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STRUCTURAL HEALTH MONITORING THROUGH DYNAMIC IDENTIFICATION TECHNIQUES: NUMERICAL SIMULATION OF A DAMAGE SCENARIO

BY

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Damage detection in civil engineering structures using changes in measured modal parameters is an area of research that has received notable attention in literature in recent years. This paper compares two different experimental techniques for predicting damage location and severity: the Change in Mode Shapes Method and the Mode Shapes Curvature Method. The comparison is established with reference to a simply supported finite element bridge model in which damage is simulated by reducing opportunely the flexural stiffness, EI. The results of this study indicate that change in modal curvature is a significant damage indicator, while indexes like MAC and COMAC–widely right for FE model updating–lose their usefulness in order to damage detection.

1. Introduction

With the term *damage* in literature [1] is indicated any change introduced into a system that negatively affects its current or future performances. The process of implementing a damage detection strategy for civil engineering structures is indicated as Structural Health Monitoring. Several research works have appointed the importance to develop techniques able either to locate damage or to determine its influence on the global mechanical features of a structure. In fact, most currently used damage detection methods are visual or localized methods using acoustic, ultrasonic, magnetic field, X-ray or thermal principles. All of these experimental techniques require that the vicinity of the damage is known in advantage and that the portion of the structure being inspected is easily accessible, therefore allowing to detect a possible damage only near the surface of the structure. Recently have been developed several techniques of dynamic identification, able to examine changes in the vibration characteristics of a structure. The basic premise is that damage will alter the stiffness, mass or energy dissipation properties of a structure, which in turn alter the measured structural dynamic response; consequently, the experimental response of a damaged structure during a dynamic investigation moves away from that theoretical waited for.

2. Description of Damage Identification Methods

2.1. Change in Mode Shapes Method

This method is based on the comparison between the two series of mode shapes deducible from the results of two different dynamic investigations executed subsequently–one after the other–on a same structure. The comparison can be executed calculating MAC and COMAC indexes, afterwards described.

The Modal Assurance Criterion (MAC) is determined by the relation [2]

$$(1)\qquad \mathrm{MAC}_{(j,k)} = \frac{\left(\sum_{i=1}^{n} \Phi^i_{Aj}\Phi^i_{Bk}\right)^2}{\sum_{i=1}^{n}\left(\Phi^i_{Aj}\right)^2 \sum_{i=1}^{n}\left(\Phi^i_{Bk}\right)^2}, \quad (j = 1, ..., m_A;\ k = 1, ..., m_B),$$

where: Φ_A and Φ_B are two series of mode shapes expressed in matrix form (eigenmode matrices), respectively of $n \times m_A$ and $n \times m_B$ class, with m_A and m_B equal to the number of investigated modes and n equal to the number of considered coordinates (that is the number of measurement points); Φ^i_{Aj} is the i-th coordinate of the j-th column of Φ_A (that is that of the j-th vibrational mode), while Φ^i_{Bk} is the i-th coordinate of the k-th column of Φ_B (that is that of the k-th vibrational mode). Those symbols can be justified remembering the form assumed by the eigenmode matrices Φ_A and Φ_B

$$[\Phi_A] = [\{\Phi_{A1}\}\{\Phi_{A2}\} ... \{\Phi_{Aj}\} ... \{\Phi_{Am}\}],$$

$$[\Phi_B] = [\{\Phi_{B1}\}\{\Phi_{B2}\} ... \{\Phi_{Bj}\} ... \{\Phi_{Bm}\}],$$

where $\{\Phi_{Aj}\}$ and $\{\Phi_{Bj}\}$ are vectors of n components (in number equal to the measurement points) and have been supposed the same number of investigated mode shapes for the two data sets in exam.

