ALGORITHM AND COMPUTER PROGRAM FOR THE DYNAMIC ANALYSIS OF HYGROTHERMAL BEHAVIOUR IN RESIDENTIAL BUILDINGS

BY
IRINA BLIUC and RODICA ROTBERG

A program for the non-steady state calculus of the thermal condition of buildings is proposed. A harmonical model has been used to creating this program. The detailed presentation of the algorithm is followed by a case study drawn up for an enclosure with load-bearing masonry structure situated at intermediate and terminal layers, at different moments: in winter, when the heating is cut-off, and in summer.

1. Theoretical Considerations

Buildings may be subjected to various thermal loads such as: fluctuating evolution of the exterior temperature, gains from the sun or central heating, energy dissipated by occupants and the interior equipment. The dynamic loads, due to life service and ventilation, determine the response of the building as temperature variation of indoor space and its enclosing areas.

These equally depend on: the functional structure of the given system, volumetric analysis, constructive characteristics, enclosing elements make-up, thermo-physical characteristics of materials.

The evaluation of the parameters or of the functions that represent the response of the building or of the enclosure involves the breaking-up of the ensemble in its components, determining their characteristics, reassembling them while preserving the characteristics of the building and enclosures. The loads can be determined separately, to which the responses could be afterwards added, following the principle of effects overlapping. In this way may be pointed out the response of a room to two or more simultaneous loads such as: sine shaped fluctuation of outdoor air temperature (which lends itself to the use of harmonic models) and limited cut-off of heating plant.

1.1. Harmonic Method Principle

The building envelope is considered the seat of linear transfers by one-dimensional conduction, air-wall convection, wall surface radiation. The building is subjected to
common loads: a) sinusoidal variation of outdoor air temperature, $T_o$; b) solar radiation on the wall sides, $F_s$; c) natural continuous ventilation, $D_c$; d) quasi-constant heating power, $F_c$, which ensure the normal value of the indoor air temperature, for average values of the outdoor temperature.

The harmonic model may be written [1]:

$$T_A(\omega) = H(\omega)S(\omega).$$

where: $T_A(\omega)$ is the Fourier transform of indoor temperatures; $H(\omega)$ - the harmonic transfer matrix of the building; $S(\omega)$ - the Fourier transform of loads.

The procedure entails two steps:
a) finding the balances at the walls interfaces;
b) finding the energy balance of the enclosure.

The harmonic transfer matrix of the building, $H(\omega)$, can be obtained by combining the equations defined in the two steps aforementioned.

a) Energy balance at air-wall interface

The energy balance at the wall sides is given by the equations:

$$\begin{aligned}
\varphi_i &= X_i T_i - Y_i T_i' = h_i(T_A - T_i) + \sum_j R_{ij}(T_j - T_i) + F_{si}, \\
\varphi_i' &= -Y_i T_i' + Z_i T_i' = h_i'(T_A' - T_i') + \sum_j R_{ij}'(T_j' - T_i') + F_{si}'.
\end{aligned}$$

where:

$$X = \frac{M_{11}}{M_{12}}; \quad Z = \frac{M_{22}}{M_{12}}; \quad Y = \frac{1}{M_{12}};$$

$M$ is the harmonic transfer matrix of the wall, (a detailed calculus is given in [3]);

$R_{ij}$ - radiative change function ($R_{ij} S_i = R_{ij} S_j$).

Equations can be rewritten for all walls surfaces (2m):

$$\begin{aligned}
\sum_{j=1}^{m} \left[ \left( \frac{\delta_{ij}}{h_i} - \frac{1 - V_i}{U_i} \right) R_{ij} \right] Q_i - \left( \frac{W_i}{U_i} \cdot \frac{R_{ij}'}{h_i'} \right) Q_j' &= \\
= S_i \left( V_i T_a - W_i T_a' - \frac{1 - V_i}{U_i} F_{si} - \frac{W_i}{U_i} F_{si}' \right),
\end{aligned}$$

where:

$$U = h_i + \sum_j R_{ij}; \quad V = \frac{(Z + U')X - Y^2}{(Z + U')(X + U) - Y^2};$$

$$W = \frac{Y U'}{(Z + U')(X + U) - Y^2}; \quad Q_i = h_i S_i (T_o - T_i).$$
The equations are valid even if the interface is in contact with the exterior, case in which $T_a$ is replaced by $T_e$.

