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CONSTRUCŢII. ARHITECTURĂ

THE PLASTIC DEFORMATION CAPACITY OF STRUCTURAL STEEL MEMBERS UNDER SEISMIC ACTION

BY
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The paper describes a calculus procedure to determine the elastic–plastic rotation capacity of steel structural members. The rotation capacity calculus is very important for the design of steel structures exposed to seismic action. The calculus procedure is based on the equivalent elastic–plastic bending moment. This calculus procedure was applied on two types of structural members (beams, columns).

1. Introduction

The aspect of plastic deformation of structural steel members appears, both in the biographic and non-linear calculus of the structures, due to seismic actions.

The calculus of plastic deformation capacity should be applied to the structural members framed into Class 1 cross-sections, defined by the limit slenderness of the section composing walls.

As regards the plastic deformation capacity, this subject has not been sufficiently defined:

a) EUROCODE 3 [1], European Norm shows that "the elements which belong to class "1" of cross sections, have *enough* capacity of plastic deformation, for the structural analysis";

b) USA Norm [2] requires rotation capacities, depending on the distance between the potential plastic articulations on the same structural member;

c) Romanian Norm P100-2006 [4] has not enough requirements regarding the plastic deformation capacity.

The performed research consists in the providing a procedure to establish, by calculus, essentially, the plastic deformation capacity, in the areas being mainly exposed to bending stress. The procedure is defined by the following aspects:

a) Determination of the potential-plastic zone length.

b) Determination of the conventional-elastic bending moment, which allows the determination of the elastic–plastic rotation using integral calculus, using the elastic inertial moment of the structural member.

c) The rotation capacity results, depending on the static scheme of the structural

d) The determination of the elastic–plastic rotation capacity allows, as well, the evaluation of the capacity of dissipation of the seismic energy in relation to the elastic limit state of the structural element behavior.

2. Classes of Cross-Sections for Structural Members

2.1. Cross-Section Class Definition

Depending on the slenderness of the compressed walls of the cross-section, four classes of sections are defined for the structural members namely

a) *Class 1 cross-sections.* Structural members having cross-sections belonging to Class 1 allow the formation and working of plastic articulations. The structures made by Class 1 members allow the *plastic–plastic* calculus model, which means that the determination of stresses inside the structure can be done in *plastic* and sections verification can be done in *plastic*, as well. The potential plastic areas are made by sections, which belong to Class 1.

b) *Class 2 cross sections.* Structural members having cross-sections belonging to Class 2 allow the formation of the plastic articulations having limited working capacity. The structures made by Class 2 members allow the *elastic–plastic* calculus model, which means that the determination of stresses inside the structure can be done in *elastic* and sections verification can be done in *plastic*. The potential plastic areas, made by sections, which belong to Class 2, allow the redistribution of a maximum of 15% of the "peaks" of the bending moment.

c) *Class 3 cross sections.* Structural members having cross sections belonging to Class 3 allow the fiber flow only in the maximum stressed fiber. The structures made by Class 3 members allow the *elastic–elastic* calculus model.

d) *Class 4 cross sections.* Structural members having cross sections belonging to Class 4 allow the fiber flow only in the maximum stressed fiber of the *active area* of the section. The structures made by Class 4 members allow the *elastic–elastic* calculus model, taking into account only the active area of the section.

2.2. Structural Members Behaviour According to the Cross-Section Class

The structural members belonging to *Class 1* cross sections enable the plastic moment of the section (M_p), having the rotation capacity, θ (Fig. 1),

$$\theta \geq 4\theta_p,$$

where θ_p is the rotation corresponding to the cross-section design plastic moment.

The structural members belonging to Class 2 cross-sections enable the plastic moment of the section (M_p), having the rotation capacity

$$\theta \geq 2\theta_p.$$

The structural members belonging to Class 3 cross-sections allow the elastic moment of the section (M_e).

The structural members belonging to Class 4 cross-sections allow the elastic bending moment of the active section (M_{ea}), having the rotation capacity corresponding to its elastic bending moment.

