steel bridge analyzed.

Effective[s] width and the shear lag at serviceability limit state (Moga *et al*., 2005; Moga *et al*., 2004).

The first step is the evaluation of the coefficients $\alpha_0$for the cantilever zone (I) and for the central zone (II) of the orthotropic plate, Fig. 6.a, and the effective length L_e, for the continuous girder with 3 spans is evaluated in Fig. 6.b:

$$\alpha'_0 = \sqrt{1+\frac{3\times 20\times 1}{120\times 1.5}} = 1.15; \qquad \alpha''_0 = \sqrt{1+\frac{0.5\times 17\times 20\times 1}{270\times 1.5}} = 1.19.$$

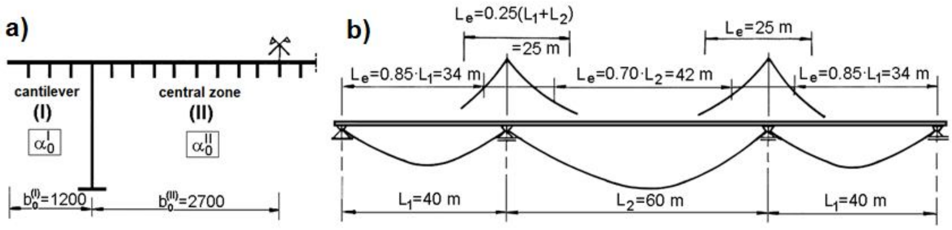

Fig. 6 – a) Parameter α_0; b) Effective length L_e.

The coefficients k, β and the effective width of the orthotropic plate are presented in Table 2.

Table 2
Design Parameters for Shear Lag

Parameter	**End spans (40 m)**	
	Cantilever (I)	Central region (II)
k	0.040	0.095
β_1	0.99	1.00
b_{eff} [cm]	119	270
Parameter	**Middle span (60 m)**	
	Cantilever (I)	Central region (II)
k	0.033	0.076
β_1	0.99	0.96
b_{eff} [cm]	119	259
Parameter	**Intermediate supports**	
	Cantilever (I)	Central region (II)
k	0.055	0.128
β_2	0.77	0.56
b_{eff} [cm]	92	151

Diagram of the β coefficients is presented in Fig. 7.a and the transverse tension stresses distribution due to shear lag on the intermediate support is presented in Fig. 7.b.

The stresses due to shear lag effect are as follows ($\sigma_{max} = \sigma_1$; $\sigma_{min} =$ $= \sigma_2$):

$$\sigma_2' = \sigma_{\min}' = 1.25(0.56 - 0.20)\sigma_{\max} = 0.45\sigma_{\max};$$

$$\sigma_2'' = \sigma_{\min}'' = 1.25(0.77 - 0.20)\sigma_{\max} = 0.71\sigma_{\max}.$$

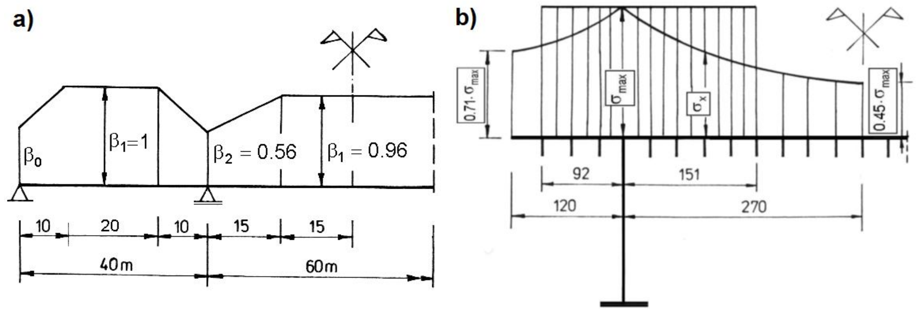

Fig. 7 – a) Diagram of parameter β; b) Transverse tension stresses distribution.