Equations (4) can be regrouped in matriceal form:

\[
AQ = BT_A + C_SF_S + C_eT_e.
\]

(6)

![Diagram of a system with heat transfer](image)

**Fig. 1.**– Calculus scheme.

b) *Energy balance of the air volumes*

The increase in indoor energy of an enclosure of volume $v_a$ and caloric efficiency $c_a$ is equal to the sum of the flows entering/getting out of the enclosure: the convective flow $Q_i$, between the walls and the surrounding air, the power $F_{ca}$, generated by the heating plant (air conditioning), $D_{e-a}$, $D_{I-a}$, the mixture terms for the refreshment of the air in enclosures.

The balance can be written as:

\[
jωc_a v_a T_a = \sum_i Q_i e_{ai} + \sum_{i=1}^n c_a D_{I-a} (T_1 - T_a) + D_{e-a} (T_e - T_a) + F_{ca}.
\]

(7)

where $e_{ai}$ equals 1 if the surface $i$ is in contact with enclosure $a$, otherwise it equals 0.

The air balance equations can be expressed under matriceal form out of which the matrix $H$ can be written:

\[
EQ + DT_A + D_e T_e + F_c = 0,
\]

(8)

\[
T_A(\omega) = \begin{bmatrix} a(\omega) & b(\omega) & c(\omega) \end{bmatrix} \begin{bmatrix} F_c \\ T_e \\ F_S \end{bmatrix},
\]

(9)

with:

\[
a(\omega) = [EA^{-1}B + D]^{-1} ; 
b(\omega) = a(\omega) [EA^{-1}C_e + D_e] ; 
c(\omega) = a(\omega) [EA^{-1}C_s].
\]

(10)
The harmonic method has many advantages in the heat analysis of the buildings. If the frequency and the amplitude characteristic of the loads are known, the dynamic behaviour of the envelope can be predicted, due to the elements of harmonic transfer matrix (amplitude, lagging ratio).

1.2. Thermal Stability in Winter Time

The thermal stability of rooms in buildings with intermittent heating, during winter, can be expressed by the decrease in the indoor air temperature when the rooms get cold due to the cut-off of the heating system. The amplitude of the indoor air temperature oscillation is given by the relation:

\[ A_{\theta t} = \Delta \theta \left(1 - e^{-\frac{\Delta \theta}{\omega_{ct}}}ight), \quad [^\circ\text{C}]. \]

where: \( \Delta \theta \) is the temperature difference between the indoor and outdoor air, \([^\circ\text{C}]; \)
\( Q \) - amount of heat lost by the room through the exterior building elements for the temperature variation of 1\(^\circ\)C. [W/K], equal to:

\[ W = W_f + W_e, \]

where: \( W_f \) is the heat accumulating capacity of the closing elements (partitions, floors and slabs, etc.), [W.h/K]; \( W_e \) - heat accumulating capacity of the exterior elements (exterior walls, roofs, etc.), [W.h/K]; \( \tau \) - time when the heating plan is switched off.

2. Application Program

The application program was used:

a) to determine the indoor temperature of an enclosure considering the sine shaped variation of the outdoor air temperature, with or without heating source (the algorithm was written using the relations (1)....(10));

b) to determine the temperature drop when the heating source was cut-off;

c) to determine the variation of the indoor air temperature as an effect of exterior temperature variation and heating plant cut-off;

d) in various plant running scenarios, the most unfavourable condition is chosen (the end of heating plant cut-off coincides with the moment when the minimum temperature is reached);

e) to determine the critical point (minimum temperature on the walls) by using the finite differences method and by using the previously measured temperature of the indoor air.

