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SPECIAL FINITE ELEMENTS USED IN REINFORCED CONCRETE ANALYSIS

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Finite element method was developed to be applied in case of the elastic materials. If this method is applied on reinforced concrete elements and structures, the results are affected by errors because this material doesn't have an elastic behavior on loads.

Because the reinforced concrete has cracks in service, the evaluation of reinforced concrete structures with finite element method requires some improvement of the method to obtain the results without errors. The paper presents a special type of finite element who can describe, with mathematical relations, the behavior of the reinforced concrete.

1. Introduction

Stress and strain analysis for the elastic materials requires a fine mesh to obtain accurate results [3]. But, for the non-elastic material who have cracks, in the near of the crack the stress gradient is very high and the results are affected by errors. In this case the conventional finite elements are not useful and some special finite elements must be used.

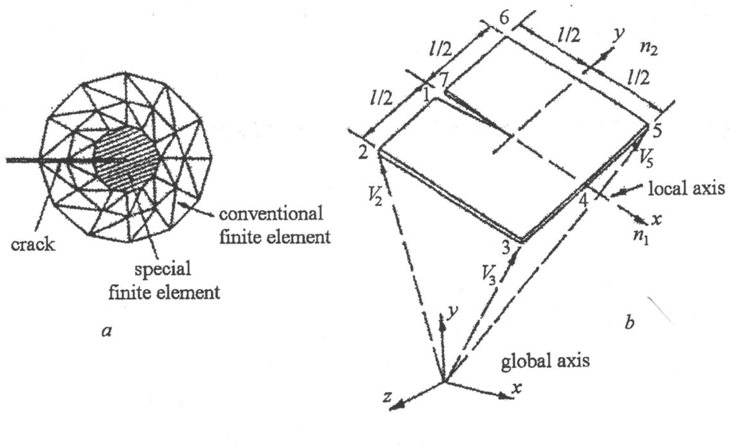

Fig. 1.– Special finite element.

The essential property of this new element must have the ability to include crack. In this case, the stress gradient will be an intrinsic characteristic of the element (Fig. 1). The big advantage consists in the explicitly definition of the rigidity matrix without any supplementary numerical integration.

To find the characteristic matrix of the special element, first must be established the stress and deformation equations for the material and to generate the functions with complex variables.

In plane elasticity problems, these equations can be described by [2]

$$\sigma_x + \sigma_y = 4\Re[\varphi'(z)], \tag{1}$$

$$\sigma_y - \sigma_x + 2\mathrm{i}\tau_{xy} = 2[z\varphi''(z) + \Psi'(z)], \tag{2}$$

$$2G(u + \mathrm{i}v) = P\varphi(z) - z\overline{\varphi'(z)} - \overline{\Psi(z)}, \tag{3}$$

where P is equal with $3...4\mu$ in plane deformation problems and $(3-\mu)/(1+\mu)$ in plane stress problems. The $\varphi(z)$ and $\Psi(z)$ are functions of complex variable $z = x + \mathrm{i}y = r\mathrm{e}^{\mathrm{i}\theta}$.

2. The Special Finite Element

For a crack oriented along x-axis (Fig. 2), the stress and deformation shape can be described with H.H. Westergaard [1] functions from type I and II namely

$$(\sigma_y)_{\mathrm{I}} = \varphi_{\mathrm{I}}'(z) + \overline{\varphi_{\mathrm{I}}''(z)} - \frac{z-\overline{z}}{2}\left[\varphi_{\mathrm{I}}''(z) - \overline{\varphi_{\mathrm{I}}''(z)}\right] - S, \tag{4}$$

$$(\sigma_y)_{\mathrm{II}} = -\frac{z-\overline{z}}{2}\left[\varphi_{\mathrm{II}}'(z) - \overline{\varphi_{\mathrm{II}}'(z)}\right]. \tag{5}$$

Similar equations can be writen for σ_x, τ_{xy}, u and v.

