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**THE THEORETICAL STUDY OF THE BEAMS SUPPORTED
ON A STRAINING ENVIRONMENT
AS AN INTERACTION PROBLEM
SOIL - STRUCTURE - INFRASTRUCTURE INTERACTION**

BY
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Between structure, infrastructure (foundation) and soil there is an effective interaction, which has to be taken into account as correctly as possible every time we do the calculation. This effective interaction can be analysed in a global form, considering on one hand the entire building, and on the other hand the soil - establishment surface, or in an analytical form: we consider first the soil - infrastructure (foundation) interaction and then the structure - infrastructure one.

Without considering the interaction, we cannot make neither the calculation (for the soil) according to the limiting deformation state which has to be compatible with the structure's resistance system, nor calculation for the limiting resistance state, because the correct distribution of efforts along the contact surface between the soil and the structure is unknown, so we cannot determine the zones of plastical equilibrium in the soil massive and the conditions of limited equilibrium.

Also, without considering the infrastructure, we cannot correctly calculate the efforts and the deformations which may occur in all resistance elements of the building. Therefore, we cannot talk about limiting state calculation without considering the interaction between the soil and the structure itself.

The problem of interaction between building, on one hand and soil foundation, on the other hand, is not approached very much in the specialized literature, because of the big difficulties raised by summarizing all the factors that describe the structure and the environment, which would be more accessible to a practical calculation.

A lot of buildings or elements of buildings standing on the soil or on another environment with finite rigidity can be taken into account as beams supported on a straining environment, (continuous foundations, resistance walls, longitudinal and transversal membranes of civil and industrial buildings, hydrotechnic works).

Therefore, in the present paper we shall analyse the problems regarding the calculation of some types of beams supported on a straining environment (foundation soil).

In general, we shall use the term "beams supported on a straining environment" and not "beams supported on an elastic environment" because the foundation soil, which is in fact the supporting environment most commonly met in practice, cannot be considered as a perfectly elastic, homogeneous and isotropic environment, and it can be at most simplified in calculation as a linear or not linear environment.

1. Introduction

The calculation of buildings supported on a straining environment has led to contradictory results due to many objective and subjective causes. In this calculation, an important role is played by the type of foundation and the calculation model used for the soil foundation.

Usually, such problems are solved when the specialist in structures and the geotechnician specialist conventionally treat the structure, the foundation and the soil as independent subsystems.

Considering and analysing (taking into account its behaviour and the response to actions) the structure and the foundation soil as a unitary whole, formed of subsystems having different geometrical and mechanical characteristics, in a continuous process of redistributing efforts, is included in the general category of interactions between systems or in the same system among subsystems.

The problems raised by this branch of science are complex and their theoretical and experimental approach is difficult. Solving these problems requires knowledge in the fields of statistics of buildings, elasticity theory, geotechnics and foundations, etc.

2. Beams Supported on a Straining Environment

A. Solving the problem of beams supported on a straining environment (foundation soil, brickwork) refers to determining the tension and the deformation of two objects that come into contact. The beam (the construction wall, the foundation) and the foundation soil form the system of objects coming into contact.

The purpose of the calculation of beams supported on a straining environment is reduced to determining the tension and the deformation of the two items (systems) having different mechanical characteristics, taking into account the effective interaction between them, in order to establish the most reasonable methods to measure and check their resistance, rigidity and stability.

However, in order to determine tension, the carrying capacity of a system, between unitary efforts and specific deformations characterizing the properties of the building's material, different diagrams effort – deformation have to be considered, which in fact makes calculation more complicated.

For the beam we can consider the characteristic curve of the specific material, which generally presents a linear dependence in the elastic field, and for the elastic – plastic calculation we can consider Prandtl's idealized diagram.

As for the foundation soil (straining environment), the effort – deformation curve will depend on the calculation model adopted, varying from one foundation soil to another, from a straining environment to another.

In fact, for the foundation soil it is impossible to set a unique calculation model, therefore for each case the model reflecting the best the environment's properties will be chosen.