It entails the use of more programs, namely:

a) calculus of harmonic transfer matrices of the closing elements (enables the creation of a library of harmonic transfer matrices for the current walls);

b) calculus of indoor temperature of the enclosure;

c) calculus of temperature on walls surface.
3. Case Study

The studied case refers to the analysis of the thermal regime of a living room in a three-roomed flat situated in a block of flats (groundfloor + two stories).

The following composition of the enclosing elements was considered:

a) brick masonry GVP 25 cm thick exterior walls and thermal insulation with cellular-expanded concrete BCA 550;

b) ceramic blocks masonry interior walls 7.5 cm thick;

c) terrace roof with thermal insulation of foam concrete;

d) reinforced concrete flooring with parquetry.

The make-up and the thermo-physical characteristics of the materials used in the closing elements are presented in Table 1.

### Table 1

<table>
<thead>
<tr>
<th>Area</th>
<th>Materials</th>
<th>Thickness layer m</th>
<th>Thermophysical characteristics</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>$\rho$ kg/m$^3$</td>
<td>$\lambda$ W/m.K</td>
</tr>
<tr>
<td>Exteriors walls:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a) in working plane</td>
<td>Mortar cement</td>
<td>0.02</td>
<td>1.76</td>
</tr>
<tr>
<td></td>
<td>GVP 1475</td>
<td>0.25</td>
<td>1.55</td>
</tr>
<tr>
<td></td>
<td>BCA GBN35</td>
<td>0.20</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>Mixed mortar</td>
<td>0.02</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td>Reinforced concrete</td>
<td>0.375</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>B.C.A 550</td>
<td>0.075</td>
<td>0.55</td>
</tr>
<tr>
<td></td>
<td>Mortar cement</td>
<td>0.02</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td>Mixed mortar</td>
<td>0.02</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>Reinforced concrete</td>
<td>0.25</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>BCA GBN 35</td>
<td>0.20</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>Mortar cement</td>
<td>0.02</td>
<td>1.80</td>
</tr>
<tr>
<td></td>
<td>Reinforced concrete</td>
<td>0.02</td>
<td>2.50</td>
</tr>
<tr>
<td></td>
<td>Mortar cement</td>
<td>1.800</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td>CMC 1243 cinders</td>
<td>0.850</td>
<td>0.35</td>
</tr>
<tr>
<td></td>
<td>Foam concrete</td>
<td>615</td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td>Tar board</td>
<td>600</td>
<td>0.17</td>
</tr>
<tr>
<td></td>
<td>Gravel</td>
<td>2,000</td>
<td>1.16</td>
</tr>
</tbody>
</table>

The dynamic characteristics of the closing and partitioning elements were determined, fact that contributes to the behaviour of the enclosure under variable conditions (Table 3). In the exterior walls, that present physical and geometrical non-homogeneousness resulting from the use of struts, girdles or lintels, the dynamic characteristics were determined on areas of different structures, the characteristics of the elements resulting as a weighted mean depending on surfaces (Table 2).

### Table 2

<table>
<thead>
<tr>
<th>Area</th>
<th>Damping</th>
<th>Phase lagging h</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>41.10</td>
<td>-6.27</td>
</tr>
<tr>
<td>2</td>
<td>16.87</td>
<td>-9.84</td>
</tr>
<tr>
<td>3</td>
<td>24.87</td>
<td>-8.24</td>
</tr>
</tbody>
</table>
The thermal characteristics of the enclosure (heat losses, heat accumulation, natural ventilation rate) are shown in Table 4. The variation of temperature on the interior surface of the exterior wall of the enclosure at a harmonic excitation with an amplitude of 9.2°C and the average April temperature of 10.6°C (living room, intermediate floor), are shown in Fig. 2.