The MAC, for a fixed mode shape, measures the correlation (the similarity) between the two corresponding series of modal amplitudes. Its values varies from 0 to 1; MAC = 1 means a perfect correlation, while MAC = 0 means that eigenmodes $\{\Phi_{Aj}\}$, $\{\Phi_{Bj}\}$ are not correlated. In theory, mode shapes remain invariant if structure doesn't suffer alterations, because they are structural intrinsic features, depending entirely on mass and stiffness distribution. The presence of damage–that is, for a fixed element, the reduction of cross-section inertial properties–involves a sensible change in those features, in a way depending on damage extent and position. This is the reason for which damage identification can be conducted paying attention to the possible differences between the two mode shapes series obtained through two different dynamic investigations executed–one after the other–on the same structure: the more MAC verges on 0, the more structure is damaged.

As MAC does not take into account local deviations of displacement, it has been introduced another index, the Coordinate Modal Assurance Criterion (COMAC), expressed by the following relation [2]:

$$\text{COMAC}_{(i)} = \frac{\left(\sum_{k=j=1}^{L} \Phi_{Aj}^{i} \Phi_{Bk}^{i}\right)^2}{\sum_{j=1}^{L} \left(\Phi_{Aj}^{i}\right)^2 \sum_{k=1}^{L} \left(\Phi_{Bk}^{i}\right)^2}, \tag{2}$$

where L is the total number of investigated modes and $i = 1, ..., n$ is the generic point of measure. This index can be used to identify the positions in which the two series, Φ_A and Φ_B, of mode shapes, are discordant, because measures the correlation between all the displacements at i-th point corresponding to the different modes. If the COMAC value is equal to 1, then no difference appears between the deflection coordinates in the intact state and the damaged one. The lower COMAC values signify the differences in coordinates and, thus, possible damage.

2.2. Mode Shapes Curvature Method

This method is based on a direct consequence of damage, as reduction of flexural stiffness of a structure in correspondence with the damaged regions; this result increases the amplitude of curvature at those regions and so can be used to detect and locate damage (*i.e.*, a crack). Change in curvature increases with reduction in flexural stiffness value and, therefore, the amount of damage can be obtained from the amplitude of curvature's change. To demonstrate this property is necessary to remind the relationship between the flexural stiffness, EI, of a simply supported prismatic beam and the frequency, ω_n, of the n-th vibrational mode

$$\omega_n = \frac{n^2\pi^2}{L^2}\sqrt{\frac{EI}{m}}, \tag{3}$$

where L is the length of the beam and m – the mass for unit of length. From the theory of structure is well-known the relation between the curvature, k, of beam's axis and the bending moment, M, at the cross-section considered

$$\frac{1}{EI} = \frac{k}{M}. \tag{4}$$

Hence, the substitution of eq. (1) in (2) gives

$$k = \frac{n^4\pi^4 M}{mL^4} \cdot \frac{1}{\omega_n^2}, \tag{5}$$

which shows the relationship between curvature and natural frequencies. This property is always valid and can be extended also to non-linear non-prismatic beams [3].

The relationship between curvature and beam deflection is then expressed by

$$k = \left[\frac{\partial^2 v}{\partial x^2}\right]\left[1 + \left(\frac{\partial v}{\partial x}\right)^2\right]^{-3/2}, \tag{6}$$

where v is the vertical displacement at the considered cross-section; in the field of small deflections and slope; eq. (6) becomes

$$k = \frac{\partial^2 v}{\partial x^2}. \tag{7}$$

The partial derivative in the second member can be computed through an approximate formula, the so-called *central difference formula*,

$$k = \frac{v_{i+1} - 2v_i + v_{i-1}}{l^2}, \tag{8}$$

where l is the distance between two successive measured locations (or the element size of the FE model in a numerical study). In this way, the curvature of the j-th mode shape can be obtained by the value of vertical displacements of the same mode shape, that is by the eigenmode matrix.