B. In establishing calculation methods, the premise should be the interaction between the beam and the soil; the deformation of the beam as an element of the foundation follows the deformation of the soil (environment) on which it stands. Between the beam and the soil (straining environment) there is a reciprocal action, emphasized by the pressures transmitted by the beam to the foundation soil, equal and in the contrary direction with the soil's reaction on the beam.

Making a section at the bottom of the foundation, we get the calculation schemes of the two elements, the beam and the soil, which are in contact (Fig. 1).

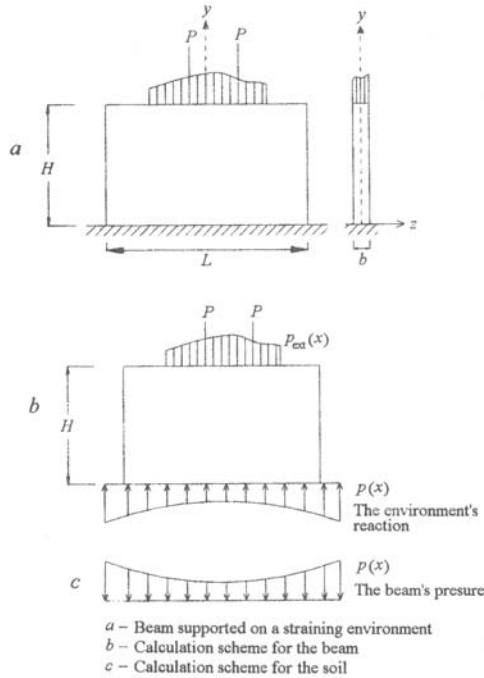


Fig. 1

Determining the tension and the deformation from the beam and the soil is conditioned by knowing the reaction of the soil (p, x, z) , representing the statically undetermined function of the studied problem.

As the width, b , of the beam is small compared to the other dimensions, we make the supposition that the reaction of the soil is constant in width, being a function of x . To determine the law of distributing pressures, $p(x)$, on the contact surface between the beam and the soil, we take into account the fact that, under normal exploitation conditions, there has to be a permanent contact between the beam (foundation) and the soil (foundation soil); therefore, movements, $W(x, y)$, of the surface of the foundation drawing under the action of $p(x)$, are equal with the composition of movement after the vertical, $V(x, y)$, of inferior surface of suspending the beam under the action of exterior forces, $p(x)$, and the reaction of the environment, $p(x)$. The contact condition will take the form

$$(1) \quad W(x, y)_{y=0} = V(x, y)_{y=0}.$$

Adding the balance relations of the beam

$$(2) \quad \sum V = 0, \quad \sum M = 0,$$

where the vertical projection was considered, as well as the moment of all active and reactive forces that action upon the beam, we shall obtain the equation system leading to the calculation of $p(x)$.

Knowing the variation law of $p(x)$ we can determine the tension and the deformation in the beam or in the soil.

As a conclusion, when calculating the beams on a straining environment, the following distinct problems come up:

1. Determining the reactions of the soil upon the beam or the pressure of the beam.

2. Determining the tension and the deformation in the beam: a) vertical movements of the soil's surface; b) vertical movements of the suspension side of the beam itself.

Regarding the expression of vertical movements of the soil, it has to be shown that it depends on the model adopted to approximate the behaviour of the foundation soil and it will not depend on the type of beam considered.

As for expressing vertical movements of the suspension side of the beam itself, it depends on the ratio between its dimensions ($\lambda = L/H$). When the ratio between the beams length and height is high ($\lambda > 5$), we can assume that the flat sections hypothesis is valid and we can use known models from the resistance of materials when calculating the movements, taking into account only the deformation produced in the bending moment, the movements of the beam's axis coinciding with the movements of the inferior edge where the suspension is made.

In the specialized literature, all problems referring to calculating beams on an elastic environment are solved in this way, both in the winklerian model hypothesis, as well as in the elastic semispace model. In other words, these are long beams, where the ratio $\lambda = L/H > 5$ and whose calculation is rather thorough.