From general case of loading, the symmetrical solutions, I, and non-symmetrical solutions, II, are summing. For a crack oriented like in Fig. 2 the general boundary condition on $\theta = \pm\pi$ ($\sigma_y = \tau_{xy} = 0$) are satisfied if for the two types φ functions we consider

$$\varphi_{\mathrm{I}}(z) = S_1 z^{1/2} + S_2 z + \sum_{n=3}^{\infty}(\mathrm{i})^{n+1} S_n z^{n/2}, \tag{6}$$

$$\varphi_{\mathrm{II}}(z) = A_1 \mathrm{i} z^{1/2} + \sum_{n=2}^{\infty}(\mathrm{i})^{n+1} A_n z^{(n+1)/2}. \tag{7}$$

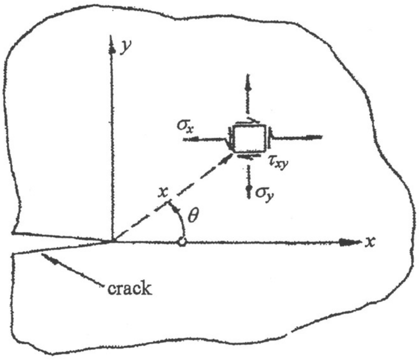

Fig. 2.- Finite element with crack along x-axis.

Next step is to write the equation between nodal forces, $\{F\}^e$, flexibility matrix, $[d]^e$, and the deformations, $\{d\}^e$,

$$\{d\}^e = [f]^e\{F\}^e, \tag{8}$$

$$[f]^e = \int_V [T_1]^T[D]^{-1}[T_1]\,\mathrm{d}V, \tag{9}$$

where $[T_1]$ is a matrix who define the stress status in static equilibrium,

$$\{\sigma\}^e = [T_1]\{F\}^e \tag{10}$$

and $[D]$ is the stress matrix.

The flexibility matrix can be written as

$$[f]^e = \frac{10^6 t}{G(P-1)}\begin{bmatrix} M_{1,1}h & M_{1,2}h^{3/2} & \dots & M_{1,11}h^{7/2} \\ & M_{2,2}h^2 & & \vdots \\ & & \ddots & \\ \text{symmetric} & & & M_{11,11}h^6 \end{bmatrix}, \tag{11}$$

where: $h=1/4$ and the factors $M_{ij}=M(i,j)$ are expressed as functions of P.

Finally, for local coordinate system, the equation is

$$\{u\} = \left[\frac{T_2}{T_0}\right]\left\{\frac{F}{u_0}\right\}^e, \tag{12}$$

where $\{u_0\}=\{u_{01}\ u_{02}\ u_{03}\}$ represents the all three degrees possible in the plane of the element.

Similarly, for global coordinate system the equation can be written

$$\{r\}^e = [Q_1]\{F_1\}^e, \tag{13}$$

where

$$[Q_1] = \frac{1}{G}\begin{bmatrix} q(1,1) & q(1,2) & \dots & q(1,11) - \frac{1}{2}n_2 & n_2 & n_1 \\ q(2,1) & q(2,2) & \dots & \vdots & & \\ \vdots & \vdots & & \vdots & & \\ q(7,1) & \dots & \dots & q(7,11) - \frac{1}{2}n_2 & n_2 & n_1 \end{bmatrix} \tag{14}$$

and one element of the $[Q_1]$ matrix is

$$q(i,j) = [R_1(i,j)P + RR_1(i,j)]l^{n(j)}n_1 + [R_2(i,j)P + RR_2(i,j)]l^{n(j)}n_2. \tag{15}$$

3. Conclusions

The non-elastic behavior of the concrete and the cracking stage developed in service for the reinforced concrete elements impose a new approach of the finite element method. Because the usual finite elements can not describe the behavior of the reinforced concrete structures, we must to establish a new type of element capable to simulate the entire process.

Initially, the first programs who describe the behavior of the reinforced concrete elements were designed for non-elastic materials but applied on continuous medium theories. The result wasn't spectacular because the concrete have cracks and is not a continuous medium. In this case the cracks must be a part from the model and the meshing process must be very fine. The proposed finite element with included cracks can solve the problem and extend the finite element method limits to all the non-elastic materials who have cracks in service stage.

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ELEMENTE FINITE SPECIALE UTILIZATE ÎN ANALIZA BETONULUI ARMAT

(Rezumat)

Metoda elementelor finite a fost dezvoltată pentru a fi utilizată pe materiale cu comportament elastic. Dacă această metodă este aplicată pentru studierea elementelor din beton armat, rezultatele vor fi afectate de erori deoarece comportarea acestui material nu este elastică sub încărcare.

Dat fiind faptul că betonul armat prezintă fisuri în exploatarea curentă, metoda elementelor finite trebuia îmbunătăţită pentru a corecta neajunsurile iniţiale. Se prezintă un tip special de element finit, capabil să descrie starea de fisurare a elementelor din beton armat.