3. Numerical Simulation

The two methods previously illustrated are now applied to the results of the modal analysis of a simply supported finite element bridge model realized using 21 beam elements. Damage is simulated by reducing the stiffness, EI, of the element at the middle of bridge deck [2]. As sample-structure has been considered the deck of the I-40 bridge over the Rio Grande in Albuquerque (NM, USA), for which in literature is available an accurate paper discussing the values to assign to the mechanical characteristics of its cross-section in order to model the same through a series of beam elements [4].

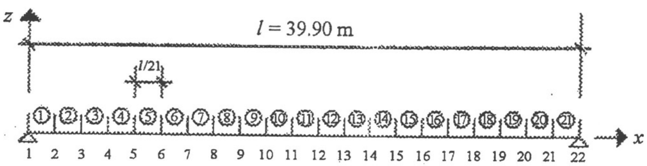

Fig. 1.– FE model for the bridge ($A = 5,684.5$ cm^2; $EI = 102.38 \times 10^{12}$ daN·cm^2).

The model considered is shown in Fig. 1; the deck is 39.9 m in length and each single element used in the mesh is 1.9 m in length and bound to the other through a continuity restraint. Modal analysis has been conducted by using SAP2000 FE code; the obtained results are shown in Fig. 2, which summarizes the mode shape of the first five flexural modes of the intact bridge. Damage has been modelled by introducing into the element 11 a 90% reduction in EI; of course this high reduction in stiffness corresponds to a very strong assumption about damage extension. However, in this way there is a clear change in the dynamic response of the bridge, as shown by the mode shape in Fig. 3. The natural frequencies of the intact and the damaged bridge are given in Table 1 for the first five modes.

Table 1
Eigenfrequencies of Mode Shapes

	Frequencies, [rad/s]	
Mode	Intact bridge	Damaged bridge
1	17.84	13.48
2	60.68	51.69
3	112.69	102.34
4	113.22	104.29
5	136.53	112.93

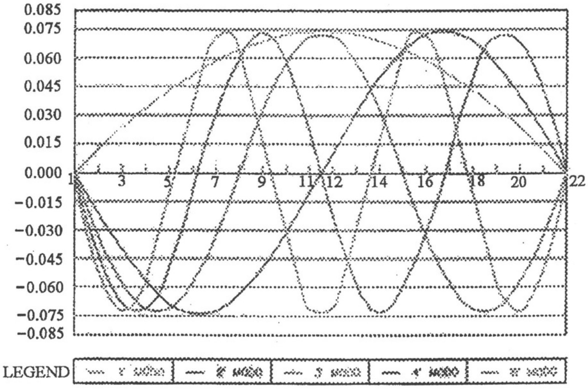

Fig. 2.– First five modal shapes of the intact bridge.

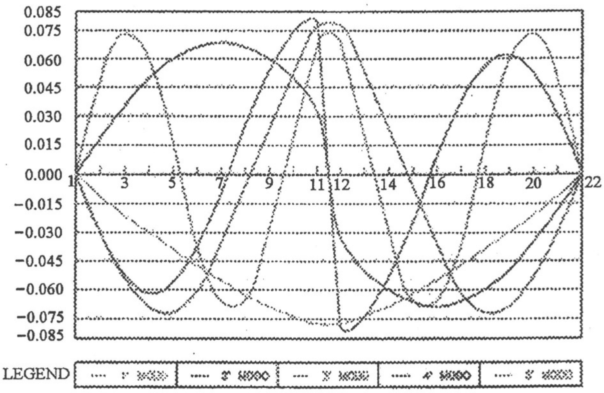

Fig. 3.– First five modal shapes of the damaged bridge.

3.1. Change In Modal Shapes Method

For the calculation of MAC index it has been applied relation (1), in which has been substituted to $\{\Phi_{A,j}\}$ the j-th column of the eigenmode matrix relative to the undamaged state and to $\{\Phi_{B,k}\}$ the k-th column of the eigenmode matrix relative to the damaged one. The results are plotted in Fig. 4, in which is reported the value of MAC index corresponding to the specific mode shape considered.