In practice, there is a whole series of elements of construction of buildings that could be considered as beams suspended on an unfavourable environment and whose ratio between dimensions (λ) is lower than 5, when the possibility of applying the methods of materials' resistance for determining both tension and deformation can be questioned. These are the beams whose ratio $\lambda < 5$, for example medium-sized beams ($2 \leq \lambda \leq 5$) and wall beams ($\lambda < 2$). In this case, the flat sections hypothesis loses ground and the calculation of movements of the beam and determining tension should be made using more rigorous methods from the elasticity theory.

Exact calculation of such elements of construction is much more difficult than in the case of ordinary beams ($\lambda \leq 5$), because we deal with a contact problem in the elasticity theory, and in specialized literature there are only some isolated attempts and only for particular cases (own weight).

A lot of researchers have emphasized the importance and the complexity of calculating the tension and the deformation in medium-sized beams and in wall beams supported on a straining environment (foundation soil).

Even if, in the calculation scheme of these beams, many elements of construction can be reduced, even whole buildings (for example, high brickwork walls or longitudinal rigidity diaphragms of civil and industrial buildings, coffered infrastructure

of high buildings, hydrotechnical buildings, brickwork walls supported on marginal beams, etc.) still, because of the many difficulties, a theoretical solution hasn't been set up yet, requiring further research, which made certain designers apply photo-elastic experimental methods, when practice required measuring or checking such constructions.

C. The calculation of medium-sized beams and of beams supported on a straining environment using the methods of the elasticity theory.

The calculation by elementary methods of the resistance of materials in the case of these construction elements, without a thorough theoretical analysis, may lead to either further security in some cases, or to insufficient dimensions or even to destroying the building in other cases.

Thus, the problem has to be solved using the methods of the elastic theory and then conclusions can be drawn regarding possible simplifications.

No matter what the model adopted for the straining environment might be (winklerian model, half-plane model, deformable linear half-space, non-linear deformable model, etc.), the medium-sized beam and the wall beam are in the category of flat issues of elasticity, when for determining the tension and the deformation level it is necessary to find the biharmonic function (Ayri's function) $F(x, y)$ that might satisfy the compatibility equation

$$(3) \quad \Delta\Delta F(x, y) = 0$$

and the conditions on the outline of the analysed field.

Once that we know the tension function, $F(x, y)$, we can determine the unitary efforts as well as the deformations and movements of any point situated in the median plan of the beam.

Thus, in the case of insignificant massic forces, the expressions of the components of the unitary effort tensor are given by the relations

$$(4) \quad \sigma_x(x, y) = \frac{\partial^2 F}{\partial y^2}, \quad \sigma_y(x, y) = \frac{\partial^2 F}{\partial x^2}, \quad \zeta_{xy}(x, y) = -\frac{\partial^2 F}{\partial x \partial y}$$

and the components of the deformation tensor are given by the relations

$$(5) \quad \varepsilon_x = \frac{1}{E}(\sigma_x - \mu\sigma_y), \quad \varepsilon_y = \frac{1}{E}(\sigma_y - \mu\sigma_x), \quad \gamma_{xy} = \frac{2(1 + \mu)}{E}\zeta_{xy}.$$

Considering the geometrical equations of flat elasticity

$$(6) \quad \varepsilon_x = \frac{\partial \mu}{\partial x}, \quad \varepsilon_y = \frac{\partial \mu}{\partial y}, \quad \gamma_{xy} = \frac{\partial \mu}{\partial y} + \frac{\partial \mu}{\partial x},$$

by integrating, using the x and, respectively, y variables we obtain the expression of the movement components according to the x - and y -axes

$$(7) \quad E\mu = \int (\sigma_x - \mu\sigma_y) dx + f_1(y), \quad E\nu = \int (\sigma_y - \mu\sigma_x) dy + f_2(x).$$

In order to specify the tension function, $F(x, y)$, in the case of a particular flat issue, for each point on the outline, we can give either tension conditions in the form

$$(8) \quad P_{nx} = \sigma_x \cos(n, x) + \zeta_{yx} \cos(n, y), \quad P_{ny} = \zeta_{xy} \cos(n, x) + \sigma_y \cos(n, y),$$

where P_{nx} and P_{ny} are the components of the intensity of exterior charges on a line element of the outline characterized by the exterior normal, \mathbf{n} , or the conditions in movements which can be presented in this form.

