BULETINUL INSTITUTULUI POLITEHNIC DIN IAŞI
Tomul LIII (LVII), Fasc. 3–4, 2007
CONSTRUCŢII. ARHITECTURĂ

OPTIMUM DESIGN TO MINIMUM COST OF SANDWICH PANELS WITH FOAM CORE AND CORRUGATED FACES

BY

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The structural arrangement of sandwich panels with foam core and corrugated faces offers unique opportunities to mix and tailor materials and cross-section proportions to meet structural design criteria. For cost effective design, economic comparisons between combinations of different materials in sandwich structures must be made. Such comparisons should be based on sandwich proportions that vary for each combination of materials and that reflect the minimum combined cost of the core and facings. Simplified relations which can be used for economic analysis are proposed in the paper.

1. Introduction

The sandwich panels are layered prefabs, made of two lightly profiled sheet metal faces in which there is ordered the foam core. The sandwich panels are used for vertical cladding elements: walls (Fig. 1) and/or slope – roofs (Fig. 2) for various industrial constructions. In Romania, these building elements are used thanks to the characteristics proved during the working, from which the following are detached:

a) the self weight is reduced, so they can be easily handled during the assembly and maintenance;

b) the thermal transfer coefficient is reduced in comparison with other cladding elements for building envelope;

c) the rigid foam core has 98% of its cells closed, which insures its impermeability to air and humidity;

d) the sandwich panels have good behavior in fire conditions; thanks to its metallic faces, powerful heat dissipation takes place and the foam core extinguished itself in about 10 s (class C2 for combustibility).

The corrugated sheet metal is made of galvanizing steel with 0.4...0.5 mm thickness, from aluminum or stainless steel. The aluminum sheet is made of Al 99.5 flat or profiled, STUCCO (tension resistant) or AlMg (semi-stiff) and it has a 0.6...1.2 mm thickness.

The sandwich panels are used to realize the building envelope that need cladding elements with reduced self-weight, high thermal isolation properties and noise attenuation.

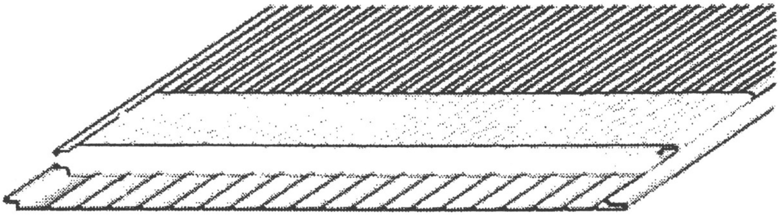

Fig. 1.– Sandwich panels for external walls.

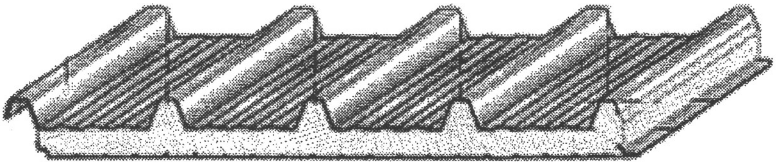

Fig. 2.– Sandwich panels for roof.

2. Optimum Design to Minimize Cost of Sandwich Panels

The structural arrangement of sandwich panels offers unique opportunities to mix and tailor materials and cross-section proportions to meet structural design criteria. For optimum design to minimize cost, economic comparisons between combinations of different materials in a sandwich panels must be made. Such comparisons should be based on sandwich proportions that vary for each combination of materials, and that reflect the minimum combined cost of the core and facings. In what follows some simplified relations are presented which can be used for economic analysis.

The principal simplifications and assumptions used in the derivation of minimum cost relations are presented subsequently:

a) the faces are thin and identical and secondary bending effects due to shear-flexible cores are negligible;

b) the costs of adhesive, other bonding or fastening processes, and surface finishes are not included in the analysis;

c) the core is assumed to be "soft" because it does not contribute significantly to the bending stiffness;

d) the unit cost per volume of core and faces does not vary with thickness.

In sandwich panels, such as external walls or roof to support normal loads in bending, cross-section proportions are usually governed by a required moment of inertia (I), bending section modulus (W) and core area (bc). These section properties may be satisfied by infinite combinations of facings (t) and core (c) thicknesses. However, only a certain set of proportions provides the required section properties at minimum cost. A procedure for determining the minimum cost of sandwich panels that have the required section properties is developed below. For simplicity, the procedure is developed for a panel strip of unit width ($b=1$).