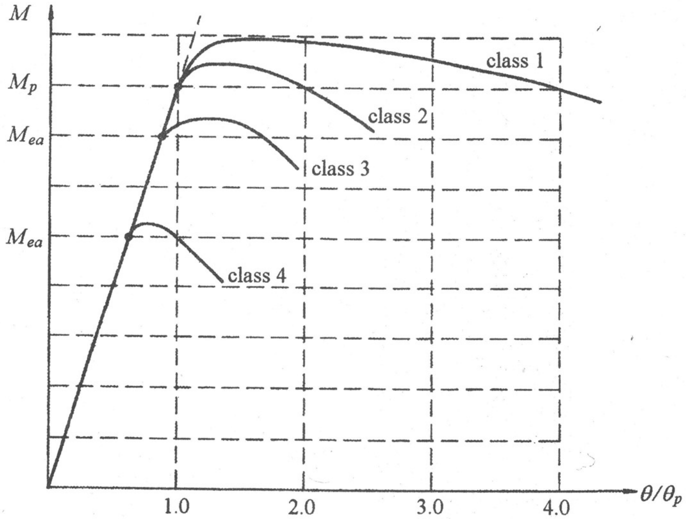

Fig. 1.– Moment *vs.* rotation dependence according to the cross section class.

3. Application Area of the Plastic Reserve of Structural Members

The structural members containing potential plastic areas are built from steel having a distinctive drift level. Mechanical characteristics of the steel shall meet the following requirements:

$$\frac{R_m}{R_c} \geq 1.25, \quad \frac{\varepsilon_m}{\varepsilon_c} \geq 20, \quad A \geq 15\%, \tag{1}$$

where: R_m is the minimum guaranteed tensile strength; R_c – minimum guaranteed yield strength; $\varepsilon_m, \varepsilon_c$ – strains, corresponding to strengths R_m, respectively R_c; A – strain at rupture on a specimen with an initial distance (L_0) between gauge points, $L_0 = 5.65\sqrt{S_0}$; S_0 – initial area of the test sample section.

The cross-section of the structural member shall comply with

a) at least a mono-symmetric cross-section, in case of mono-axial bending; symmetric axis shall be in the same plan as the load;

b) bi-symmetric section in case of bi-axial bending.

The structural member is not subjected to twist strength.

The loads don't belong to the category of cyclic loads, which determine the fatigue behavior. In the potential plastic areas concentrated loads shall not be applied.

The structural members, analysed in this paper, have *a double T welded cross section*, bi-symmetric, and are shown in several options, regarding the static scheme.

4. Plastic Calculus Due to Combined Stresses

4.1. Limit Slenderness of the Structural Elements Walls

The plastic potential areas are made in the Class 1 cross-sections.

The limit slenderness of the bending elements (Fig. 2) shall meet the following conditions:

$$\frac{b'}{t} \leq 9\,\varepsilon, \quad \frac{h_i}{t_i} \leq 66\,\varepsilon, \tag{2}$$

where: b' is the free width of the flange; t – the thickness of flange; h_i, t_i – the web depth and thickness, respectively, $\varepsilon = \sqrt{240/R_c}$.

The limit slendernesses of the compressed and bent elements (Fig. 3) shall meet the following requirements:

$$\frac{b'}{t} \leq 9\,\varepsilon, \quad \frac{h_i}{t_i} \leq \frac{33}{\alpha}\varepsilon, \tag{3}$$

where α bounds the compressed area of the cross-section.

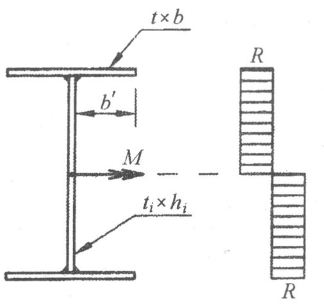

Fig. 2.– Bent element – distribution of stresses.

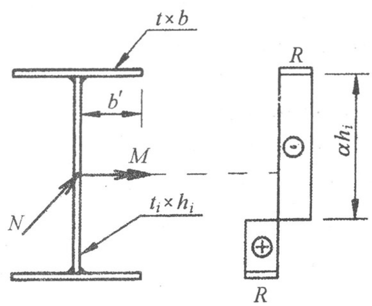

Fig. 3.– Compressed and bent element – distribution of stresses.

4.2. Design Stress due to Single Loads

The design plastic shear resistance due to single shear load is determined with relation

$$T_p = A_l R_f, \tag{4}$$

where: R_f is the share design stress, A_l – the cross sectional area of the web.

The design plastic compression resistance due to single compression load is determined with relation

(5) $$N_p = AR,$$

where: A is the cross sectional area, R – the design strength of steel.

The design plastic resistance moment due to bending load is obtained with

(6) $$M_p = W_p R,$$

where W_p is the plastic section modulus.