It can be noticed that on the support zone the bending stresses being of tension, the effective[p] width is not necessary to be evaluated ($A_{\text{eff}} = A_f$).

The effect of plate buckling in the elastic global analysis and ULS may be also neglected for the central zones of the spans because is fulfilled the condition (Beg *et al*., 2010), $A_{\text{eff}} > \rho_{\text{lim}} A_f = 0.5A_f$.

In these conditions is not interaction between shear lag and local buckling on the girder.

4.2. Example 2: Stiffened Tension Flange of a Composite Steel-Concrete Girder

The effective[s] width of the tension flange and the shear lag at the ultimate limit state effects are analyzed for a composite steel-concrete girder.

The composite box girder for a footbridge has been designed with a span of 58.0 m, a width of 6.0 m which includes two lateral pedestrian ways of 1.60 m width and a central cycle way of 2.50 m width, Fig. 8 (Moga *et al*., 2016; Moga, 2018).

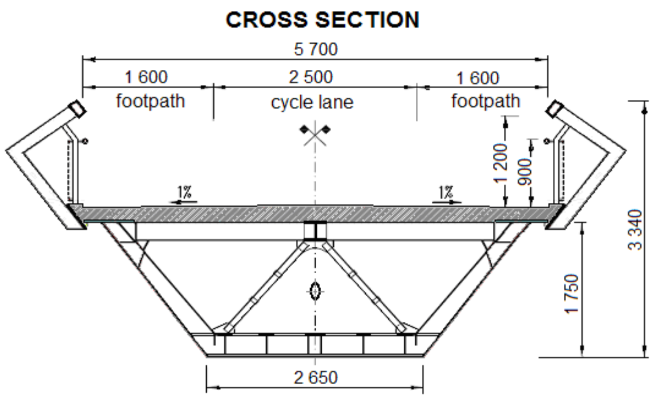

Fig. 8 – Pedestrian bridge cross-section.

According to the recommendation of SR EN 1993-1-5 §3.3 at the ultimate limit states shear lag effects may be determined using the method *c* – elastic-plastic shear lag effects allowing for limited plastic strains.

The relation (9) can be applied for flange in tension in which case $A_{c\,\mathrm{eff}}$ should be replaced by the gross area of the tension flange A_f:

$$A_{\mathrm{eff}} = A_f \beta^k;\ \alpha_0 = \sqrt{1+\frac{\sum A_{sl}}{b_0 t_f}} = \sqrt{1+\frac{0.5\times 300}{(265/2)\times 3.5}} = 1.1,$$

where, Fig. 9: $\sum A_{sl} = 4A_{sl.1} + A_{sl.2} = 300\ \mathrm{cm}^2$.

It yields:

$$k = \frac{\alpha_0 b_0}{L_e} = \frac{1.15\times 265/2}{5{,}800} = 0.026 \in (0.02-0.70);$$

$$\beta = \beta_1 = \frac{1}{1+6.4k^2} = 0.99.$$

It is obtained: $\rho^k = 0.99^{0.026} \approx 1.0 \Rightarrow A_{\mathrm{eff}} = A_f$.

It can be noticed that the entire section of the bottom flange is effective.

This conclusion also results from the simplified relation:

$$b_{\mathrm{eff}} \min\left(b_0;\frac{L_i}{8}\right) = \min\left(\frac{2{,}650}{2};,\frac{58000}{8}\right) = 1{,}350\ \mathrm{mm}$$

(2,650 mm for the entire flange).