According to the known data of the problem, when it comes to flat elasticity (flat tension level or flat deformation level), several situations can be encountered regarding the outline conditions. Thus, most of the time the exterior charges operating on the outline are known; then in each point on the outline there has to be a mechanical balance between interior tensions and exterior charges. The above mentioned problem, where outline conditions are given only for the tensions, is called *the first fundamental problem of the elasticity theory*.

When the outline conditions are given only for movements, we have *the second fundamental problem of the elasticity theory*, less commonly encountered in practice. It has to be noticed that such movement conditions are different from the contact conditions (fixity conditions) of the body, represented by points in order to determine the rigidity movement of the body.

When the outline conditions are given for tensions in some places and for movements in other places, we are dealing with a *mixed problem of the elasticity theory*.

In specialized literature, most of the solutions were given for the first fundamental problem, which can also be more easily put into practice. Regarding the second elasticity problem – the mixed problem, even if it is quite commonly encountered, it has got the fewest solutions, because of its difficulties of solving, especially by analytical, exact methods. In this respect, we can mention the solutions given by P.P. Teodorescu, M.I. Dlugaci, A.S. Kalamank, regarding beams built-in on one or two sides.

Considering medium-sized beams and wall beams supported on a straining environment, usually operated on the horizontal superior side by charges from the superstructure or directly from superior elements of the building, the calculation methods have to derive from the same fundamental idea expressing the interaction between the beam and the straining environment: the movements of the inferior side of the beam follow the movements of the land on which it stands. In other words, the calculation of medium-sized beams and of beams supported on a straining environment is reduced to determining the tension and deformation in a connective-simple field (whole beam) or connective-multiple (beam with holes), when on three sides the conditions of outline in efforts are given (we know exterior active forces) and on the fourth side (the supported one) the conditions of movement outline have to be given.

It is a mixed problem of flat elasticity, when mathematically we have to find a biharmonic function, $F(x, y)$, which satisfies the compatibility equation (3) and the

following outline conditions:

$$(9) \quad \begin{array}{l} \text{For } y = H : \\ \left\{ \begin{array}{l} (\sigma_y)_{y=H} = -p(x), \\ (\zeta_{xy})_{y=H} = 0; \end{array} \right. \end{array} \quad \begin{array}{l} \text{For } x = \pm \frac{L}{2} \\ \left\{ \begin{array}{l} (\sigma_x)_{x=\pm L/2} = 0, \\ (\zeta_{xy})_{x=\pm L/2} = 0; \end{array} \right. \end{array} \quad \begin{array}{l} \text{For } x = 0 \\ \nu(x, y)_{y=0} = W(x, y)_{y=0}. \end{array}$$

The great difficulty in the calculation of such beams ($\lambda < 5$), in comparison with ordinary beams ($\lambda \geq 5$), consists in determining movements (ν) vertically on their supporting surface, under the action of exterior active forces given and under the action of the reaction ($p(x)$), whose variation law is unknown.

3. Calculation Method

For determining the tension and the deformation in the medium-sized beams and the wall beams supported on a straining environment, one can try the various methods currently used for solving flat elasticity problems. However, we shall try to insist on those which may lead to practical results using a more accessible design.

Skipping elementary methods of elasticity theory when the tension function can be a polynomial one, but which do not allow us to write the outline conditions on the vertical sides of the beam, only in global form, so they are not used for wall beams ($\lambda < 2$) because the principle of Saint-Venant cannot be applied, we shall concisely draft some ways of solving the connective-simple field which we shall analyse.

From a mathematical point of view, we can make an exact calculation or an approximate one.

From the beginning, it has to be noticed that it is not very common that exact calculation should lead to the result, because physical phenomena – like the one we analyse – are usually complex and can be only approximately comprised in different mathematical formulae. This is why most of the calculation methods used in practical solutions are approximate, often numerical methods.

Approximations can derive either from choosing the tension function so as it approximates the most its real form (for example a trigonometric series) and which wholly fulfils the biharmonic condition and the outline conditions, or from looking for a tension function approximately meeting one of these conditions (finite differences method, variational method, conditions in outline points method, etc).