The cost per unit surface area, C_p, of a sandwich panel meeting the assumptions given earlier, is expressed as follows:

$$C_p = 2tC_f + cC_c, \tag{1}$$

where: C_f is the cost per volume unit of face material and C_c – the cost per volume unit of core material. This expression will be used below in the development of relationships for minimum cost of sandwich panels.

2.1. The Criterion 1 of Optimum Design: Moment of Inertia

In a specific design situation, stiffness requirement may dominate the design problem and govern sandwich proportions. Thus, criterion 1 results in a cross-section that provides the required moment of inertia (I^*) at minimum cost. The resulting section is adequate only if it's bending section modulus (W) and shear area ($1c$) are also adequate for the specific requirements of the application

$$W \geq W^* \quad \text{and} \quad 1c \geq 1c^*, \tag{2}$$

where W^* is the bending section modulus and $1c^*$ – the shear area at minimum cost. Equations satisfying criterion 1 are developed below.

The total material cost per surface area unit of the sandwich panel is obtained in terms of I^* and unit costs of materials, by combining relationship for moment of inertia, $I = btd^2/2$, with equation (1), for a unit width of section ($b=1$)

$$C_p = \left(2C_f - C_c\right) t + C_c \left(\frac{2I^*}{t}\right)^{1/2}. \tag{3}$$

Performing the derivative of this equation with respect to t and setting the result equal to zero, *i.e.*

$$\frac{\partial C_p}{\partial t} = 0, \tag{4}$$

one obtains the optimum thicknesses for facings (t_{opt}) and core (c_{opt}) required for minimum sandwich panel cost, in terms of I^* and costs unit of materials, C_f, C_c

$$t_{\text{opt}} = \left[\frac{C_c^2 I^*}{2\left(2C_f - C_c\right)^2}\right]^{1/3}, \tag{5}$$

$$c_{\text{opt}} = \left(\frac{4C_f}{C_c} - 3\right) t. \tag{6}$$

Substituting the optimum thicknesses of facing from eq. (5) and core from eq. (6), it results the bending section modulus

$$W = \left(\frac{4C_f}{C_c} - 3\right)\left[\frac{C_c^2 I^*}{2\left(2C_f - C_c\right)^2}\right]^{2/3}. \tag{7}$$

Substituting the relationship (6) in the eq. (1) and performing the calculus it results

$$\min C_p \cong 2tC_f + 4tC_f, \quad \text{if} \quad \frac{C_f}{C_c} \geq \frac{3}{4}. \tag{8}$$

Eq. (8) shows that for a cross-section determined for moment of inertia at minimum cost, the faces comprise one-third of the sandwich panel cost and the core comprises two-third of the sandwich panel cost. This result is true for many practical sandwich panels where the cost per volume unit of the faces material is significantly greater than of the core, *i.e.* $C_f/C_c \geq 3/4$.

2.2. The Criterion 2 of Optimum Design: Bending Section Modulus

In contrast to the above, situations can arise where strength requirements dominate the design problem and govern sandwich proportions. Thus, criterion 2 results in a cross section which provides the required section modulus (W^*) at minimum cost. The resulting section is adequate only if it's moment of inertia (I) and shear area ($1c$) are also adequate for the specific requirements of the application

$$I \geq I^* \quad \text{and} \quad 1c \geq 1c^*, \tag{9}$$

where: I^* is moment of inertia and $1c^*$ – shear area at minimum cost. Equations satisfying criterion 2 are developed below.

The total material cost per surface area unit of the sandwich panel is obtained in terms of W^* and costs unit of materials, by combining relationship for bending section modulus of sandwich elements: $W = I/z_{\max} = btd$ with eq. (1) for a unit width of section ($b = 1$)

$$C_p = \left(2C_f - C_c\right)t + C_c\frac{W^*}{t}. \tag{10}$$

By reasoning in a similar way as described above for criterion 1, the following relationships can be derived for situations where W^* governs design:

$$t_{\text{opt}} = \left(\frac{C_c W^*}{2C_f}\right)^{1/2}, \tag{11}$$

$$c_{\text{opt}} = \frac{2C_f t}{C_c} = \left(\frac{2C_f W^*}{C_c}\right)^{1/2}. \tag{12}$$

Substituting the optimum thicknesses of facing from eq. (11) and core from eq. (12), it results the moment of inertia

$$I = \left(1 + \frac{C_c}{C_f}\right)\left(\frac{C_f W^{*3}}{2C_c}\right)^{1/2}. \tag{13}$$

Substituting the relationship (12) in the eq. (1) and performing the calculus, it results

$$\min C_p = 2tC_f + 2tC_f. \tag{14}$$

Eq. (14) shows that for a cross-section determined to obtain minimum cost of materials for a given bending section modulus, the total material cost is divided equally between the faces and the core.