### Table 3
**The Dynamic Characteristics of Exterior Walls and Terrace**

<table>
<thead>
<tr>
<th>Element</th>
<th>Surface m²</th>
<th>Composition</th>
<th>Damping h⁻¹</th>
<th>Phase lagging h⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall 1</td>
<td>6.16</td>
<td>Area 1</td>
<td>33.67</td>
<td>-7.30</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Area 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wall 2</td>
<td>7.95</td>
<td>Area 1</td>
<td>33.72</td>
<td>-7.27</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Area 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Area 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wall 3</td>
<td>15.68</td>
<td>Area 1</td>
<td>39.22</td>
<td>-6.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Area 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Area 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Wall 4</td>
<td>3.93</td>
<td>Area 1</td>
<td>33.88</td>
<td>-7.28</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Area 2</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Area 3</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Terrace</td>
<td>20.8</td>
<td>Area 1</td>
<td>58.59</td>
<td>-6.112</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Area 3</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Table 4
**The Thermal Characteristics of the Enclosure**

<table>
<thead>
<tr>
<th>Characteristics</th>
<th>Value Store 1</th>
<th>Value Store 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat losses, [W/°C]</td>
<td>46.5</td>
<td>45.3</td>
</tr>
<tr>
<td>Heat accumulation, [W.h/°C]</td>
<td>3.872</td>
<td>5.494</td>
</tr>
<tr>
<td>Natural ventilation rate, [h⁻¹]</td>
<td>0.94</td>
<td>0.70</td>
</tr>
<tr>
<td>Volume of enclosure, [m³]</td>
<td>54.9</td>
<td>54.9</td>
</tr>
</tbody>
</table>

Fig. 2. - The variation of temperature on the interior surface of the exterior wall (exterior harmonic excitation with the amplitude 9.2°C and average April temperature 10.6°C): -×-×-×- wall 1; -○-○- wall 2; -●-●-●- terrace.
The response of the enclosure to a harmonic excitation of the outdoor temperature of 5°C amplitude and average value of \(-15^\circ\text{C}\) (December month, City of Jassy), given by the variation of the indoor air temperature if the heating source provides constant heat so that the air temperature should be maintained at standard norms (20°C) is presented in Fig. 3.

![Graph showing temperature variation over time.](image)

**Fig. 3.** Variation of indoor temperature of enclosure to a harmonic excitation of the outdoor temperature of 5°C amplitude and average value \(-15^\circ\text{C}\): \(\times\times\times\) indoor air temperature (store 1); \(\circ\) indoor air temperature (store 2).

The obtained results so far allow us to determine the damping coefficient depending on the amplitudes of indoor and outdoor temperatures variation

\[
v = \frac{A_{T_2}}{A_{T_1}}.
\]

(13)

It can be seen that the value of the damping capacity for the top floor is higher than the one for the intermediate floor, due to the terrace roof.

When 12 h cut-off of the heating plant is considered, a decrease of the indoor air temperature is recorded (relation (11)) as well as of the temperature on the interior surface of the exterior wall (found by using finite differences method) – Fig. 4.

![Graph showing temperature variation over time.](image)

**Fig. 4.** Variation of temperature indoors and on the exterior wall when the heating plant is switched off (the same enclosure): \(\times\times\times\) indoor air temperature; \(\circ\circ\circ\) temperature on interior surface of the exterior wall.
4. Conclusions

Creating a comfortable micro-climate inside buildings is one of the essential exigencies of buildings hygiene. The indoor air temperature has a vital role in ensuring the thermal comfort and involves a certain correlation function of the temperature of the room closing surfaces. This is important both in identifying the condensation hazard where the thermal resistance is minimal, and in evaluating the extent to which comfort requirements are met. The algorithm we built enables us to determine the necessary values when the interdependence between the two aspects is being assessed. The program ensures the thermal analysis of enclosures depending on their position within the building, the thermal characteristics of the component elements, the random character of the exterior parameters and of the service conditions.

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Technical University “Gh. Asachi”, Jassy, Department of Civil and Industrial Engineering

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ALGORITM ȘI PROGRAM DE CALCUL PENTRU ANALIZA REGIMULUI HIGROTERMIC ÎN LOCUNŢE

(Rezumat)

Se propune un program pentru calculul nestăţionar al regimului termic al clădirilor. La întocmirea programului s-a folosit un model armonic. Prezentarea detaliată a algoritmului de calcul este urmată de un studiu de caz întocmit pentru o incintă cu structură din zidărie portantă situată la nivel intermediar și terminal, în regim de iarnă, la întreruperea funcționării instalației de încălzire și în regim de vară.