The importance of approximate methods, especially numerical ones, consists in their wide range of application, as there are almost no limits, the results can be transposed in a simple language, without a complicated mathematical terminology, the solution can be given with any accuracy, according to the importance of the specific construction.

3.1. Variational Methods

Detailed presentations of variational methods, as well as applications in the field of construction mechanics, can be encountered in many specialized works.

The principle of variational methods utilized to solve a differential equation with partial derivatives of the flat elasticity consists in replacing the function $F(x, y)$, that satisfies the differential equation for given limit conditions, with an approximate analytic expression, chosen as to best approximate this function, meaning that the deviation from the real value of the function should be minimum.

Accordingly, in the case of the wall beam supported on a straining environment, we choose an analytic expression, depending on a number (theoretically infinite, practically finite) of random parameters, for the function that has to be approximated (Airy's function) and we determine the condition that best approximates its real value. When using the variational method, two problems are usually encountered:

- a) the problem of choosing the form of the approximation function;
- b) the problem of the approximation method of the function.

For the flat elasticity issues we are interested in, the easiest form of expressing the approximation function is the following:

$$(10) \quad F = \varphi_0 + \varphi = \varphi_0 + \sum_{i=1}^n a_i \varphi_i,$$

where φ_0 and φ_i are suitable given functions which, on the whole, best express the analysed function, and a_i are constant undetermined parameters, varying according to the approximation method used.

Three methods of creating the series can be distinguished:

a) We choose the functions φ_0 and φ_i so that each of them satisfies a part of the limit conditions, but they do not verify the differential equation that they satisfy, F . The a_i parameters can be determined from the condition that the expression approximates best the function F , inside the field as well as on the outline.

b) We choose the functions φ_0 and φ_i so that each of them satisfies a part of the outline conditions, but they do not verify the differential equation. The a_i parameters can be determined from the condition that the expression approximates best the function F inside the field.

c) We choose the functions φ_0 and φ_i so that each of them satisfies the differential equation, being a particular integral of this equation.

The outline conditions are not satisfied. The a_i parameters are determined from the condition that the entire series in its whole should best approximate the function F on the outline of the analysed field.

Of the three methods of creating the series, the first one is the easiest, because it imposes the smallest number of limitations on the function φ_i , being more difficult to determine the a_i parameters.

When the functions φ_0 and φ_i can be chosen without any difficulty in order to meet the conditions required in methods b) and c), the determination of a_i parameters becomes easier.

Without giving a detailed analysis, we emphasize that several approximation methods can be used, helping to establish the equations that lead us to the calculation of undetermined parameters. There is the method of uniform approximation

We choose φ_i functions in order that tensions on the outline be null and satisfy the equations

$$(15) \quad \begin{cases} \sigma_x = \frac{\partial^2 \varphi_i}{\partial y^2} = 0, & \zeta_{xy} = -\frac{\partial^2 \varphi_i}{\partial x \partial y} = 0, & \text{for } x = \pm \frac{L}{2}; \\ \sigma_y = \frac{\partial^2 \varphi_i}{\partial x^2} = 0, & \zeta_{xy} = -\frac{\partial^2 \varphi_i}{\partial x \partial y} = 0, & \text{for } y = 0 \text{ and } y = H. \end{cases}$$

The number of equations depends on the number of undetermined parameters imposed when expressing $F(x, y)$. In general, two or maximum four parameters are enough, determining it by solving a system of two, maximum four equations.

When expressing the a_i parameters, in the case of the beam on a straining environment, the parameters b_i of the series approximating the distributions of the environment's reaction appear as unknown. To determine the b_i parameters we shall write the contact condition (interaction) between the beam and the contact, $V(x, y)_{y=0} = W(x, y)_{y=0}$, in a number of points equal to the number of parameters from the approximation series of reactions, minus two (balance equations for the beam with active and reactive forces). For this purpose it is necessary to find out the vertical movements (V) of the inferior side of the beam, using the elasticity theory, when the tension function is known, as well as vertical movements (W) of the supporting surface of the straining environment operated by the pressures $p(x)$.

The movements, W , are calculated according to the rules of earth mechanics and the adopted model for the foundation soil.