2.3. The Criterion 3 of Optimum Design: Moment of Inertia and Bending Section Modulus Satisfied Simultaneously

In certain cases, a cross section that provides the required moment of inertia (I^*) and the required bending section modulus (W^*), simultaneously, also provides the minimum sandwich panel cost. In order to be adequate, the cross-section determined in accordance with criterion 3 must also provide adequate shear area

$$1c \geq 1c^*, \tag{15}$$

where $1c^*$ is shear section at minimum cost. Equations satisfying criterion 3 are developed below.

Proportions that provide section properties in accordance with criterion 3 are as derived from relationships of moment of inertia and bending section modulus, for sandwich panels with reasonably thin faces

$$c_{\text{opt}} = \frac{2I^*}{W^*} - \frac{W^{*2}}{WI} \cong \frac{2I^*}{W^*}, \quad \text{for thin faces}, \tag{16}$$

$$t_{\text{opt}} = \frac{W^{*2}}{2I^*}. \tag{17}$$

Thus, substituting eqs. (16) and (17) into (1), it results the following panel cost:

$$C_p = \frac{W^{*2}}{2I^*}\left(2C_f - C_c\right) + \frac{2I^*}{W^*}C_c. \tag{18}$$

This equation gives the sandwich panel cost directly from the minimum required section properties.

2.4. The Criterion 4 of Optimum Design: Shear Area

In addition to the requirements for bending section properties, the cross-section must also provide sufficient core thickness (c^*) or greater to develop required shear strength. This serves as a final check on the strength capacity of the cross-section meeting the other criteria given above.

2.5. The Graphical Determination of Decisive Criterion

A graphical presentation of both required and cost-effective proportions demonstrates, quantitatively, which of the above criterions governs in a given design situation. Fig. 3 presents such graphs for a specific set of I^*, W^* and c^* requirements, which will be used in following numerical solution.

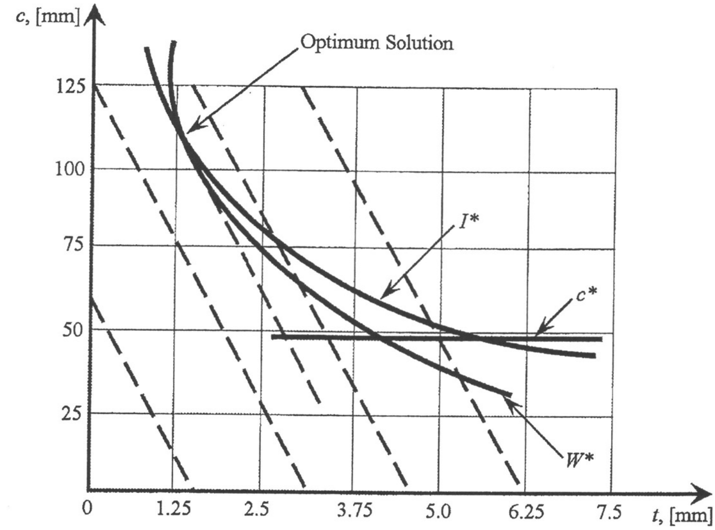

Fig. 3.– Criteria for selecting minimum cost for sandwich panel.