4.3. The Calculus of the Bent and Sheared Members

The bent and sheared members shall comply with the following checking conditions (in the potential plastic areas)

(7) $$\frac{T}{T_p} \leq 0.5; \quad \frac{M}{M_p} \leq k_r,$$

where: T and M are the effective shear force and effective bending moment, respectively, in the section under consideration, and k_r – the reduction coefficient due to the presence of the shear load.

The k_r coefficient is defined using the following method:

$$v = \frac{T}{T_p}; \quad k = (1 - v^2)^{0.5}$$

and consequently

(8) $$k_r = \frac{2\beta}{1 + 2\beta}\left(1 + \frac{1}{2\beta}k\right),$$

where: $\beta = 2A_t/A_i$ is the material distribution coefficient on the cross section; A_t – area of a single flange; A_i – web area.

4.4. The Calculus of Compressed and Bent Members

The compressed and bent members shall comply with the following checking conditions (in the potential plastic areas):

(9) $$\frac{N}{N_p} \leq 0.15; \quad \frac{M}{M_p} \leq k_r,$$

where: N is the effective compression force and k_r – the reduction coefficient due to the presence of axial force.

The k_r coefficient is defined as it follows:

$$n = \frac{N}{N_p},$$

$$k_r = \frac{2\beta}{1+2\beta}\left[1 + \frac{1}{2\beta}\alpha_1(2-\alpha_1)\right], \tag{10}$$

where

$$\alpha_1 = 1 + (1+\beta)n.$$

4.5. The Calculus of Compressed, Bended and Shared Members

The members having the internal member forces N, T, M shall meet, in the potential plastic areas, the following checking criteria

$$\frac{N}{N_p} \leq 0.15; \quad \frac{T}{T_p} \leq 0.5; \quad \frac{M}{M_p} \leq k_r, \tag{11}$$

where k_r is the reduction coefficient. This coefficient is defined using the following formulas:

$$v = \frac{T}{T_p}; \quad k = (1-v^2)^{0.5}; \quad n = \frac{N}{N_p}; \quad k_r = \frac{2\beta}{1+2\beta}\left[1 + \frac{1}{2\beta}\alpha_1(2-\alpha_1)\frac{1}{k}\right].$$

with

$$\alpha_1 = k + (1+\beta)n.$$

5. Simply Supported Member Having Concentrated Load

5.1. The Calculus of the Elastic Displacements (Fig. 4)

The design elastic moment and the design elastic force shall be calculated with relations

$$M_e = \frac{P_e\, l}{4}, \qquad P_e = \frac{4M_e}{l}.$$

The moment expressions for the displacements calculus are

$$M = M_e - \frac{2M_e}{l}\xi\, l = (1-2\xi)M_e,$$

$$m_1 = 0.25 - 0.5\,\xi\, l = (1-2\,\xi)\,0.25\,l, \qquad m_2 = 1$$

The elastic deflection is

$$f_e = \frac{2l}{EI}\int_0^{0.5}(1-2\xi)M_e(1-2\xi)\,0.25\,l\,\mathrm{d}\xi = \frac{0.5M_e\,l^2}{EI}\int_0^{0.5}(1-4\xi+4\xi^2)\,\mathrm{d}\xi$$

Performing the calculus it results

$$f_e = \frac{0.25}{3}\cdot\frac{M_e\,l^2}{EI}. \tag{12}$$

The elastic rotation is

$$2\theta = \frac{2l}{EI}\int_{0}^{0.5}(1-2\xi)M_e\,\mathrm{d}\xi \tag{13}$$

resulting

$$\theta_e = 0.250\frac{M_e\,l}{EI}\cdot$$

5.2. The Calculus of the Elastic–Plastic Displacements (Fig. 5)

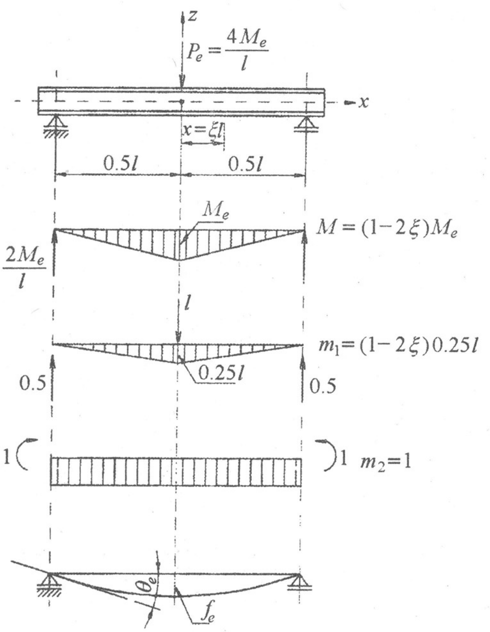

Fig. 4.– Elastic displacements for a simply supported member.