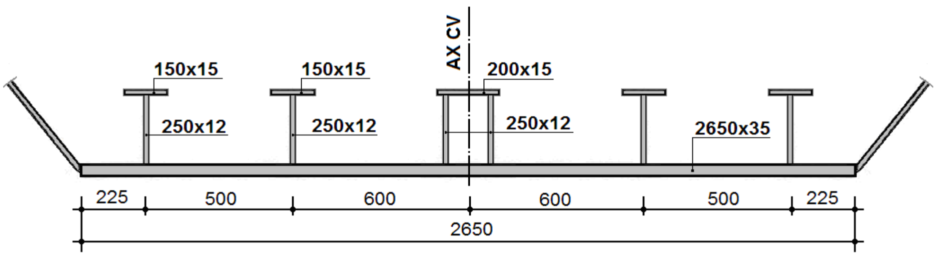

Fig. 9 – Bottom flange for the central segment.

In Fig. 10 the stresses distribution on the bottom flange width for β > 0.2 is presented.

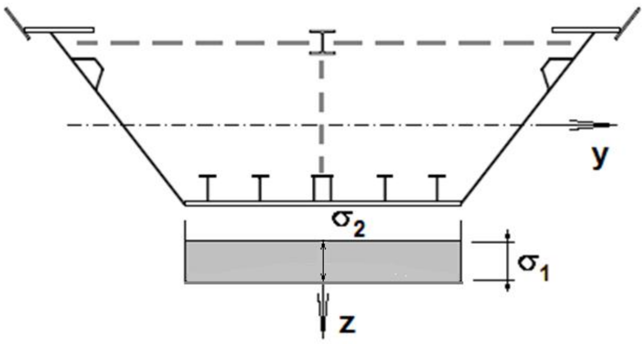

Fig. 10 – Shear lag effect in tension flange.

The stress at the middle of the flange is

$$\sigma_2 = 1.25(\beta - 0.20)\sigma_1 = 0.99\sigma_1 \approx \sigma_1.$$

5. Conclusions and Final Remarks

The term shear lag is related to some of the discrepancies between the approximate theory of the bending of beams and their real behaviour and refers to the increases of the bending stresses near the flange-to-web junctions, and the corresponding decreases in the flange stresses away from these junctions.

In relation to the effective width method, SR EN 1993-1-5. §3.3 introduces three different designations for three types of effective width:

- Effective^s width – shear lag effects;
- Effective^p width – local buckling of plates;
- Effective width – interaction of shear lag and local buckling.

When designing plated structures, the effects of shear lag, plate buckling and interaction of both effects should be taken into account at the ultimate, serviceability or fatigue limit states, because the shear lag and plate buckling reduce the stiffness of the plated structures.

The two design examples presented in this paper are useful in the design activity.

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LĂȚIMEA EFECTIVĂ A TĂLPII GRINZILOR METALICE ÎN CORELARE CU FENOMENUL SHEAR LAG

(Rezumat)

În teoria clasică de încovoiere, deformațiile specifice din forfecare sunt neglijate, astfel încât o secțiune plană rămâne plană şi după deformația produsă sub efectul încărcării.

Prin acceptarea acestei ipoteze, rezultă distribuția liniară a deformațiilor şi tensiunilor din încovoiere, respectiv distribuția cunoscută a eforturilor unitare tangențiale.

Fenomenul „shear lag" este legat de discordanța între teoria clasică de încovoiere a grinzii şi comportarea reală a acesteia, în particular cea referitoare la creşterea eforturilor unitare din încovoiere la nivelul legăturii talpă-inimă şi corespunzător, scăderea acestora în talpă, la o anumită distanță de această zonă de conexiune.

În cazul structurilor cu tălpi dezvoltate, efectul shear lag este luat în considerare prin introducerea unei lățimi efectives reduse, pe lățimea căreia eforturile unitare normale din încovoiere se consideră constante şi egale cu valoarea maximă a acestora calculată prin teoria clasică de încovoiere (formula lui Navier).

În lucrare se prezintă unele aspecte teoretice privind fenomenul shear lag şi evaluarea lățimii efectives, precum şi două aplicații (exemple) numerice pentru o structură metalică de pod rutier şi o pasarelă pietonală pe o grindă casetată compusă oțel-beton.