The following observations are key elements shown in Fig. 3:

a) The curves in Fig. 3 define section proportions which satisfy the required values of I^*, W^* and c^*. The portions of the curves define the bounds over which proportions are governed by W^*, I^* and c^*, respectively.

b) The dashed straight-lines are contours of constant sandwich panel unit cost. The minimum cost-cross section occurs at the point of tangency of the cost contour and the I^* and W^* curves.

c) The points of tangency, as obtained by criterions 1, 2 and 3, are marked on Fig. 3. For this specific set of criterions, these happen to points occur on approximately the same cost, but this is not the general rule.

d) Both criterion 1 and criterion 2 fail to provide the minimum sandwich panel cost because, in each instance, the alternate requirement for section properties is not satisfied. For example, for the same depth, the criterion 1 thickness (and cost) must be increased to provide the needed W^* value. The result is similar for I^* and criterion 2.

e) Criterion 3, by definition, satisfies both bending section modulus and moment of inertia requirements. It is the only point on the boundary where meet both criterions. Hence, it provides the minimum cost (*i.e.* tangent to the left-most cost contour).

f) Criterion 4 forms a fourth bound on cross-section proportions, which represents the minimum depth required for shear strength. Core depth provided by the above criterions must greater than c^* in order to meet the shear strength criterion.

For specific set of design criterion examined above, criterions 1, 2 and 3 all produce similar minimum sandwich panel costs since they are approximately the same cost contour. However, they produce significantly different minimum cost proportions. In this instance, criterion 3 prevails, since it satisfies both bending section modulus and moment of inertia criterions.

Clearly, the minimum cost solution examined above is not subject to generalization. Depending upon shifts in the relative values, I^*, W^* and c^*, in a specific design situation, or a change in slope of the cost contours, as dictated by relative cost unit of face and core materials, either criterion 1 or criterion 2 could govern in bending. Furthermore, if the core is especially "soft", criterion 4 might prove to govern as a result of shear strength considerations.

While the above graphical approach is useful in understanding how the governing criterion is selected, a direct numerical design procedure is useful in determining optimum designs. The following procedure provides a direct approach to the determination of minimum cost sandwich panel.

3. Example

As example, it is considered a sandwich wall panel (Fig. 4) than spans $L = 2.40$ m and has fiberglass-mat-reinforced polyester faces of equal thickness and an extruded polystyrene structural foam core.

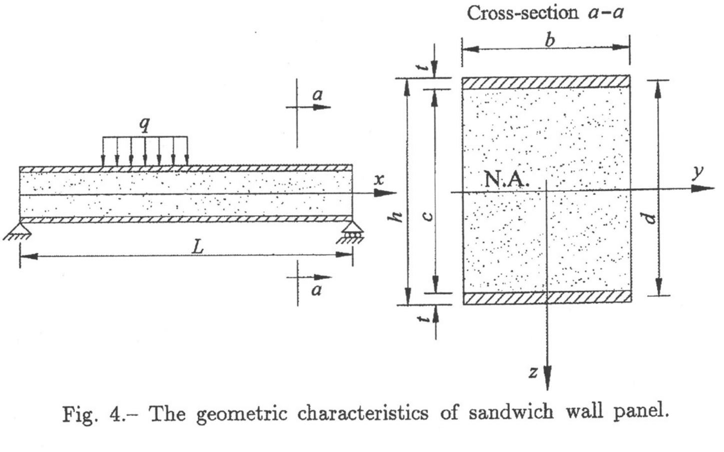

Fig. 4.– The geometric characteristics of sandwich wall panel.

A. *Given data:*

a) wind load, $q = 0.002$ N/mm^2;

b) safety factor on strength, $\gamma = 5$;

c) deflection limit, $w_a = L/150 = 16$ mm;

d) design value of tensile strength, $R^t_f = 13.8$ N/mm^2;

e) design value of shear strength, $R^f_c = 0.0483$ N/mm^2;

f) modulus of elasticity for faces material, $E_f = 6{,}900$ N/mm^2;

g) modulus of elasticity for core material, $E_c = 10.35$ N/mm^2;

h) shear modulus for core material, $G_c = 6.9$ N/mm^2.

B. *Required properties:*

a) Wrinkling strength of face

$$\sigma_v = 0.5\left(E_f E_c G_c\right)^{1/3}\left(\frac{1}{\gamma}\right) \cong 7.9 \text{ N/mm}^4 < R^t_f.$$

b) Required bending section modulus

$$W^* = \frac{M}{\sigma_v} = \frac{qL^2}{8\sigma_v} = 182.28 \text{ mm}^3.$$

c) Required moment of inertia

$$I^* = \frac{5qL^4}{384 E_f w_a} = 7{,}826.09 \text{ mm}^4, \text{ for bending only.}$$

d) Try adding 25% to the moment of inertia required for bending to compensate for shear deflection. Therefore, try $I^* = 9{,}782.6$ mm^4 as first cut.