In expressions of V and W , the b_i parameters are unknown; they can be determined by solving the algebraic linear equation system obtained by writing the contact condition. Except the equations expressing the interaction beam-contact, we shall also use the two equations of static equilibrium.

Knowing the analytic expression $p(x)$ approximating the reactions of the supporting environment, we can go back to the tension function, $F(x, y)$, and to the unitary efforts characterizing the tension of the beam itself, replacing the a_i parameters with their numerical values. Thus, we can consider that the calculation of the medium-sized beam and the wall beam supported on a straining environment is finished.

Also according to the known analytic expression $p(x)$, approximating the reactions of the environment, we can determine real movements and the tension in the land on which the beam stands.

For the analysed case, the advantages of using the variational method are the following:

a) we obtain an analytic, approximate, expression for the tension function, which allows us to determine the unitary efforts and the movements in any point of the beam;

b) choosing the tension function is not too difficult in case of some experience in calculation and if indications from specialized literature are used and the parameters of such functions are deduced by solving a system of two, maximum four algebraic equations;

c) the accuracy of the results can be increased, according to the importance of the work, by increasing the amount of calculations.

The disadvantages of the variational method are those generally known and also the difficulties in the case of contact problems (beams on a straining environment):

a) it cannot be applied for any outline of the analysed field (that can be analytically expressed);

b) determining the parameters of the tension function, $F(x, y)$, is made using a system of two or four equations, but solving them is difficult, as a_i parameters will be expressed according to the other n parameters b_i from the expression approximating the distribution of reactions.

Determining b_i parameters is possible by solving a system of n algebraic equations with n unknown terms, $n - 2$ being the number of contact points and $n -$ the number of the parameters of the polynomial $p(x)$. Considering the information in the specialized literature concerning ordinary beams ($\lambda \geq 5$) on a straining environment, it results that the number of equations can be of at least 11 when the charge is unsymmetrical and 6 equations for antisymmetrical charge. The great difficulty does not consist in solving these systems of algebraic equations, but in creating them. This is because the vertical movements (V) of the inferior edge of the beam, when we know the expression of the tension function which contains a series of literal unknown parameters (b_i), are determined by a very thorough calculation, inadequate for the design and the movements, W , of the land in case of a random charge, $p(x)$, and of the adopted model, in the form of a half-plane or linear deformable half-space – most commonly used in practice – raise rather difficult calculation problems.

4. Conclusion

We consider that the variational method can be more successfully applied in the case of medium-sized beams and of wall beams supported on a straining environment, when the rigidity of the beam is bigger than the rigidity of the environment, as the beam can be considered as absolutely rigid. In this case, the distribution of $p(x)$ reactions is known and the determination of the tension function could be made by solving at most four algebraic equations.

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REZOLVAREA TEORETICĂ A PROBLEMEI GRINZILOR REZEMATE
PE MEDIU DEFORMABIL CA O PROBLEMĂ DE CONLUCRARE
Conlucrarea dintre infrastructură, structură și teren

(Rezumat)

Calculul construcțiilor rezemate pe terenuri deformabile a condus la rezultate contradictorii datorită mai multor cauze obiective și subiective. Un rol însemnat în acest calcul îl joacă tipul de fundație și modelul de calcul considerat pentru terenul de fundare.

De regulă astfel de probleme sunt soluționate după o idee conform căreia structura, fundația și terenul de fundare sunt în mod convențional tratate ca subsisteme independente, de către specialistul de structuri și de specialistul geotehnician.

Considerarea ansamblului structură-teren de fundare și analizarea sa, privind comportarea și răspunsul la acțiuni, ca un tot unitar, compus din subsisteme cu caracteristici geometrice și mecanice diferite între care există un proces continuu de redistribuire a eforturilor, se include în categoria generală a interacțiunilor dintre sisteme sau în cadrul aceluiași sistem, între subsisteme.

Problemele propuse de această disciplină sunt complexe și abordarea lor din punct de vedere teoretic și experimental este dificilă. Rezolvările fac apel la cunoștințe de Statica construcțiilor, Teoria elasticității, Geotehnică și fundații etc.