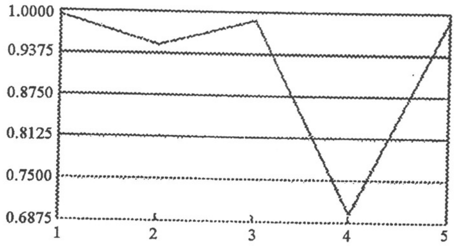

Fig. 4.– MAC values for the first five flexural modes.

From this graph is clear that the lowest values appear at the second and fourth vibrational mode: $MAC_{(j=2)}=0.949$ and $MAC_{(j=4)}=0.708$. According to the physical meaning of MAC, this involves that the lower correlation between modal shapes is connected with the second and fourth vibrational mode, while the other modes are substantially invariant (MAC > 0.98). Since that deviation occurs near the middle of the span, a first supposition about damage localization could be advanced. However, to obtain an analytical information (not only qualitative) about damage identification is necessary to calculate the COMAC index. In order to do it, is necessary to apply relation (2) by substituting to $\{\Phi_{iA}\}$ the i-th row of the modal shape matrix of the intact structure and to $\{\Phi_{iB}\}$ the i-th row of the analogous matrix of the damaged one. The results are plotted in Fig. 5.

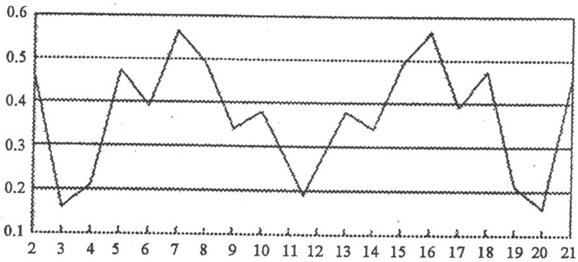

Fig. 5.– COMAC values for the first five flexural modes.

From this graph is clear that the lowest values appear at the section 3-20 and at the middle of span, $COMAC_{(3)}=0.16151$ and $COMAC_{(11)}=0.25894$ (because of symmetry the same values appear at the sections 20 and 12). According to the physical meaning of COMAC, this involves that the highest deviation between modal shapes occurs at sections 3, 11, 12, 20.

In absence of any kind of *a priori* information regarding damage detection, the analysis of that graph inclines to assert that the bridge is damaged in correspondence of elements 11, 2-3 and 19-20. In reality, damage has been introduced only in element 11, so the information provided by COMAC is, partially, at least, wrong (false-positive error).

3.2. Mode Shapes Curvature Method

This method is based on the graphical representation, along the axis of the deck, of the absolute difference between the modal curvatures of the intact and the damaged structure; the graph obtained in this way shows a high peak at the damaged elements, indicating the presence of a fault. Modal curvatures can be calculated by using the central difference relation (8), where now the element size of the FE adopted is $l = 1.9$ m, while v_i is the deflection at the considered cross-section. The subsequent operation concerned with the calculation, for all vibrational modes investigated, of the following difference: $|k_{\text{int}} - k_{\text{dam}}|_i$, where i is referred to the i-th cross-section (result is plotted in Fig. 6).