e) Required minimum core thickness for shear

$$c^* = \frac{qL}{2R^f_c} = 49.69 \text{ mm}^2.$$

C. *Determine applicable criterion for minimum cost proportions*

a) Bending section modulus furnished by criterion 1:

$$W = \left(\frac{4 \times 5.360}{268} - 3\right)\sqrt[3]{\left[\frac{268^2 \times 9{,}782.6}{2(2 \times 5{,}360 - 268)^2}\right]^2} = 167.76 \text{ mm}^3 < W^*.$$

b) Moment of inertia furnished by criterion 2:

$$I = \left(1 + \frac{268}{5{,}360}\right)\sqrt{\frac{5{,}360 \times 182.28^3}{2 \times 268}} = 8{,}171.425 \text{ mm}^4 < I^*.$$

Since neither criterion 1 or 2 satisfies both W^* and I^*, criterion 3 governs the bending cross-section.

D. *Proportions and cost for criterion 3*

$$c_{\text{opt}} = \frac{2I^*}{W^*} - \frac{W^{*2}}{2I^*} = 105.64 \text{ mm} > c^*.$$

Note that the second term is small for thin faces.

$$t_{\text{opt}} = \frac{W^{*2}}{2I^*} = 1.70 \text{ mm}.$$

The minimum cost of sandwich panel results

$$C_p = 2tC_f + cC_m = 46.535 \text{ euro/m}^2.$$

From criterion 3 it results the core thickness which check criterion 4 also.

Minimum cost sandwich panel which meets design criteria has 105.64 mm core and 1.70 mm faces. Sandwich panel cost is 46.535 euro/m^2 plus cost of bonding adhesive.

4. Conclusions

The paper presents the optimum design to minimum cost of sandwich panels with foam cores and corrugated faces. The building envelope of different industrial constructions can be made from wall and roof sandwich panels. This sandwich panels can be optimized to minimum cost accordingly to many criterions. The sandwich panels satisfy the strength, stiffness and local stability exigencies if these four criterions are carried out regarding the moment of inertia, the bending section modulus and the shear area.

The theoretical and numerical values determined in this paper are used at the optimum values design for the facing and core thicknesses, which determine minimum cost of materials, utilized to made constitutive layers of the cladding sandwich panels. The graphical method of both required and cost-effective proportions demonstrates, quantitatively, which of the four criteria governs in a given design situation.

The obtained results for optimum facings and core thicknesses may be adjusted in accordance with training possibilities for constitutive layers manufacture of sandwich panels.

Received, February 21, 2008

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PROIECTAREA OPTIMĂ LA COST MINIM A PANOURILOR SANDVIŞ CU MIEZ DIN SPUME PLASTICE ŞI FEŢE CUTATE

(Rezumat)

Panourile sandviş cu miezul continuu din spume plastice şi feţe subţiri, nervurate sau cutate, se utilizează pe scară largă la realizarea anvelopelor pentru construcţii industriale cu destinaţii variate şi regimuri de exploatare diferite. Elementele de închidere executate din panouri sandviş trebuie să satisfacă atât exigenţele de capacitate portantă: rezistentă, rigiditate, stabilitate locală, configuraţie geometrică corespunzătoare elementelor stratificate tip sandviş etc. cât şi exigenţele de confort interior: termic, higrig şi acustic, economie de energie, igienă, protecţia mediului, corelate permanent cu costuri minime pentru materialele utilizate la realizarea straturilor componente – feţe şi miez.

Panourile sandviş cu straturi exterioare subţiri şi miez continuu se pot proiecta raţional răspunzând diverselor criterii de eficientizare structurală, funcţională şi economică. Se descrie modul de optimizare a elementelor sandviş bazat pe criteriul costului minim, respectând cerinţele specifice de rigiditate şi rezistenţă în raport cu valorile caracteristicilor mecano-geometrice. Soluţiile prezentate se folosesc la calculul valorilor optime pentru grosimile feţelor şi ale miezului, ce conduc la costul minim al materialelor din care se execută straturile componente ale panourilor sandviş. Rezultatele obţinute pentru grosimile optime ale miezului şi feţelor se pot ajusta în raport cu posibilităţile practice de fabricare a acestor straturi.