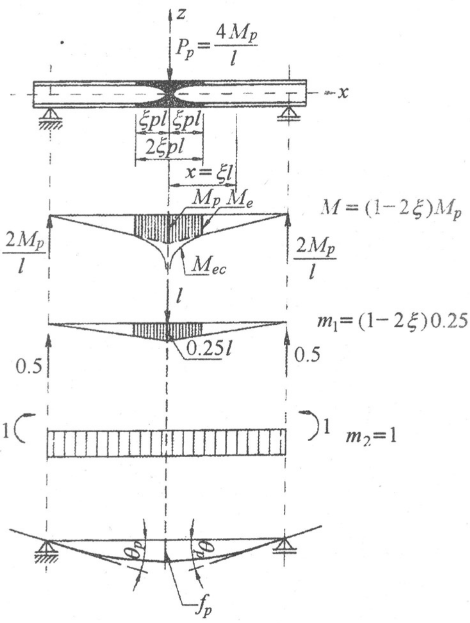

Fig. 5.– Elastic – plastic displacement for a simple supported beam.

The bending moment and the capable-elastic load have the following equations, respectively:

$$M_p = \frac{P_p\,l}{4}; \qquad P_p = \frac{4M_p}{l}.$$

The moment expressions for the displacements calculus are

$$M = (1-2\xi)M_p; \qquad m_1 = (1-2\xi)\,0.25\,l; \qquad m_2 = 1.$$

The length of the elastic–plastic area results from relation

$$M_p - \frac{2M_p}{l}\,\xi_p\,l = M_e, \qquad \text{where} \qquad 1-2\xi_p = \frac{M_e}{M_p}$$

and consequently

$$\xi_p = 0.5\left(1 - \frac{M_e}{M_p}\right) = 0.5\left(1 - \frac{2}{3}\cdot\frac{1+3\beta}{1+2\beta}\right) \tag{14}$$

The elastic–plastic bending moment (Fig. 6) in the range $[0,\ \xi_p]$ is done by

$$M_{ep} = \left[1 - \frac{1}{3(1+2\beta)}\left(\frac{c}{h}\right)^2\right] M_p.$$

The elastic area height (Fig. 6) in the range $[0,\ \xi_p]$ results from relation

$$M_{ep} = M; \qquad 1 - \frac{1}{3(1+2\beta)}\left(\frac{c}{h}\right)^2 = 1 - 2\xi,$$

that is

$$\frac{c}{h} = \sqrt{6(1+2\beta)\,\xi}\,.$$

The expression of conventional elastic bending moment (Fig. 7) in the range $[0,\ \xi_p]$ results from

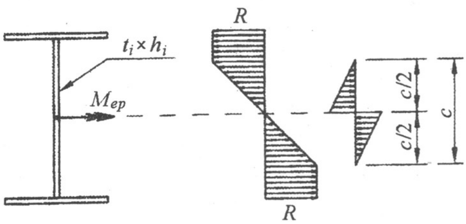

Fig. 6.– Distribution of stresses from the elastic–plastic bending moment.

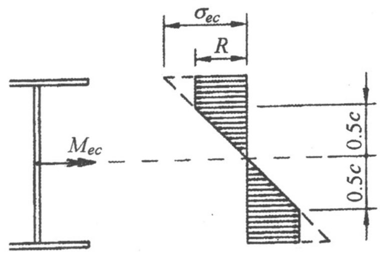

Fig. 7.– Distribution of stresses from the conventional elastic bending moment.

$$\frac{\sigma_{ec}}{0.5h} = \frac{R}{0.5c}, \quad \text{with} \quad \sigma_{ec} = \frac{R}{c/h} = \frac{M_{ec}}{W_e};$$

consequently

$$M_{ec} = \frac{W_e R}{c/h} = \frac{M_e}{c/h} = \frac{(1-\xi_p)M_p}{\sqrt{6(1+2\xi)}}\cdot\frac{1}{\sqrt{\xi}}.$$