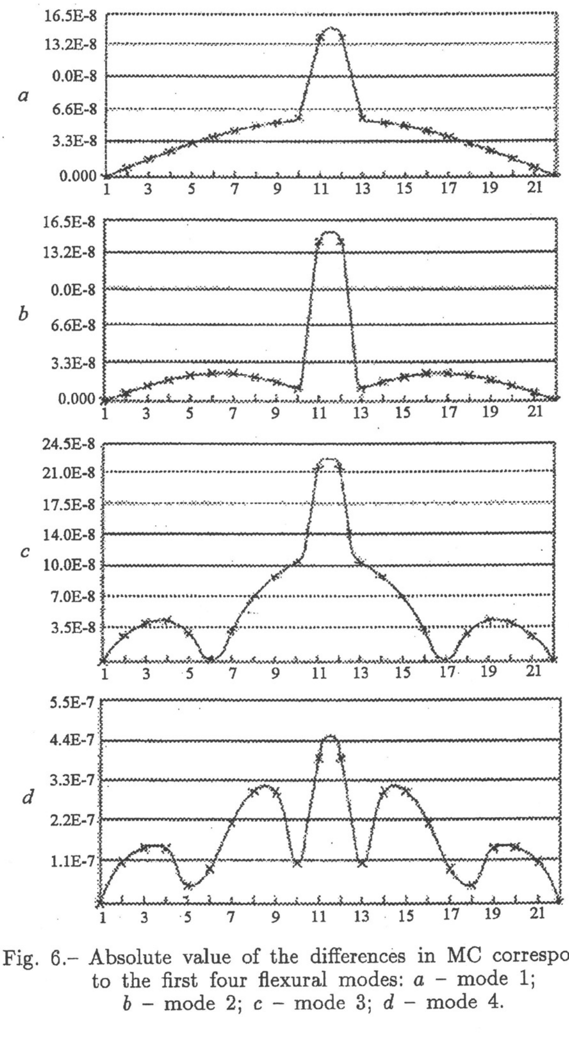

Fig. 6.– Absolute value of the differences in MC corresponding to the first four flexural modes: *a* – mode 1; *b* – mode 2; *c* – mode 3; *d* – mode 4.

It can be observed that for the higher modes (especially mode 4), the absolute difference in modal curvatures shows several peaks not only at the position of the damaged element but also at different undamaged locations. This can cause confusion in all those practical applications in which the location of faults is unknown in advantage.

In order to eliminate these misleading information and to summarize the results for all modes, is necessary to calculate the Curvature Damage Factor (CDF), [3], which can be written as

$$\text{CDF} = \frac{1}{N}\sum_{i=1}^{N} |v''_{0i} - v''_{di}|, \tag{9}$$

where N is the total number of modes considered, v''_0 – the curvature mode shape of the intact structure and v''_d – that of the damaged structure. The result is plotted in Fig. 7; the position of damage appears clearly at the element 11.

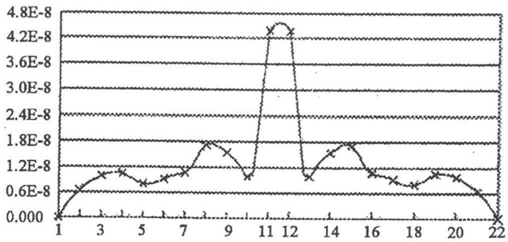

Fig. 7.– Curvature Damage Factor (CDF) along beam model.

4. Conclusions

Two methods proposed in literature for damage identification in bridges have been tested using simulated data for a simply supported bridge structure. The Change in Mode Shapes Method, based on indexes like MAC and COMAC, is not properly sensitive to damage since the change in displacement mode shapes generally is too small to be used without generating false-positive errors. The Mode Shapes Curvature Method allows to identify the damage with precision, since the change in frequency is intimately related to the change in curvature.

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MONITORIZAREA INTEGRITĂŢII STRUCTURALE PRIN TEHNICI DE IDENTIFICARE DINAMICĂ: SIMULAREA NUMERICĂ A UNUI SCENARIU DE CEDARE

(Rezumat)

Detectarea cedării în structurile inginereşti folosind schimbările care apar în parametrii modali măsuraţi reprezintă un domeniu de cercetare căruia i se acordă o atenţie notabilă în literatura de specialitate a ultimilor ani.

Se compară două tehnici experimentale diferite pentru a prevedea localizarea cedării şi severitatea acesteia: „*Scimbarea* – în metoda formei proprii" şi „*Curbura* formei proprii". Comparaţia a fost stabilită cu referire la un model în element finit pentru un pod simplu rezemat, în care cedarea este simulată prin reducerea rigidităţii flexurale, EI. Rezultatele studiului indică faptul ca schimbarea care apare în curbura formei proprii este un indicator important al cedării.