Denoting

$$k_1 = \frac{1-\xi_p}{\sqrt{6(1+2\beta)}} \tag{15}$$

it results finally

$$M_{ec} = \frac{k_1}{\sqrt{\xi}} M_p.$$

The elastic–plastic deflection in the range $[0,\ \xi_p]$ is

$$f_{ep} = \frac{2}{EI}\int\limits_0^{\xi_p} \frac{k_1}{\sqrt{\xi}} M_p(1-2\xi)0.25\,l\,\mathrm{d}\xi = \frac{0.5k_1 M_p l^2}{E}\int\limits_0^{\xi_p}\frac{1-2\xi}{\sqrt{\xi}}\,\mathrm{d}\xi.$$

Performing the calculus it results

$$f_{ep} = k_1\sqrt{\xi_p}\left(1-\frac{2}{3}\xi_p\right)\frac{M_p l^2}{EI}.$$

Denoting

(16) $$k_2 = \sqrt{\xi_p}\left(1-\frac{2}{3}\xi_p\right)$$

it results

$$f_{ep} = k_1 k_2 \frac{M_p l^2}{EI}.$$

The elastic deflection in the range $[\xi_p,\ 0.5]$ is

$$f_e = \frac{2l^2}{EI}\int\limits_{\xi_p}^{0.5}(1-2\xi)M_p(1-2\xi)\,0.25\,\mathrm{d}\xi.$$

Denoting

(17) $$k_3 = \frac{0.25}{3} - 0.5\xi_p + \xi_p^2 - \frac{2}{3}\xi^3$$

we obtain

$$f_e = k_3\frac{M_p l^2}{EI}.$$

The elastic–plastic deflection in the range $[-0.5,\ +0.5]$ is

(18) $$f_p = f_{ep} + f_e = (k_1 k_2 + k_3)\frac{M_p l^2}{EI}.$$

The elastic–plastic rotation in the interval $[0,\ \xi_p]$ is

$$2\theta_{ep} = \frac{2l}{EI}\int\limits_0^{\xi_p}\frac{k_1}{\sqrt{\xi}}M_p\,\mathrm{d}\xi,\ \ i.e.\ \theta_{ep} = \frac{k_1 M_p l}{EI}\int\limits_0^{\xi_p}\frac{1}{\sqrt{\xi}}\,\mathrm{d}\xi,$$

namely

(19) $$\theta_{ep} = 2k_1\sqrt{\xi_p}\,\frac{M_p l}{EI}.$$

The elastic rotation in the range $[\xi_p, 0.5]$ is

$$2\theta_{ep} = \frac{2l}{EI}\int_{\xi_p}^{0.5}(1-2\xi)M_p\,\mathrm{d}\xi \quad i.e.\ \theta_e = \frac{M_pl}{EI}\int_{\xi_p}^{0.5}(1-2\xi)\,\mathrm{d}\xi.$$

and

(20) $$k_4 = 0.25 - \xi_p + \xi_p^2.$$

The elastic–plastic rotation in the range [0, 0.5] is

(21) $$\theta_p = \left(2k_1\sqrt{\xi_p} + k_4\right)\frac{M_pl}{EI}.$$

6. Conclusions

A procedure for evaluating by calculus the capacity of elastic–plastic deformation of the steel structural members is described. The procedure underlines the parameters which determine the elastic–plastic deformation capacity: the cross-section geometry, the static and the load scheme of the structural member. In the future, the research can be developed by analysing the plastic deformation capacity, by shearing and by experimental analysis, which shall demonstrate the hypothesis of the calculus procedure summarized in this paper. Numerical examples based on the theoretical development obtained will be published in a future paper.

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CAPACITATEA DE DEFORMARE PLASTICĂ A ELEMENTELOR STRUCTURALE DIN OŢEL SUPUSE LA ACŢIUNEA SEISMICĂ

(Rezumat)

Se propune un procedeu de determinare prin calcul a capacităţii de rotire elasto-plastică a elementelor structurale din oţel. Calculul capacităţii de rotire este foarte important pentru proiectarea structurilor din oţel, expuse acţiunii seismice. Procedeul de calcul se bazează pe momentul încovoietor elastic-convenţional în evaluarea capacităţii de rotire elasto-plastică. Procedeul a fost aplicat pe două tipuri de elemente structurale iar unele exemplificări vor fi prezentate într-o lucrare viitoare.